Modeling and analysis of transients in periodic gratings. I. Fully absorbing boundaries for 2-D open problems

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Frequency domain methods allow us to simulate and study efficiently only some of the periodic structures that are widespread in optics and spectroscopy. Time domain approaches could be more effective, but their deployment is held back by a number of unsolved problems associated mainly with a proper truncation of the computation space in the so-called open problems. This paper is devoted to analysis of these problems in the 2-D case (infinite one-dimensionally periodic semitransparent and reflecting gratings in the field of pulsed E- and H-polarized waves). © 2010 Optical Society of America

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1. INTRODUCTION

Study of transient electromagnetic waves [1–8] is a dominating trend in theoretical radiophysics today, because communications, electronics, and radiolocation are impossible without profound insights into spatial–temporal and spatial–frequency field transformations in sophisticated electromagnetic structures. Also the potential of traditional frequency domain approaches is exhausted to a large extent. Additionally the time domain approaches

- are free from some frequency domain idealizations;
- are universal, because limitations imposed on geometrical and constitutive parameters of considered objects are minimal;
- make it possible to construct explicit and straightforward computational routines (without inversion of any operators) that are rather efficient (limited time and memory resources required) and solve the problems within a reasonable time;
- deliver results that are easy to translate into a standard set of frequency domain characteristics.

However in time domain approaches we still face problems for which complete and justified solution takes a lot of analytic effort. For example there are problems of (a) correct and effective truncation of the computation space in the so-called open problems in which the domain of analysis tends to infinity along one or several spatial directions, (b) far-zone treatment, (c) large and distant sources of the fields, etc. [7,8]. Problems of this kind are considered in this paper.

Available heuristic and approximate solutions of problems associated with finite domain transition in the analysis of open time domain problems are mostly based on the so-called absorbing boundary conditions (ABCs) [9–11] and the perfectly matched layers (PMLs) [12–14]. A weak point of these solutions is the unpredictable behavior of computational errors when the observation time is large. As a consequence, the obtained results are not safe to rely on in the case of resonant wave scattering.

In this paper an alternative approach will be elaborated. It will allow us to estimate evenly and minimize errors caused by translation of open initial boundary problems into corresponding closed problems. It is based on exact absorbing conditions (EACs), specifically on constructing and embedding them into the standard finite-difference method. Incorporation of EACs into the initial boundary problem turns it into the equivalent closed problem. The history of this approach dates back to 1986, when Maykov *et al.* first formulated [15] the exact nonlocal conditions for virtual boundaries across a regular semi-infinite hollow waveguide. This approach is based on the use of the radiation condition for spatial–time amplitudes of partial components (modes) of nonsinusoidal waves emitted from effective sources and scatterers. Afterwards this approach was modified and adapted (refer, e.g., to [8,16–24]) for a great variety of problems in theoretical and applied radiophysics. Its validity and efficiency has been proved repeatedly by numerical experiments and special tests.

2. TWO-DIMENSIONAL INITIAL BOUNDARY VALUE PROBLEMS

Let us present the scalar problems for analysis of the spatial–time transformations of E- and H-polarized fields in a near zone of a 1-D periodic grating (see Fig. 1 the structures are uniform along the x axis and periodic with a period l along the y axis) in the following form:

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$$\begin{cases} P_{\varepsilon,\mu,\sigma}[U] \equiv \left[-\varepsilon\mu\dfrac{\partial^2}{\partial t^2} - \sigma\mu\dfrac{\partial}{\partial t} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2} \right] U(g,t) = F(g,t); & g=\{y,z\} \in \mathbf{Q}, \quad t>0 \\ U(g,0)=\phi(g), \quad \dfrac{\partial}{\partial t}U(g,t)\Big|_{t=0} = \psi(g); & g \in \bar{\mathbf{Q}} \\ E_{tg}(p,t)|_{p=\{x,y,z\}\in \mathbf{S}} = 0; & t \geqslant 0 \\ E_{tg}(p,t) \text{ and } H_{tg}(p,t) \text{ are continuous at the surfaces } \mathbf{S}^{\varepsilon,\mu,\sigma}; & t \geqslant 0 \end{cases}. \quad (1)$$

Here, $U=E_x$, $E_y=E_z=H_x\equiv 0$ in the case of the E-polarized field and $U=H_x$, $H_y=H_z=E_x\equiv 0$ in the H case; $\vec{E}\equiv\vec{E}(g,t)=\{E_x,E_y,E_z\}$ and $\vec{H}\equiv\vec{H}(g,t)=\{H_x,H_y,H_z\}$ are vectors of electrical and magnetic field strengths; $\sigma=\eta_0\sigma_0$, $\sigma_0\equiv\sigma_0(g)\geqslant 0$ is the specific conductivity of a locally inhomogeneous medium; $\varepsilon\equiv\varepsilon(g)\geqslant 1$ and $\mu\equiv\mu(g)\geqslant 1$ are its relative permittivity and magnetic permeability; ($\sigma_0(g)$, $\varepsilon(g)$, and $\mu(g)$ are piecewise constant functions); $\eta_0=(\mu_0/\varepsilon_0)^{1/2}$ is the free-space impedance; ε_0 and μ_0 are electric and magnetic vacuum constants; $p=\{x,y,z\}$ is a point in $\mathbf{R}^3$ space; and x, y, and z are Cartesian coordinates. The S.I. metric system is used for all physical parameters except time t, which is measured in meters—it is the product of the natural time and the velocity of light in vacuum.

The surfaces $\mathbf{S}=\mathbf{S}_x\times[|x|\leqslant\infty]$ of perfectly conducting elements in the geometry of the gratings and the surfaces $\mathbf{S}^{\varepsilon,\mu,\sigma}$ of discontinuities of material properties of the medium are assumed to be sufficiently smooth. The domain of analysis in the problems (1) coincides with a part of the $\mathbf{R}^2$ plane limited by the contours $\mathbf{S}_x$: $\mathbf{Q}=\mathbf{R}^2\setminus\overline{\text{int}\,\mathbf{S}_x}$; $\overline{\text{int}\,\mathbf{S}_x}$ is the closure of int $\mathbf{S}_x$ domains that are occupied by ideal conductors.

In the case of the classical statement of the problems (1) (all the equations are satisfied in each point of the relevant domain), the solutions should have as many continuous derivatives as are present in the equations, and that implies strict limitations on the smoothness for all the entries. The generalized statements and solutions are more suitable for a description of physical phenomena that are governed by differential equations, and they make analysis of the problem much simpler. It is known [25] that initial boundary value problems (1) can be formulated in such a way (source functions $\phi(g)$, $\psi(g)$ and $F(g,t)$ for all $t>0$ are finite in $\bar{\mathbf{Q}}$, and so on) that they will be unambiguously resolved in Sobolev's space $\mathbf{W}_2^1(\mathbf{Q}^T)$; $\mathbf{Q}^T=\mathbf{Q}\times(0;T)$, $(0;T)=\{t:0<t<T<\infty\}$. Let us assume that all necessary conditions are fulfilled and consider the following problem.

The analysis domain $\mathbf{Q}=\mathbf{R}^2\setminus\overline{\text{int}\,\mathbf{S}_x}$ comprises all of the space $\mathbf{R}^2$. For such domain the problems (1) can be resolved efficiently only in two cases:

- The problem (1) degenerates into a conventional Cauchy problem ($\text{int}\,\mathbf{S}=\varnothing$, medium is homogeneous, and the supports of functions $F(g,t)$, $\phi(g)$, and $\psi(g)$ are bounded). With some restriction on the source functions a classical and generalized solution of the Cauchy problem does exist, is unique, and is described by the well-known Poisson formula [26].
- Functions $F(g,t)$, $\phi(g)$, and $\psi(g)$ have the same displacement symmetry as periodic structure. In this case the domain of analysis can be reduced to $\mathbf{Q}^{new}=\{g\in\mathbf{Q}:0<y<l\}$, completing problems (1) with periodicity conditions [8] on lateral surfaces of the plane-parallel Floquet channel $\mathbf{R}=\{g\in\mathbf{R}^2:0<y<l\}$.

The domain of analysis can be reduced to $\mathbf{Q}^{new}$ in a more general case also. The objects of analysis in this case are not quite physical (complex sources, waves, and fields). However by simple mathematical transformations all the results can be presented in the usual, physically correct form. There are many reasons why the modeling of physically realizable situations in the electromagnetic theory of gratings should start with the analysis of initial boundary value problems for the images $f^{new}(g,t,\Phi)$ of the functions $f(g,t)$ describing the true sources:

$$\begin{aligned} f(g,t) &= \int_{-\infty}^{\infty} \tilde{f}(z,t,\Phi)e^{2\pi i\Phi(y/l)}d\Phi \\ &= \int_{-\infty}^{\infty} f^{new}(g,t,\Phi)d\Phi \leftrightarrow f^{new}(g,t,\Phi) \\ &= \frac{\exp(2\pi i\Phi y/l)}{l}\int_{-\infty}^{\infty} f(\bar{y},z,t)e^{-2\pi i\Phi(\bar{y}/l)}d\bar{y}. \end{aligned} \quad (2)$$

From Eq. (2) it follows that

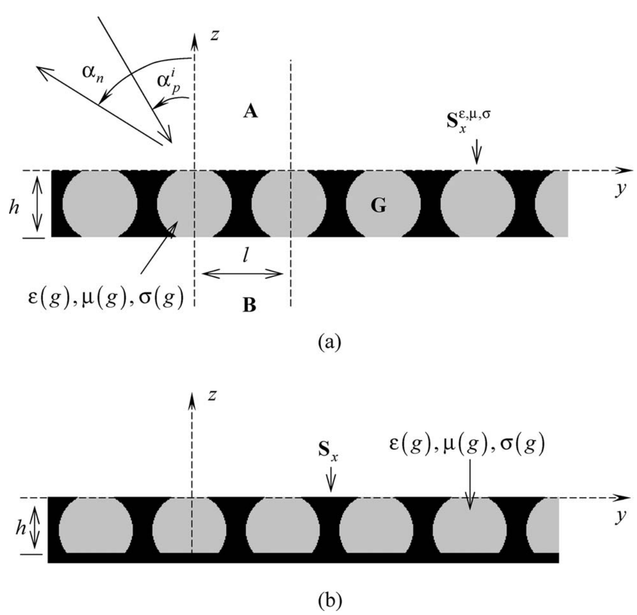

Fig. 1. Geometry of model problems: (a) semi-transparent and (b) reflecting plane gratings. All structures are homogeneous along the x axis.

$$f^{new}\left\{\frac{\partial f^{new}}{\partial y}\right\}(y+l,z,t,\Phi)=e^{2\pi i\Phi}f^{new}\left\{\frac{\partial f^{new}}{\partial y}\right\}(y,z,t,\Phi).$$

The use of the foregoing conditions restricts the analysis domain to the domain $\mathbf{Q}^{new}$, which is a part of Floquet channel $\mathbf{R}$, and this allows us to rewrite the problems (1) in the following form:

$$U(g,t)=\int_{-\infty}^{\infty}U^{new}(g,t,\Phi)d\Phi,$$

$$\begin{cases}\left[-\varepsilon\mu\frac{\partial^2}{\partial t^2}-\sigma\mu\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]U^{new}(g,t)=F^{new}(g,t); & g=\{y,z\}\in\mathbf{Q}^{new},\quad t>0\\ U^{new}(g,0)=\phi^{new}(g),\qquad \left.\frac{\partial}{\partial t}U^{new}(g,t)\right|_{t=0}=\psi^{new}(g); & g\in\overline{\mathbf{Q}^{new}}\\ E_{tg}^{new}(p,t)\big|_{p=\{x,y,z\}\in\mathbf{S}}=0; & t\geq 0\\ E_{tg}^{new}(p,t)\text{ and }H_{tg}^{new}(p,t)\text{ are continuous at the surfaces }\mathbf{S}^{\varepsilon,\mu,\sigma}; & t\geq 0\\ U^{new}\left\{\frac{\partial U^{new}}{\partial y}\right\}(l,z,t)=e^{2\pi i\Phi}U^{new}\left\{\frac{\partial U^{new}}{\partial y}\right\}(0,z,t); & t\geq 0\end{cases}. \quad (3)$$

In Sections 3–6, we will obtain important results required for correct truncation of the computational domain in the open 2-D initial boundary value problems (3). Such results are indispensable in construction of reliable and effective computational schemes, and allow us to study transformations of the field in resonant cases [8].

3. TRANSFORMATION OF EVOLUTIONARY BASIS OF A SIGNAL IN A REGULAR FLOQUET CHANNEL

Let us consider (from this point on) problems (3) for semitransparent infinite gratings (see Fig. 2) in the simplified form for which indices *new* are dropped. We assume also that the supports of source functions $F(g,t)$, $\phi(g)$, and $\psi(g)$ belong to the set $\bar{\mathbf{Q}}_L\setminus\mathbf{L}$; $\mathbf{Q}_L=\{g\in\mathbf{Q}:-L_2-h<z<L_1\}$, $L_1\geq 0$ and $L_2\geq 0$. The regular parts $\mathbf{A}$ and $\mathbf{B}$ of the channel $\mathbf{R}$ (the parts $z>L_1$ and $z<-L_2-h$ of the domain ${}_L\mathbf{Q}=\mathbf{Q}\setminus(\mathbf{Q}_L\cup\mathbf{L})=\mathbf{A}\cup\mathbf{B}$) are free from sources and scatterers. Here, $\mathbf{L}=\mathbf{L}_1\cup\mathbf{L}_2$ is an artificial boundary that separates domain $\mathbf{Q}_L$ from domain ${}_L\mathbf{Q}$. It is denoted by dashed lines in Fig. 2. The field formed by the grating propagates infinitely far along $\mathbf{A}$ and $\mathbf{B}$.

Let us take, for the sake of definiteness, the upper ($z>L_1$) regular part of the $\mathbf{R}$ channel. Here, $\varepsilon(g)=\mu(g)\equiv 1$ and $\sigma(g)=\phi(g)=\psi(g)=F(g,t)\equiv 0$. Assuming that the excitation $U(g,t)$ in domain $\mathbf{Q}_L$ has not yet reached boundary $\mathbf{L}_1$ ($z=L_1$) by the time $t=0$, we obtain via the separation of variables the following representation for the solutions $U(g,t)$ of problems (3):

$$U(g,t)=\sum_{n=-\infty}^{\infty}u_n(z,t)\mu_n(y);\quad z\geq L_1,\quad 0\leq y\leq l,\quad t\geq 0. \quad (4)$$

The orthonormal system $\{\mu_n(y)\}$ of transversal, complete in the space $\mathbf{L}_2(0;l)$ functions of the form $\mu_n(y)=l^{-1/2}\exp(i\Phi_n y)$; $\Phi_n=(n+\Phi)2\pi/l$, $n=0,\pm 1,\ldots$, comes from the nontrivial solutions of the homogeneous (spectral) problem

$$\begin{cases}\left[\frac{d^2}{dy^2}+\Phi_n^2\right]\mu_n(y)=0;\quad 0<y<l\\ \mu_n\left\{\frac{d\mu_n}{dy}\right\}(l)=e^{2\pi i\Phi}\mu_n\left\{\frac{d\mu_n}{dy}\right\}(0)\end{cases}. \quad (5)$$

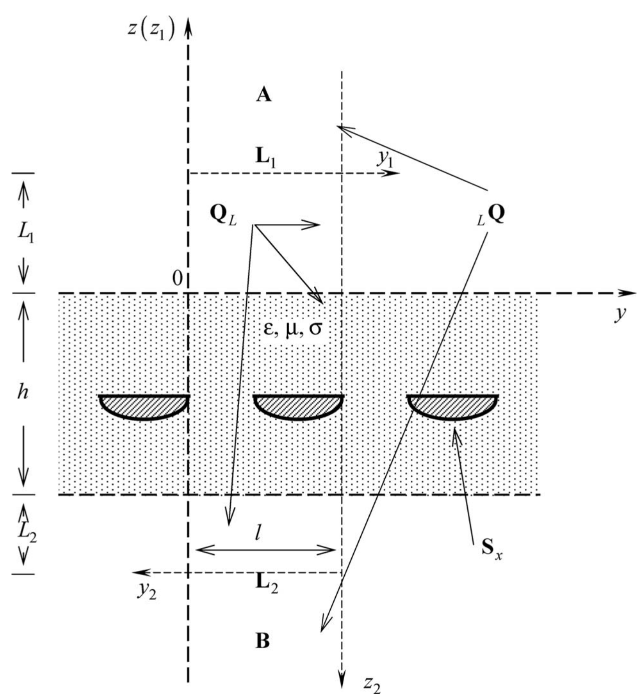

Fig. 2. Geometry of model problems (3).

The spatial–time amplitudes $\{u_n(z,t)\}$ (evolutionary basis) of the signal $U(g,t)$ are obtained from the solutions of the initial boundary value problems

$$\begin{cases} \left[-\dfrac{\partial^2}{\partial t^2}+\dfrac{\partial^2}{\partial z^2}-\Phi_n^2\right]u_n(z,t)=0; & t>0 \\ u_n(z,0)=0, \quad \left.\dfrac{\partial}{\partial t}u_n(z,t)\right|_{t=0}=0 \end{cases};$$

$$z \geqslant L_1, \quad n=0,\pm 1,\pm 2,\ldots . \tag{6}$$

The cosine Fourier transform of problems (6) with respect to $\bar{z}=z-L_1$ on semi-axis $\bar{z}\geqslant 0$ (image$\leftrightarrow$original), namely,

$$\tilde{f}(\omega)=F_c[f](\omega)\equiv\sqrt{\frac{2}{\pi}}\int_0^\infty f(\bar{z})\cos(\omega\bar{z})d\bar{z}\leftrightarrow$$

$$\leftrightarrow f(\bar{z})=F_c^{-1}[\tilde{f}](\bar{z})\equiv\sqrt{\frac{2}{\pi}}\int_0^\infty \tilde{f}(\omega)\cos(\omega\bar{z})d\omega, \tag{7}$$

results in the following Cauchy problems for images $\tilde{u}_n(\omega,t)$:

$$\begin{cases} D(\sqrt{\Phi_n^2+\omega^2})[\tilde{u}_n(\omega,t)]\equiv\left[\dfrac{\partial^2}{\partial t^2}+(\Phi_n^2+\omega^2)\right]\tilde{u}_n(\omega,t)=-\sqrt{\dfrac{2}{\pi}}\bar{u}_n{}'(0,t); & \omega>0, \quad t>0 \\ \tilde{u}_n(\omega,0)=0, \quad \left.\dfrac{\partial}{\partial t}\tilde{u}_n(\omega,t)\right|_{t=0}=0; & \omega\geqslant 0 \end{cases}. \tag{8}$$

Here, $\tilde{u}_n(\omega,t)\leftrightarrow\bar{u}_n(\bar{z},t)=u_n(z,t)$, and $\bar{u}_n'(0,t) = \partial\bar{u}_n(\bar{z},t)/\partial\bar{z}|_{\bar{z}=0}$. It has also been considered that

$$-\omega^2\tilde{f}(\omega)-\sqrt{\frac{2}{\pi}}\left.\left[\frac{d}{d\bar{z}}f(\bar{z})\right]\right|_{\bar{z}=0}\leftrightarrow\frac{d^2}{d\bar{z}^2}f(\bar{z}),$$

and that the wave $U(g,t)$ in the region $\mathbf{A}$ does not contain components propagating in the sense of decreasing z. The components emitted toward $z=\infty$ are equal to zero for sufficiently large z at any finite instant of time $t=T$.

Extending functions $\tilde{u}_n(\omega,t)$ with zero on semi-axis $t<0$, let us pass to the generalized statement of the Cauchy problems (8) [26]:

$$\begin{aligned} D(\sqrt{\Phi_n^2+\omega^2})[\tilde{u}_n(\omega,t)] &\equiv\left[\frac{\partial^2}{\partial t^2}+(\Phi_n^2+\omega^2)\right]\tilde{u}_n(\omega,t) \\ &=-\sqrt{\frac{2}{\pi}}\bar{u}_n{}'(0,t)+\delta^{(1)}(t)\tilde{u}_n(\omega,0)+\delta(t)\left.\frac{\partial}{\partial t}\tilde{u}_n(\omega,t)\right|_{t=0} \\ &=-\sqrt{\frac{2}{\pi}}\bar{u}_n{}'(0,t); \quad \omega>0, \quad -\infty<t<\infty. \end{aligned} \tag{9}$$

Here, $\delta(\ldots)$ is the Dirac delta function and $\delta^{(m)}(\ldots)$ is its generalized mth derivative. The convolution of the fundamental solution $G(\lambda,t)=\chi(t)\lambda^{-1}\sin\lambda t$ of the operator $D(\lambda)$ (see [8]) with the right-hand side of Eq. (9) gives the following representation of $\tilde{u}_n(\omega,t)$:

$$\tilde{u}_n(\omega,t)=-\sqrt{\frac{2}{\pi}}\int_0^t \sin[(t-\tau)\sqrt{\Phi_n^2+\omega^2}]\frac{\bar{u}_n{}'(0,\tau)}{\sqrt{\Phi_n^2+\omega^2}}d\tau;$$

$$\omega\geqslant 0, \quad t\geqslant 0. \tag{10}$$

Applying inverse Fourier transform (7) to Eq. (10), we obtain

$$\bar{u}_n(\bar{z},t)=-\int_0 J_0[\Phi_n((t-\tau)^2-\bar{z}^2)^{1/2}]\chi[(t-\tau)-\bar{z}]\bar{u}_n{}'(0,\tau)d\tau; \quad \bar{z}\geqslant 0, \quad t\geqslant 0, \tag{11}$$

from which it follows that

$$u_n(z,t)=-\int_0 J_0[\Phi_n((t-\tau)^2-(z-L_1)^2)^{1/2}]\chi[(t-\tau)-(z-L_1)]u_n{}'(L_1,\tau)d\tau;$$

$$z\geqslant L_1, \quad t\geqslant 0. \tag{12}$$

Expressions (12) display the general property of the solutions $U(g,t)$ of problems (3) in the subdomain ${}_L\mathbf{Q}$, namely, the solutions satisfying zero initial conditions and being free of the components (modes) propagating toward compact inhomogeneity of the channel $\mathbf{R}$ (toward the domain $\mathbf{Q}_L$). These expressions define diagonal transport operator $Z_{L_1\to z}(t)$ (see [8,17,20,27,28]), which operates according to the rule

$$u(z,t)=\{u_n(z,t)\}=Z_{L_1\to z}(t)[u'(L_1,\tau)]; \quad u'(b,\tau)$$

$$=\{u_n{}'(b,\tau)\}, \quad z\geqslant L_1, \quad t\geqslant\tau\geqslant 0;$$

and enables us to trace changes of the transient wave field during its free propagation along a finite regular section of the $\mathbf{R}$ channel. Here, $J_m(\ldots)$ is a Bessel cylindrical function, $\chi(\ldots)$ is the Heaviside step function,

$$u_n{}'(b,t)=\left.\frac{\partial u_n(z,t)}{\partial z}\right|_{z=b}=\int_0^l\left.\frac{\partial U(g,t)}{\partial z}\right|_{z=b}\mu_n^*(y)dy, \tag{13}$$

and asterisk $*$ stands for complex conjugation.

4. NONLOCAL ABSORBING CONDITIONS

Let us consider the case when the observation point in expressions (12) lies on the artificial boundary $\mathbf{L}_1$ ($z=L_1$). Then

$$u_n(L_1,t) = -\int_0 J_0[\Phi_n(t-\tau)]\chi(t-\tau)u_n{}'(L_1,\tau)d\tau; \quad t \geq 0. \tag{14}$$

Differentiation of Eq.(14) with respect to t gives

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right] u_n(z,t)\bigg|_{z=L_1} = \Phi_n \int_0 J_1[\Phi_n(t-\tau)]\chi(t-\tau)u_n{}'(L_1,\tau)d\tau; \quad t \geq 0 \tag{15}$$

on account of the familiar relationships $dJ_0(x)/dx = -J_1(x)$, $J_0(0)=1$, and $\chi^{(1)}(t-\tau)=\delta(t-\tau)$, where $\chi^{(1)}(\ldots)$ is the generalized derivative of $\chi(\ldots)$.

Next, the application of the Laplace transform regarding t (image $\leftrightarrow$ original),

$$\tilde{f}(s) = L[f](s) \equiv \int_0^\infty f(t)\mathrm{e}^{-st}dt \leftrightarrow f(t) = L^{-1}[\tilde{f}](t) \equiv \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} \tilde{f}(s)\mathrm{e}^{st}ds, \tag{16}$$

in view of the familiar formulas $\tilde{f}_1(s)\tilde{f}_2(s) \leftrightarrow \int_0^t f_1(t-\tau)f_2(\tau)d\tau$ (the convolution theorem), $\lambda^2[\sqrt{s^2+\lambda^2}(\sqrt{s^2+\lambda^2}+s)]^{-1} \leftrightarrow \lambda J_1(\lambda t)$ [29], and $s\tilde{f}(s)-f(0) \leftrightarrow df(t)/dt$ gives [in the space of images $\tilde{u}_n(z,s)$]

$$\left[\frac{\partial}{\partial z} + s\right]\tilde{u}_n(z,s)\bigg|_{z=L_1} = \frac{\Phi_n^2 \tilde{u}_n{}'(L_1,s)}{\sqrt{s^2+\Phi_n^2}(\sqrt{s^2+\Phi_n^2}+s)}, \tag{17}$$

and finally

$$\tilde{u}_n{}'(L_1,s) = -\left(s + \frac{\lambda_n^2}{s+\sqrt{s^2+\lambda_n^2}}\right)\tilde{u}_n(L_1,s). \tag{18}$$

The inverse Laplace transform of Eq.(18), in view of $(s+\sqrt{s^2+\lambda^2})^{-1} \leftrightarrow (\lambda t)^{-1}J_1(\lambda t)$ [30], allows us to return to the original functions $u_n(z,t)$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right] u_n(z,t)\bigg|_{z=L_1} = -\Phi_n \int_0 J_1[\Phi_n(t-\tau)](t-\tau)^{-1}\chi(t-\tau)u_n(L_1,\tau)d\tau; \quad t \geq 0. \tag{19}$$

The translation of Eq.(15) into Eq.(19) [the truth of transformations (16)] rests on the assertion [31] that at some points g of any bounded subdomain inside a $\mathbf{Q}$ domain, the field $U(g,t)$ from a set of compact support sources cannot grow faster than $\exp(\alpha t)$ as $t\to\infty$, where $\alpha>0$ is a constant. The assertion is true for any electromagnetic structure whose spectrum Ω_k does not contain points $\bar{k}$ of the upper half-plane of the first (physical) sheet of the surface giving the natural variation range of the complex frequency parameter k. This holds for all the gratings under consideration (see, for example, [8]).

In terms of Eqs. (4) and (13), expressions (14), (15), and (19) become

$$U(y,L_1,t) = -\sum_{n=-\infty}^{\infty}\left\{\int_0^t J_0[\Phi_n(t-\tau)] \times \left[\int_0^l \frac{\partial U(\tilde{y},z,\tau)}{\partial z}\bigg|_{z=L_1} \mu_n^*(\tilde{y})d\tilde{y}\right]d\tau\right\}\mu_n(y) = V_1(y,t); \quad 0 \leq y \leq l, \quad t \geq 0, \tag{20}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right] U(y,z,t)\bigg|_{z=L_1} = \sum_{n=-\infty}^{\infty}\left\{\int_0^t J_1[\Phi_n(t-\tau)] \times \left[\int_0^l \frac{\partial U(\tilde{y},z,\tau)}{\partial z}\bigg|_{z=L_1} \mu_n^*(\tilde{y})d\tilde{y}\right]d\tau\right\}\Phi_n\mu_n(y) = V_2(y,t); \quad 0 \leq y \leq l, \quad t \geq 0, \tag{21}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right] U(y,z,t)\bigg|_{z=L_1} = -\sum_{n=-\infty}^{\infty}\left\{\int_0^t J_1[\Phi_n(t-\tau)](t-\tau)^{-1} \times \left[\int_0^l U(\tilde{y},L_1,\tau)\mu_n^*(\tilde{y})d\tilde{y}\right]d\tau\right\}\Phi_n\mu_n(y) = V_3(y,t); \quad 0 \leq y \leq l, \quad t \geq 0. \tag{22}$$

Let us consider the possibility of Eqs. (20)–(22) being boundary conditions for restriction of the analysis domain $\mathbf{Q}$ of open problems (3). Using the results from [8,15,25,26] we can prove the following statement.

Statement 1. *Problems (3), and problems (3) supplemented with any one of conditions (20)–(22), are equivalent. The requirements that ensure their unique solvability (correctness classes) are identical.*

Formulas (20)–(22) are exact. Hence their addition to the original problems does not actually increase the computation error or distort the process of simulation.

Relations (14), (15), and (19)–(22) constitute the exact radiation conditions for the outgoing transient waves formed by the grating. Formulas (14), (15), and (19) describe behavior of spatial–temporal amplitudes of all partial components (modes) of the waves guided by the regular channel $\mathbf{R}$ in direction $z\to\infty$. Behavior of these wave fields as a whole is governed by formulas (20)–(22). Therefore the open problems (3) are equivalent to the problems

(3) whose analysis domain $\mathbf{Q}_L$ is finite, with any condition of (20)–(22) met on the virtual boundaries $\mathbf{L}_1$ and $\mathbf{L}_2$. By the same reasoning, conditions (20)–(22) can be regarded as exact absorbing conditions: the wave field $U(g,t)$ neither undergoes deformation across $\mathbf{L}_1$ boundary nor reflects back into the $\mathbf{Q}_L$ domain, the wave $U(g,t)$ fully transmitting to the upper $(z>L_1)$ regular part of the $\mathbf{R}$ channel as if it were absorbed by the domain $\mathbf{A}$ or its boundary $\mathbf{L}_1$.

At $V_2(y,t)=V_3(y,t)=0$, the nonlocal conditions (21) and (22) coincide with the simplest local classical ABC of the first-order approximation [9,10]. This means that functions $V_2(y,t)$ and $V_3(y,t)$ determine the ABC's closing error, or the difference between the exact values of the function $[\partial/\partial t+\partial/\partial z]U(y,z,t)|_{z=L_1}$ and the corresponding results given in the computational schemes using this approximate absorbing condition. The availability of the closing error allows us to estimate the accuracy of the corresponding computational scheme as a whole.

5. LOCAL ABSORBING CONDITIONS

Finite-difference algorithms employing the nonlocal (both in space and time variables) absorbing conditions (20)–(22) call for substantial memory resources as the $V_j(y,t)$ function databases grow progressively with time. They are all stored to make the next step, proceeding through time layers [32]. The problem can be solved in the following manner. We will turn to the local conditions by applying the following scheme which is easy to realize. In view of the representation [33]

$$J_0(x)=\frac{2}{\pi}\int_0^{\pi/2}\cos(x\sin\phi)d\phi,$$

rewrite relations (14) as

$$u_n(L_1,t)=-\frac{2}{\pi}\int_0^{\pi/2}\left\{\int_0\cos[\Phi_n(t-\tau)\times\sin\phi]\chi(t-\tau)u_n{}'(L_1,\tau)d\tau\right\}d\phi;\quad t\geqslant 0. \tag{23}$$

Introduce

$$w_n(t,\phi)=-\int_0\frac{\sin[\Phi_n(t-\tau)\sin\phi]\chi(t-\tau)u_n{}'(L_1,\tau)}{\Phi_n\sin\phi}d\tau;\quad t\geqslant 0,\quad 0\leqslant\phi\leqslant\pi/2. \tag{24}$$

Then

$$\frac{\partial w_n(t,\phi)}{\partial t}=-\int_0\cos[\Phi_n(t-\tau)\sin\phi]\chi(t-\tau)u_n{}'(L_1,\tau)d\tau,$$

and from relations (23) we have

$$u_n(L_1,t)=\frac{2}{\pi}\int_0^{\pi/2}\frac{\partial w_n(t,\phi)}{\partial t}d\phi;\quad t\geqslant 0. \tag{25}$$

The integral form (24) is equivalent to the differential formulation

$$\left\{\begin{array}{l}\left[\dfrac{\partial^2}{\partial t^2}+\Phi_n^2\sin^2\phi\right]w_n(t,\phi)=-u_n{}'(L_1,t);\quad t>0\\ w_n(0,\phi)=\left.\dfrac{\partial w_n(t,\phi)}{\partial t}\right|_{t=0}=0\end{array}\right.. \tag{26}$$

Indeed, passing from formulation (26) to the generalized Cauchy problem and using the fundamental solution $G(\lambda,t)=\chi(t)\lambda^{-1}\sin\lambda t$ of the operator $D(\lambda)\equiv[d^2/dt^2+\lambda^2]$ (see [8]), one easily learns that relations (24) and (26) define the same functions $w_n(t,\phi)$.

Now multiply relations (25) and (26) by $\mu_n(y)$ and sum over $n=0,\pm1,\pm2,\ldots$. On account of

$$\sum_{n=-\infty}^{\infty}\Phi_n^2 w_n(t,\phi)\mu_n(y)=-\frac{\partial^2 W(y,t,\phi)}{\partial y^2}$$

for

$$W(y,t,\phi)=\sum_{n=-\infty}^{\infty}w_n(t,\phi)\mu_n(y)$$

[see problem (5)], we obtain

$$U(y,L_1,t)=\frac{2}{\pi}\int_0^{\pi/2}\frac{\partial W(y,t,\phi)}{\partial t}d\phi;\quad t\geqslant 0,\quad 0\leqslant y\leqslant l,$$

$$\left\{\begin{array}{ll}\left[\dfrac{\partial^2}{\partial t^2}-\sin^2\phi\dfrac{\partial^2}{\partial y^2}\right]W(y,t,\phi)=-\left.\dfrac{\partial U(y,z,t)}{\partial z}\right|_{z=L_1}; & 0<y<l,\quad t>0\\ W(y,0,\phi)=\left.\dfrac{\partial W(y,t,\phi)}{\partial t}\right|_{t=0}=0; & 0\leqslant y\leqslant l\\ W\left\{\dfrac{\partial W}{\partial y}\right\}(l,t,\phi)=e^{i2\pi\Phi}W\left\{\dfrac{\partial W}{\partial y}\right\}(0,t,\phi); & t\geqslant 0\end{array}\right.. \tag{27}$$

This local (both in space and time variable) exact absorbing condition (EAC) enables us to truncate efficiently the computation domain when solving problems (3) numerically. From here on, $W(y,t,\phi)$ is an auxiliary function coming from the solution of the separate initial boundary value problem, which is the inner problem with respect to the corresponding condition, and $0\leqslant\phi\leqslant\pi/2$ is a numerical parameter.

A similar treatment for relations (15) and (19) gives the following local EACs, different from relations (27):

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right]U(y,z,t)\bigg|_{z=L_1}=\frac{2}{\pi}\int_0^{\pi/2}W(y,t,\phi)\cos^2\phi d\phi;\quad t\geqslant 0,\quad 0\leqslant y\leqslant l,$$

$$\left\{\begin{array}{ll}\left[\dfrac{\partial^2}{\partial t^2}-\cos^2\phi\dfrac{\partial^2}{\partial y^2}\right]W(y,t,\phi)=-\dfrac{\partial^2}{\partial y^2}\left[\dfrac{\partial}{\partial z}U(y,z,t)\bigg|_{z=L_1}\right]; & 0<y<l,\quad t>0\\ W(y,0,\phi)=\dfrac{\partial W(y,t,\phi)}{\partial t}\bigg|_{t=0}=0; & 0\leqslant y\leqslant l\\ W\left\{\dfrac{\partial W}{\partial y}\right\}(l,t,\phi)=e^{i2\pi\Phi}W\left\{\dfrac{\partial W}{\partial y}\right\}(0,t,\phi); & t\geqslant 0\end{array}\right.,\tag{28}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right]U(y,z,t)\bigg|_{z=L_1}=\frac{2}{\pi}\int_0^{\pi/2}\frac{\partial W(y,t,\phi)}{\partial t}\sin^2\phi d\phi;\quad t\geqslant 0,\quad 0\leqslant y\leqslant l,$$

$$\left\{\begin{array}{ll}\left[\dfrac{\partial^2}{\partial t^2}-\cos^2\phi\dfrac{\partial^2}{\partial y^2}\right]W(y,t,\phi)=\dfrac{\partial^2 U(y,L_1,t)}{\partial y^2}; & 0<y<l,\quad t>0\\ W(y,0,\phi)=\dfrac{\partial W(y,t,\phi)}{\partial t}\bigg|_{t=0}=0; & 0\leqslant y\leqslant l\\ W\left\{\dfrac{\partial W}{\partial y}\right\}(l,t,\phi)=e^{i2\pi\Phi}W\left\{\dfrac{\partial W}{\partial y}\right\}(0,t,\phi); & t\geqslant 0\end{array}\right..\tag{29}$$

Expression (28) was obtained by virtue of the formula [34]

$$J_1(x)=\frac{2}{\pi}\int_0^{\pi/2}\sin(x\cos\phi)\cos\phi d\phi$$

with the substitutions

$$w_n(t,\phi)=\Phi_n\int_0\frac{\sin[\Phi_n(t-\tau)\cos\phi]\chi(t-\tau)u_n{}'(L_1,\tau)}{\cos\phi}d\tau;$$
$$t\geqslant 0,\quad 0\leqslant\phi\leqslant\pi/2.$$

The derivation of expression (29) was through the Poisson integral [33]

$$J_1(x)=\frac{2x}{\pi}\int_0^{\pi/2}\cos(x\cos\phi)\sin^2\phi d\phi$$

and

$$w_n(t,\phi)=-\Phi_n\int_0\frac{\sin[\Phi_n(t-\tau)\cos\phi]\chi(t-\tau)u_n(L_1,\tau)}{\cos\phi}d\tau;$$
$$t\geqslant 0,\quad 0\leqslant\phi\leqslant\pi/2.$$

Under the assumption $W(y,t,\phi)\equiv 0$ (which cannot be justified), expressions (28) and (29) reduce to the classical ABC of the first-order approximation. Using the trapezoidal rule, the integral in expression (28) is replaced by a finite sum and we end up with an approximate condition that agrees well with [11].

By invoking formulas [34]

$$J_0(x)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\exp(ix\sin\phi)d\phi$$

and

$$J_1(x)=\frac{1}{\pi}\int_0^{\pi}\sin(x\sin\phi)\sin\phi d\phi,$$

one also arrives at the following local EACs:

$$U(y,L_1,t) = -\frac{1}{2\pi}\int_{-\pi}^{\pi} W(y,t,\phi)d\phi; \quad 0 \leqslant y \leqslant l, \quad t \geqslant 0,$$

$$\begin{cases} \left[\frac{\partial}{\partial t} - \sin\phi\frac{\partial}{\partial y}\right]W(y,t,\phi) = \left.\frac{\partial}{\partial z}U(y,z,t)\right|_{z=L_1}; & 0 < y < l, \quad t > 0 \\ W(y,0,\phi) = \left.\frac{\partial W(y,t,\phi)}{\partial t}\right|_{t=0} = 0; & 0 \leqslant y \leqslant l \\ W\left\{\frac{\partial W}{\partial y}\right\}(l,t,\phi) = \mathrm{e}^{i2\pi\Phi}W\left\{\frac{\partial W}{\partial y}\right\}(0,t,\phi); & t \geqslant 0 \end{cases}, \tag{30}$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial z}\right]U(g,t)\bigg|_{z=L_1} = \frac{1}{\pi}\int_0^{\pi} W(y,t,\phi)d\phi; \quad 0 \leqslant y \leqslant l, \quad t \geqslant 0,$$

$$\begin{cases} \left[\frac{\partial^2}{\partial t^2} - \sin^2\phi\frac{\partial^2}{\partial y^2}\right]W(y,t,\phi) = -\sin^2\phi\frac{\partial^2}{\partial y^2}\left[\left.\frac{\partial U(g,t)}{\partial z}\right|_{z=L_1}\right]; & 0 < y < l, \quad t > 0 \\ W(y,0,\phi) = \left.\frac{\partial W(y,t,\phi)}{\partial t}\right|_{t=0} = 0; & 0 \leqslant y \leqslant l \\ W\left\{\frac{\partial W}{\partial y}\right\}(l,t,\phi) = \mathrm{e}^{i2\pi\Phi}W\left\{\frac{\partial W}{\partial y}\right\}(0,t,\phi); & t \geqslant 0 \end{cases}. \tag{31}$$

Conditions (30) and (31) are concerned with relations (14), (15), (20), and (21) in the same way as relations (27) and (28). Here, we have made the substitutions

$$w_n(t,\phi) = \int_0^t \exp[i\Phi_n(t-\tau)\sin\phi]u_n{}'(L_1,\tau)d\tau; \quad |\phi| \leqslant \pi, \tag{32}$$

$$w_n(t,\phi) = \Phi_n \sin\phi\int_0^t \sin[\Phi_n(t-\tau)\sin\phi]u_n{}'(L_1,\tau)d\tau; \quad 0 \leqslant \phi \leqslant \pi,$$

to deduce conditions (30) and (31), respectively. Note the new technical detail—the differential form $[\partial/\partial t - i\Phi_n \sin\phi]w_n(t,\phi) = u_n'(L_1,\tau)$ [from which follows the equation with respect to $W(y,t,\phi)$ in the inner initial boundary value problem in condition (30)] that is equivalent to the integral form (32) has been constructed with the help of the fundamental solution $G(\lambda,t)=\chi(t)\exp(-\lambda t)$ of the operator $[d/dt+\lambda]$ (see [8]).

We return now to the representation (4) for the lower regular part $z<-L_2-h$ of the $\mathbf{R}$ channel as well as for its upper part and construct the following initial boundary value problems similar to problems (6)

$$\begin{cases} \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial z^2} - \Phi_n^2\right]u_n(z,t) = 0, \quad t > 0 \\ u_n(z,0) = 0, \quad \left.\frac{\partial}{\partial t}u_n(z,t)\right|_{t=0} = 0 \end{cases}; \quad z \leqslant -L_2 - h, \quad n = 0, \pm 1, \pm 2, \ldots \tag{33}$$

for the evolutionary basis elements $u_n(z,t)$ of the signal $U(g,t)$, $g \in \mathbf{B}$. Problems (6) generate three types of nonlocal EACs [formulas (20)–(22)] and five types of local EACs [formulas (27)–(31)]. A comparison between problems (6) and problems (33) shows how all these EAC formulas can be rewritten in terms of boundary $\mathbf{L}_2$. In what follows, conditions (21) and (28) will be used. Taking into account the change both in the direction of free propagation of pulsed waves (toward $z\to-\infty$ instead of $z\to+\infty$) and in the position of the artificial boundary $\mathbf{L}_2$ ($z=-L_2-h$ instead of $z=L_1$), we have

$$\left[\frac{\partial}{\partial t} - \frac{\partial}{\partial z}\right]U(y,z,t)\bigg|_{z=-L_2-h} = -\sum_n \left\{\int_0^t J_1[\Phi_n(t-\tau)]\left[\int_0^l \left.\frac{\partial U(\tilde{y},z,\tau)}{\partial z}\right|_{z=-L_2-h}\mu_n^*(\tilde{y})d\tilde{y}\right]d\tau\right\}\Phi_n\mu_n(y);$$

$$0 \leqslant y \leqslant l, \quad t \geqslant 0 \tag{34}$$

and

$$\left[\frac{\partial}{\partial t}-\frac{\partial}{\partial z}\right]U(y,z,t)\bigg|_{z=-L_2-h}=\frac{2}{\pi}\int_0^{\pi/2}W(y,t,\phi)\cos^2\phi d\phi;\quad t\geq 0,\quad 0\leq y\leq l,$$

$$\begin{cases}\left[\frac{\partial^2}{\partial t^2}-\cos^2\phi\frac{\partial^2}{\partial y^2}\right]W(y,t,\phi)=\frac{\partial^2}{\partial y^2}\left[\frac{\partial}{\partial z}U(y,z,t)\bigg|_{z=-L_2-h}\right]; & 0<y<l,\quad t>0\\ W(y,0,\phi)=\frac{\partial W(y,t,\phi)}{\partial t}\bigg|_{t=0}=0; & 0\leq y\leq l\\ W\left\{\frac{\partial W}{\partial y}\right\}(l,t,\phi)=e^{i2\pi\Phi}W\left\{\frac{\partial W}{\partial y}\right\}(0,t,\phi); & t\geq 0\end{cases}. \tag{35}$$

Now we can formulate the main result of this section.

Statement 2. *Open problems (3) with analysis domain* $\mathbf{Q}$ *are equivalent to closed problems (3) with analysis domain* $\mathbf{Q}_L$ *and with any one of nonlocal or local EACs (20)–(22), (27)–(31), (34), and (35) on its outer boundary* $\mathbf{L}=\mathbf{L}_1\cup\mathbf{L}_2$. *For auxiliary functions* $W(y,z,\phi)$, *the inner initial boundary value problems in (27)–(31) and (35) are well posed*.

6. PROBLEMS OF LARGE AND DISTANT FIELD SOURCES

In formulating earlier the initial boundary value problem and specifying $\mathbf{Q}$ and $\mathbf{Q}_L$ domains, we assumed that the functions describing the sources that excite the gratings are finitary in the closure of the complete analysis domain $\mathbf{Q}$, and their supports belong to $\overline{\mathbf{Q}_L}\setminus\mathbf{L}$ for all the time $0\leq t\leq T$. The advantage is the following: conditions on virtual boundaries $\mathbf{L}$ can be formulated in terms of the total field $U(g,t)$. The limitations introduced by these assumptions can be partially or completely removed by enclosing a certain part of the current $(F(g,t))$ and/or the momentary sources ($\phi(g)$ and $\psi(g)$) in the ${}_L\mathbf{Q}$ domain. The only concern is the following: one should exclude the incoming primary wave $U^i(g,t)$ generated by this source from the field $U(g,t)$ on $\mathbf{L}$. To this end, the scattered (secondary) field $U^s(g,t)=U(g,t)-U^i(g,t)$ is introduced. The final equations for the modified problem can be formulated either in terms of the total field $U(g,t)$ or in terms of the secondary field $U^s(g,t)$. The first alternative is preferred, as the formally true separation of the field $U(g,t)$ into $U^s(g,t)$ and $U^i(g,t)$ can be physically invalid in partial subdomains of the domain $\mathbf{Q}$.

The problems (36) stated as

$$\begin{cases}\left[-\varepsilon\mu\frac{\partial^2}{\partial t^2}-\sigma\mu\frac{\partial}{\partial t}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right]U(g,t)=F(g,t)+\tilde{F}(g,t); & g=\{y,z\}\in\mathbf{Q},\quad t>0\\ U(g,0)=\phi(g)+\tilde{\phi}(g),\quad \frac{\partial}{\partial t}U(g,t)\bigg|_{t=0}=\psi(g)+\tilde{\psi}(g); & g\in\bar{\mathbf{Q}}\\ E_{tg}(p,t)|_{p=\{x,y,z\}\in\mathbf{S}}=0; & t\geq 0\\ E_{tg}(p,t)\text{ and }H_{tg}(p,t)\text{ are continuous at the surfaces }\mathbf{S}^{\varepsilon,\mu,\sigma}; & t\geq 0\\ U\left\{\frac{\partial U}{\partial y}\right\}(l,z,t)=e^{2\pi i\Phi}U\left\{\frac{\partial U}{\partial y}\right\}(0,z,t); & t\geq 0\end{cases} \tag{36}$$

differ from problems (3) by the existence of the functions $\tilde{F}(g,t)$, $\tilde{\phi}(g)$, and $\tilde{\psi}(g)$ that are finitary in the domain $\mathbf{Q}$. It is assumed that the supports of these functions and the corresponding sources belong to the domain $\mathbf{A}=\{g\in\mathbf{Q}:z>L_1\}$ (see Fig. 2). As before, domain $\mathbf{B}=\{g\in\mathbf{Q}:z<-L_2-h\}$ carries no sources or efficient scatterers.

In $\mathbf{A}$ the total field can be written as $U(g,t)=U^i(g,t)+U^s(g,t)$, where $U^i(g,t)$ is the field in the channel $\mathbf{R}$ from the sources $\tilde{F}(g,t)$, $\tilde{\phi}(g)$, and $\tilde{\psi}(g)$:

$$\begin{cases} \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] U^i(g,t) = \tilde{F}(g,t); & g \in \mathbf{R}, \quad t > 0 \\ U^i(g,0) = \tilde{\phi}(g), \quad \frac{\partial}{\partial t} U^i(g,t) \Big|_{t=0} = \tilde{\psi}(g); & g \in \bar{\mathbf{R}} \\ U^i \left\{ \frac{\partial U^i}{\partial y} \right\} (l,z,t) = e^{2\pi i \Phi} U^i \left\{ \frac{\partial U^i}{\partial y} \right\} (0,z,t); & t \geqslant 0 \end{cases} . \tag{37}$$

In order to find $U^s(g,t)$ in **A** and $U(g,t)$ in **B**, we may consider now the following homogeneous initial boundary value problems:

$$\begin{cases} \left[-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right] \begin{Bmatrix} U^s(g,t) \\ U(g,t) \end{Bmatrix} = 0; & g \in \begin{Bmatrix} \mathbf{A} \\ \mathbf{B} \end{Bmatrix}, \quad t > 0 \\ \begin{Bmatrix} U^s(g,0) \\ U(g,0) \end{Bmatrix} = 0, \quad \frac{\partial}{\partial t} \begin{Bmatrix} U^s(g,t) \\ U(g,t) \end{Bmatrix} \Bigg|_{t=0} = 0; & g \in \begin{Bmatrix} \bar{\mathbf{A}} \\ \bar{\mathbf{B}} \end{Bmatrix} \\ \begin{Bmatrix} U^s \\ U \end{Bmatrix} \left\{ \begin{Bmatrix} U^s \\ U \end{Bmatrix} \Big/ \partial y \right\} (l,z,t) = e^{2\pi i \Phi} \begin{Bmatrix} U^s \\ U \end{Bmatrix} \left\{ \begin{Bmatrix} U^s \\ U \end{Bmatrix} \Big/ \partial y \right\} (0,z,t); & \begin{Bmatrix} z > L_1 \\ z < -L_2 - h \end{Bmatrix}, \quad t \geqslant 0 \end{cases} . \tag{38}$$

It is assumed that the perturbation caused by the sources concentrated in $\mathbf{Q}_L$ have not reached boundaries $\mathbf{L}_1$ and $\mathbf{L}_2$ of domains **A** and **B** at time $t=0$. The solutions of problems (38) are function $U(g,t)$ in **B** and function $U^s(g,t)$ in **A**. They determine the outgoing waves traveling in the $z \to -\infty$ and $z \to +\infty$ directions, respectively. Therefore we will prove that (see Sections 4 and 5)

$$U(y,L_1,t) - U^i(y,L_1,t) = -\sum_{n=-\infty}^{\infty} \left\{ \int_0^t J_0[\Phi_n(t-\tau)] \left[\int_0^l \frac{\partial [U(\tilde{y},z,\tau) - U^i(\tilde{y},z,\tau)]}{\partial z} \Bigg|_{z=L_1} \mu_n^*(\tilde{y}) d\tilde{y} \right] d\tau \right\} \mu_n(y);$$

$$0 \leqslant y \leqslant l, \quad t \geqslant 0, \tag{39}$$

$$U(y,-L_2-h,t) = \sum_{n=-\infty}^{\infty} \left\{ \int_0^t J_0[\Phi_n(t-\tau)] \left[\int_0^l \frac{\partial U(\tilde{y},z,\tau)}{\partial z} \Bigg|_{z=-L_2-h} \mu_n^*(\tilde{y}) d\tilde{y} \right] d\tau \right\} \mu_n(y);$$

$$0 \leqslant y \leqslant l, \quad t \geqslant 0, \tag{40}$$

and

$$U(y,L_1,t) - U^i(y,L_1,t) = \frac{2}{\pi} \int_0^{\pi/2} \frac{\partial W(y,t,\phi)}{\partial t} d\phi; \quad t \geqslant 0, \quad 0 \leqslant y \leqslant l,$$

$$\begin{cases} \left[\frac{\partial^2}{\partial t^2} - \sin^2 \phi \frac{\partial^2}{\partial y^2} \right] W(y,t,\phi) = -\frac{\partial [U(y,z,t) - U^i(y,z,t)]}{\partial z} \Bigg|_{z=L_1}; & 0 < y < l, \quad t > 0 \\ W(y,0,\phi) = \frac{\partial W(y,t,\phi)}{\partial t} \Bigg|_{t=0} = 0; & 0 \leqslant y \leqslant l \\ W \left\{ \frac{\partial W}{\partial y} \right\} (l,t,\phi) = e^{i2\pi \Phi} W \left\{ \frac{\partial W}{\partial y} \right\} (0,t,\phi); & t \geqslant 0 \end{cases} , \tag{41}$$

$$U(y,-L_2-h,t) = \frac{2}{\pi} \int_0^{\pi/2} \frac{\partial W(y,t,\phi)}{\partial t} d\phi; \quad t \geqslant 0, \quad 0 \leqslant y \leqslant l,$$

$$\begin{cases} \left[\dfrac{\partial^2}{\partial t^2}-\sin^2\phi\dfrac{\partial^2}{\partial y^2}\right]W(y,t,\phi)=\left.\dfrac{\partial U(y,z,t)}{\partial z}\right|_{z=-L_2-h}; & 0<y<l, \quad t>0 \\ W(y,0,\phi)=\left.\dfrac{\partial W(y,t,\phi)}{\partial t}\right|_{t=0}=0; & 0\leqslant y\leqslant l \\ W\left\{\dfrac{\partial W}{\partial y}\right\}(l,t,\phi)=e^{i2\pi\Phi}W\left\{\dfrac{\partial W}{\partial y}\right\}(0,t,\phi); & t\geqslant 0 \end{cases}. \tag{42}$$

As before, $W(y,t,\phi)$ are certain auxiliary functions here.

The couples (39) and (41) are exact (nonlocal and local) absorbing conditions on the boundary $\mathbf{L}_1$ in region $\mathbf{A}$ with cross section at $z=L_1$. The couples (40) and (42) represent the same conditions for the boundary $\mathbf{L}_2$ in region $\mathbf{B}$ with cross section at $z=-L_2-h$. They are direct analogues of conditions (20) and (27) constructed in Sections 4 and 5. It is evident that other nonlocal and local conditions from these sections may be adjusted for the situation considered here.

In $\mathbf{Q}_L$ function $U(g,t)$ is governed by the equations

$$\begin{cases} \left[-\varepsilon\mu\dfrac{\partial^2}{\partial t^2}-\sigma\mu\dfrac{\partial}{\partial t}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}\right]U(g,t)=F(g,t); & g\in\mathbf{Q}_L, \quad t>0 \\ U(g,0)=\phi(g), \quad \left.\dfrac{\partial}{\partial t}U(g,t)\right|_{t=0}=\psi(g); & g\in\overline{\mathbf{Q}_L} \\ E_{tg}(p,t)|_{p=\{x,y,z\}\in\mathbf{S}}=0; & t\geqslant 0 \\ E_{tg}(p,t) \text{ and } H_{tg}(p,t) \text{ are continuous at the surfaces } \mathbf{S}^{\varepsilon,\mu,\sigma}; & t\geqslant 0 \\ U\left\{\dfrac{\partial U}{\partial y}\right\}(l,z,t)=e^{2\pi i\Phi}U\left\{\dfrac{\partial U}{\partial y}\right\}(0,z,t); & t\geqslant 0 \end{cases}. \tag{43}$$

Statement 3. *Problems (36) and problems (43) with conditions (39) and (40) or (41) and (42) in the domain $\mathbf{Q}_L$ have the same solutions $U(g,t)$ for an arbitrary observation time $t\in[0;T]$. In the modified problems, functions $U^i(g,t)$, which are involved in the EACs of (39) and (41) type for the virtual boundary $\mathbf{L}_1$, act as real sources outside the bounded analysis domain $\mathbf{Q}_L$.*

EACs of (39) and (41) type allow us to truncate the calculation space $\mathbf{Q}_L$ to a reasonable size when dealing with the problem of the type of (36) with large and/or distant sources of transient waves $U^i(g,t)$. These sources—$\tilde{F}(g,t)$, $\tilde{\phi}(g)$, and $\tilde{\psi}(g)$—are merely enclosed in the $_L\mathbf{Q}$ domain. Their contribution to the total field $U(g,t)$ is considered via the boundary values of the functions $U^i(g,t)$, $t\in[0;T]$ and their normal derivatives on the $\mathbf{L}$ boundaries. All information relevant for the realization of the scheme is provided by the solution of problem (37), which is quite simple computationwise. Also the problem is explicitly solved using the mirror image technique. The Poisson formula governing a given source field in a free 2-D space (in space $\mathbf{R}^2$) readily admits the boundary wall condition of the Floquet channel $\mathbf{R}$ [26,35].

Where and how the primary $U^i(g,t)$ wave is excited is usually not a problem for standard scattering analysis of infinite periodic gratings. Nor does it need $U^i(g,t)$ values at all observation time $t\in[0;T]$ and at all points g from $\mathbf{A}$. A proper numerical experiment needs only $U^i(g,t)|_{g\in\mathbf{L}_1}$ and $[\partial U^i(g,t)/\partial z]|_{g\in\mathbf{L}_1}$ values for all times $t\in[0;T]$. But these values must be in agreement with the boundary values of some function $U^i(g,t)$ that governs in domain $\mathbf{A}$ a transient electromagnetic wave running on the virtual boundary $\mathbf{L}_1$ (principle of causality). On the $\mathbf{L}_1$ boundary separating domains $\mathbf{Q}_L$ and $\mathbf{A}$ this requirement complies with the functions

$$U_p^i(y,L_1,t)=v_p(L_1,t)\mu_p(y), \quad [\partial U_p^i(y,z,t)/\partial z]|_{z=L_1}$$
$$=v_p{}'(L_1,t)\mu_p(y),$$
$$0\leqslant y\leqslant l, \quad p=0,\pm 1,\pm 2,\ldots, \tag{44}$$

whose amplitudes $v_p(L_1,t)$ and $v_p{}'(L_1,t)$ are related as

$$v_p(L_1,t)=\int_0 J_0[\Phi_p(t-\tau)]\chi(t-\tau)v_p{}'(L_1,\tau)d\tau; \quad t\geqslant 0. \tag{45}$$

It is evident that relations (44) and (45) give boundary values of the function $U_p^i(g,t)=v_p(z,t)\mu_p(y)$ describing a transient wave running on the boundary $\mathbf{L}_1$ from region $\mathbf{A}$. This is so because relation (45) comes from relation (12), taking into account changes in the direction of propagation of the wave.

7. CONCLUSION

This paper offers novel exact absorbing conditions that allow one to truncate efficiently the computation domain of finite-difference algorithms [7] applied for open initial boundary-value problems in the theory of gratings. An analytical approach to the problem of equivalent replacement of open problems with closed ones is developed. Near-to-far field relations are obtained for periodic gratings. Operators of these relations act in the space of amplitudes of the outgoing impulse waves. These operators represent exactly all changes in the field during its free propagation in regular plane-parallel Floquet channels. The problem of large and distant sources is solved. Relations required for correct formulation and algorithmization of the problems for gratings excited by their spatial spectrum's impulse harmonics are deduced.

The efficiency and correctness of the approach, and the validity of the results, are attributed to the rigorous mathematical methods used, and have been proved many times [8,17,21–24,32,36–40].

In our companion paper in this issue we will use the results obtained here to solve a number of actual gratings problems.

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