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MICROWAVE ELECTRODYNAMICS

The Strong Approach to Analysis of Transients in the Axially Symmetrical Waveguide Units*

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ABSTRACT: The exact "absorbing" conditions for virtual boundaries in the cross-sections of regular circular and coaxial circular waveguides are constructed, which allow one to truncate efficiently the computational domain of finite-difference methods as applied to the simulation of transients in open axially-symmetric waveguide resonators. The "open" initial-boundary value problems, describing the impulse TE_{0n}- and TM_{0n}-waves in the structure of this kind, are reduced to the equivalent "closed" ones. The so-obtained conditions are embodied in the computer programs for the analysis and model synthesis of functional units of impulse-wave radiators.

Axially symmetric waveguide units are important functional elements of many up-to-date devices and instruments of a microwave range. They are capable to withstand high-Q free magnetic field oscillations to change in wide ranges a mode and spectral structure of signals, to respond controllably to signals of different polarization and duration. Their electrodynamic analysis, detection and optimization of different anomalous and resonance modes of wave scattering can be appreciably accelerated by the approaches basing on the mathematical simulation and computational experiment. Here the frequency-domain methods realizing the idea of analytic regularization were and remain unique as regards their efficiency and the content of results obtained [1,2].

In the time domain, the approaches of which are directed to studying the physics of transients and regularities in the space-time pulsed field transformations, the finite-difference method [3] could play the same role. But there is one and very difficult theoretical problem on this way - this is the

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problem of effective truncation of the computational domain by descritization of “open” initial-boundary value problems, i.e., the problems for which the analysis domain is not truncated in one or several space directions. The most simple solving of this problem is based on the use of a known radiation condition for outgoing waves: perturbation propagates with a finite velocity, therefore for any finite value of the observation time t it is always possible to construct the virtual boundaries L, being at a rather large distance, in the points of which the field strength will be equal to zero. The main disadvantage of such an approach is the necessity in significant extension of the computational domain at high T values, determining the upper limit in the interval $0<t<T$ of the observation time t change.

Classic approximated boundary “absorbing” conditions (ABCs: see [4-6]) and perfectly matched layers (PMLs: see [7,8]) allow one to “close” the “open” initial-boundary value problems with the near virtual boundaries L, but it is not possible to evaluate analytically the level of errors that they introduce into the sought-for field characteristics. When investigating the resonance situations and in the case of analysis of small-sized domains and high T values this level can exceed the critical one when there is none true significant digit in the data obtained [9].

In the present paper the problem of computational domain truncation is solved using the exact “absorbing” conditions the theory of which is developed since the late of nineties of the last century [9-11]. These conditions do not provoke, in the analysis domain, false signals formed by the outgoing wave reflection from the imperfect transparent virtual boundaries L. They permit to change original equivalent “open” initial-boundary value problems by equivalent “closed” ones. An additional error of computations, caused by their inclusion into the schemes of the finite difference method, is by an order of magnitude lower then the ordinary errors in the approximation of original initial-boundary value problems [9].

STATEMENT OF INITIAL BOUNDARY VALUE PROBLEMS

Transformation of impulse TE_{0n}- ($\partial/\partial\phi\equiv 0$ and $E_\rho = E_z = H_\phi \equiv 0$) and TM_{0n} -waves ($H_\rho = H_z = E_\phi \equiv 0$) by the axial-symmetric open waveguide resonators (an example of the geometry of a similar structure is given in Fig.1) is described by the following two-dimensional (in the half-plane of the variable $g=\{\rho\geq 0, z\}$) scalar initial-boundary value problems:

$$\begin{cases} \left[\frac{\partial}{\partial\rho}\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\right)+\frac{\partial^2}{\partial z^2}-\varepsilon(g)\frac{\partial^2}{\partial t^2}-\sigma(g)\frac{\partial}{\partial t}\right]U(g,t)=0; \quad t>0, \quad g\in Q; \\ U(g,t)\big|_{t=0}=\varphi(g), \quad \frac{\partial}{\partial t}U(g,t)\Big|_{t=0}=\psi(g); \quad g\in\bar{Q}; \\ E_{tg}(g,t)\big|_{g\in S}=0, \quad U(0,z,t)=0; \quad t\geq 0. \end{cases} \qquad (1)$$

Here $\vec{E}=\{E_\rho,E_\phi,E_z\}\equiv\vec{E}(g,t)$ and $\vec{H}=\{H_\rho,H_\phi,H_z\}\equiv\vec{H}(g,t)$ are the vectors of the electric- and magnetic field strength vectors; $\sigma(g)=\eta_0\sigma_0(g)$, $\eta_0=(\mu_0/\varepsilon_0)^{1/2}$ is the free space impedance; ε_0 and μ_0 are the vacuum electric and magnetic constants; $\varepsilon\equiv\varepsilon(g)\geq 1$ and $\sigma_0\equiv\sigma_0(g)\geq 0$ is the relative dielectric permeability and specific conductivity of the locally nonuniform (isotropic, nonmagnetic and nondisperse) wave propagation medium: the time t has the dimensions of length – this is the product of true time by the light propagation velocity in vacuum R^2; ρ, ϕ, z are the cylindrical coordinates

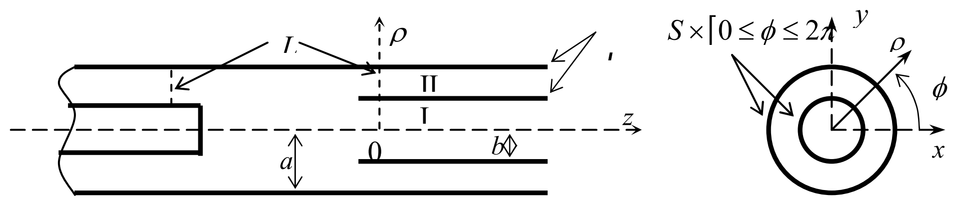


FIGURE 1. Resonator in the gap of a circular coaxial waveguide $\phi=\pi/2$

The last of the conditions in (1) is given by the problems symmetry: the axis p=0 coincides with the circular symmetry axis, therefore, here, only E_z- and H_z-field components can differ from zero. At $U(g,t)=E\phi$ the problems (1) describe the space-time transformations of TE_{0n}- wave and at U(g, t)=$H\phi$ - of TM_{0n}-waves. It will be recalled that [9] in the case TM_{0n}- wave the $H\phi$-field component satisfies the telegraph equation of (1) only for the piecewise constants ε (g) and σ (g). The analysis domain Q of the problems (1) is the part of the half-plane $\rho 0z$ truncated by the contour S. $S\times[0\leq\phi\leq 2\pi]$ is the surface of perfect conductors. It is assumed that the functions, finite in the domain Q closure, $\varphi(g)=U^i(g,0)$,

$\psi(g)=\partial U^i(g,t)/\partial t\big|_{t=0}$ ($U^i(g,t)$ is the incident wave), $\sigma(g)$ and $\varepsilon(g)-1$ satisfy the conditions on the single-valued solvability of problems (1) in the Sobolev space $W_2^1(Q^T)$, $Q^T=Q\times(0;T)$, $T<\infty$ [12]. In the regular and coaxial circular waveguides (in the domain ${}_LQ=Q\setminus(Q_L\cup L)$ by which the node-formed field can propagate infinitely far, the effective scatterers do not take place. Here $\varepsilon(g)\equiv 1$, $\sigma(g)\equiv 0$ and $U(g,t)=U^s(g,t)+U^i(g,t)$. It is assumed also that at a zero time $t=0$ the wave $U^i(g,t)$; $g\in {}_LQ$, exiting the resonator Q_L, did not reach yet its virtual boundary L (in Fig.1 it is marked by the dashed lines) that lies in the plane of regular waveguide cross-sections. The geometry of the domain ${}_LQ$ in problems (1) is such that the general solutions of the last ones in the corresponding regular frequency domains (it is evident that we can restrict ourselves to consideration of domains I and II with $z>0$) can be represented in the following form

$$\begin{cases} U^s(z,\rho,t)=\sum_n u_n(z,t)\,\mu_n(\rho) \\ U^i(z,\rho,t)=\sum_n v_n(z,t)\,\mu_n(\rho) \end{cases};\quad t\ge 0, \tag{2}$$

where the orthonormalized bases $\{\mu_n(\rho)\}$; $n\in\{n\}$ are given by the nontrivial solutions of the standard "transverse" uniform boundary-value problems:

$$\begin{cases} \begin{cases} \mu_n(\rho)=J_1(\lambda_n\rho)\sqrt{2}\left[bJ_0(\lambda_n b)\right]^{-1}; \quad n=1,2,... \\ \lambda_n>0 - \text{are the roots of the equation}\quad J_1(\lambda b)=0; \end{cases} \quad \rho<b \\ \begin{cases} \mu_n(\rho)=\dfrac{G_1(\lambda_n,\rho)\sqrt{2}}{\sqrt{a^2G_0^2(\lambda_n,a)-b^2G_0^2(\lambda_n,b)}}; \quad n=1,2,... \quad (TE_{0n}-\text{waves}); \\ \lambda_n>0 - \text{are the roots of the equation}\quad G_1(\lambda,a)=0; \quad b<\rho<a \\ G_q(\lambda,\rho)=J_q(\lambda\rho)N_1(\lambda b)-N_q(\lambda\rho)J_1(\lambda b); \end{cases} \end{cases}$$

$$\begin{cases} \begin{cases} \mu_n(\rho) = J_1(\lambda_n \rho)\sqrt{2}\left[bJ_1(\lambda_n b)\right]^{-1}; \ n=1,2,... \\ \lambda_n > 0 - \text{are the roots of the equation} \quad J_0(\lambda b) = 0; \end{cases} & \rho < b \\ \begin{cases} \mu_0(\rho) = \left[\rho\sqrt{\ln(a/b)}\right]^{-1}; & (TM_{0n} - \text{waves}); \\ \mu_n(\rho) = \dfrac{\tilde{G}_1(\lambda_n,\rho)\sqrt{2}}{\sqrt{a^2\tilde{G}_1^2(\lambda_n,a) - b^2\tilde{G}_1^2(\lambda_n,b)}}; \ n=1,2,... & b<\rho<a \\ \lambda_0 = 0; \ \lambda_n > 0 \ (n=1,2,...) - \text{are the roots of the equation} \\ \tilde{G}_0(\lambda,b) = 0, \ \tilde{G}_q(\lambda,\rho) = J_q(\lambda\rho)N_0(\lambda a) - N_q(\lambda\rho)J_0(\lambda a) \end{cases} \end{cases}$$

Here J_m and N_m are the cylindrical Bessel and Neuman functions.

As in the domain ${}_LQ$ [9]

$$\partial\begin{Bmatrix} H_\rho \\ E_\rho \end{Bmatrix} \Big/ \partial t = \pm\eta_0^{\mp 1}\frac{\partial U}{\partial z}$$

$$\partial\begin{Bmatrix} H_z \\ E_z \end{Bmatrix} \Big/ \partial t = \mp\eta_0^{\mp 1}\frac{1}{\rho}\frac{\partial(\rho U)}{\partial \rho}$$

$$\begin{Bmatrix} TE_{0n} - \text{waves} \\ TM_{0n} - \text{waves} \end{Bmatrix}$$

then (see representation (2))

$$\begin{Bmatrix} H^s_{\rho(z)} \\ E^s_{\rho(z)} \end{Bmatrix} = \sum_n u_n^{\rho(z)}(z,t)\,\mu_n^{\rho(z)}(\rho),$$

$$\begin{Bmatrix} H^i_{\rho(z)} \\ E^i_{\rho(z)} \end{Bmatrix} = \sum_n v_n^{\rho(z)}(z,t)\,\mu_n^{\rho(z)}(\rho); \qquad (3)$$

$$\begin{Bmatrix} TE_{0n} - \text{waves} \\ TM_{0n} - \text{waves} \end{Bmatrix}.$$

The space-time amplitudes $u_n(z,t)$, $v_n(z,t)$ and so on in representations (2), (3) are sometimes named as elements of evolutional bases of corresponding signals [9]. They completely determine the dynamics of propagating impulse waves on any finite sections of regular waveguides.

“ABSORBING” CONDITIONS

The exact absorbing conditions for virtual boundaries L, permitting to substitute the “open” initial-boundary value problems (1) by the equivalent “closed” ones will be constructed using the technique approved in [9,11]. Below we consider only a part of such conditions and only for a part of the virtual boundary in the plane $z=0$. The full spectrum of conditions, which can be used for correct restriction of the computational domain for solving the scalar and vector initial-boundary value problems in the theory of open waveguide resonators, is presented in details in [13]. The simplest of these conditions for the problems under consideration has the form

$$U^s(\rho,0,t) = -\sum_n \left\{ \int_0^t J_0\left[\lambda_n(t-\tau)\right] \left[\int_{\rho_1}^{\rho_2} \left.\frac{\partial U^s(\tilde{\rho},z,\tau)}{\partial z}\right|_{z=0} \mu_n(\tilde{\rho})\tilde{\rho}\, d\tilde{\rho} \right] d\tau \right\} \mu_n(\rho);$$

$$\rho_1 \le \rho \le \rho_2,\ t \ge 0. \quad (4)$$

Representation (4) is obtained by solving the problems (1) in the domain ${}_LQ$ that does not contain effective scatterers. Essentially, formula (4) gives the analytical description of the process of free propagation of outgoing impulse waves formed by the node Q_L. It is an exact near-zone analog of the radiation condition related with the finite velocity of propagation of perturbations determined by the solutions of problems (1). The presence in condition (4) of integral operators by the space (ρ) variable and the time variable allows us to refer it to the class of nonlocal conditions. For the transition from (4) to the local condition

$$U^s(\rho,0,t) = \frac{2}{\pi} \int_0^{\pi/2} \frac{\partial W(\rho,t,\varphi)}{\partial t} d\varphi;$$

$$t \ge 0,\ \rho_1 \le \rho \le \rho_2,$$

$$\begin{cases} \left[\frac{\partial^2}{\partial t^2} - \sin^2 \varphi \frac{\partial}{\partial \rho} \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \right] W(\rho,t,\varphi) = - \left. \frac{\partial U^s(\rho,z,t)}{\partial z} \right|_{z=0} ; \\ \qquad \rho_1 < \rho < \rho_2, \quad t > 0 \\ W(\rho,0,\varphi) = \left. \frac{\partial W(\rho,t,\varphi)}{\partial t} \right|_{t=0} = 0; \\ \qquad \rho_1 \le \rho \le \rho_2 \end{cases} \tag{5}$$

it is necessary to use the integral representation of the Bessel function J_0 and to perform the equivalent change of integral forms, which arise in this case, by the differential ones (see [11,13]).

In (4) and (5) for the circular waveguide it is necessary to put $\rho_1 = 0$ and $\rho_2 = b$, and for the coaxial waveguide $\rho_1 = b$ and $\rho_2 = a$. The interior initial-boundary value problems in (5) should be complemented by the following boundary conditions:

$$\begin{cases} W(0,t,\varphi) = W(b,t,\varphi) = 0 \quad (TE_{0n} - \text{waves}), \\ W(0,t,\varphi) = \left. \frac{\partial(\rho W(\rho,t,\varphi))}{\partial \rho} \right|_{\rho=b} = 0 \\ \qquad (TM_{0n} - \text{waves}) \end{cases}$$

(for domain **I** corresponding to the circular waveguide) and

$$\begin{cases} W(b,t,\varphi) = W(a,t,\varphi) = 0 \\ \qquad (TE_{0n} - \text{waves}), \\ \left. \frac{\partial(\rho W(\rho,t,\varphi))}{\partial \rho} \right|_{\rho=b} = \left. \frac{\partial(\rho W(\rho,t,\varphi))}{\partial \rho} \right|_{\rho=a} = 0 \\ \qquad (TM_{0n} - \text{waves}) \end{cases}$$

(for domain **II**, corresponding to the coaxial circular waveguide). These conditions are the direct consequence of the boundary conditions of problems (1).

Problems (1) with the nontruncated analysis domain Q and problems (1) with the truncated domain of analysis Q_L complemented by any of conditions (4) or (5) are equivalent [13]. Thus the analysis domain (the domain of discretization of original initial-boundary value problems) can be correctly narrowed to Q_L. Realization of the corresponding algorithm leads to the same simple and exact numerical solution of input problems at any instant of observation time t, as in the case of physically "closed" domains Q_L.

We should note the following important differences between nonlocal and local exact "absorbing" conditions. The first of them (see, for example (4)) require the complete information on the systems of "transverse" eigenfunctions of waveguides loaded with the open waveguide resonator. In the general case the solution of corresponding spectral problems can be very clumsy and it will need significant non-productive expenditures of machine resources. The second (local) conditions are without this disadvantage - they should have a preference in analysis of units with power take of channels, for which the "transverse" functions can be determined analytically. But even in the case when the "transverse" functions of a regular waveguide transmission line are known (the situation under consideration is this case too), the use of local conditions makes it possible to execute the computation more quickly and with much less capacities of the employed computer [13].

TESTING OF AN ALGORITHM

The algorithm of solving the problems (1), (4) and (1), (5), basing on their standard discretization by the finite difference method, is represented in the computer programs for calculation of impulse TE_{0n} and TM_{0n} waves in the axial-symmetric structures with, practically, any geometrical and material parameters. Also, there are not strict limitations on the parameters of initial signals $U^i(z,\rho,t)$. Numerical testing was carried out in the framework of a standard scheme permitting to evaluate the real error of results [9]. An exact (strict) solution of model problems has been compared with the solution obtained using exact conditions and approximated classic ABCs. Besides, testing was carried by the results obtained by the strict methods of frequency domain [2]. Below an example of such work is given.

In the book [3], in Fig.3(b), for extension of the circular waveguide with $\theta = b/a = 0.8$ and $L = l/a = 0.85$ (see Fig.2), we find two points ($\kappa_1 \approx 0.98$ and $\kappa_2 \approx 1.05$), in the range $0.6 < \kappa < 1.2$ of relative frequencies $\kappa = a/\lambda$ ($\lambda = 2\pi/k$ is the wavelength in the free space), in which the incident TM_{01} wave is all reflected by the semitransparent structure.

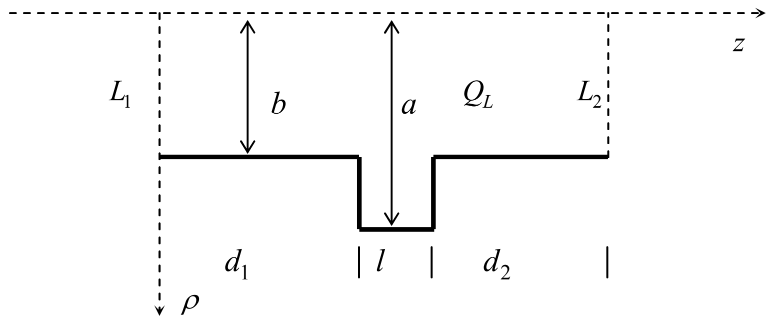


FIGURE 2. Extension of the circular waveguide $L = L_1 \cup L_2$

Let us sumilate a similar situation in the framework of the initial-boundary value problem (1), (4): the axial-symmetric nonuniformity with a = 1, b = 0.8, l = 0.85 and $d = d_1 + l + d_2$ = 0.6 + 0.86 + 0.55 = 2 from the left is irradiated with the pulse TM_{01}-wave $\{E\rho,\ H\phi,\ Ez\}$ with the space-time amplitude of the electromagnetic field $E\rho$-component (see representation (3)) equal to

$$v_1^\rho(0,t) = A\frac{\sin\left[\Delta k\left(t-\tilde{T}\right)\right]}{\left(t-\tilde{T}\right)}\cos\left[\tilde{k}\left(t-\tilde{T}\right)\right] = f(t) \qquad (6)$$

($\tilde{T} = 70$, $A = 1$, $\Delta k = 1.9$, $\tilde{k} = 5.6$). Here $\tilde{T}$ is the signal time delay, $\tilde{k}$ is its central frequency and Δk determines the effective frequency band occupied by the signal. The regular circular waveguides on the left and right of the nonuniformity are single-mode ones in the frequency range 3 < k < 6.9. The extension supports propagation of the one wave at 2.4 < k < 5.52 and of two waves at 5.52 < k < 8.65. The function $v_\rho^1(0,t)$ and moduli $\tilde{v}_\rho^1(0,k)$ of its corresponding amplitudes

$$\tilde{v}_1^\rho(0,k) = \frac{1}{2\pi}\int_0^T v_1^\rho(0,t)\mathrm{e}^{ikt}\,dt \ \leftrightarrow\ v_1^\rho(0,t)$$

(image ↔ original; T is the upper limit in the interval of observation time t) in the frequency range 2 < k < 9 are presented in Fig.3.

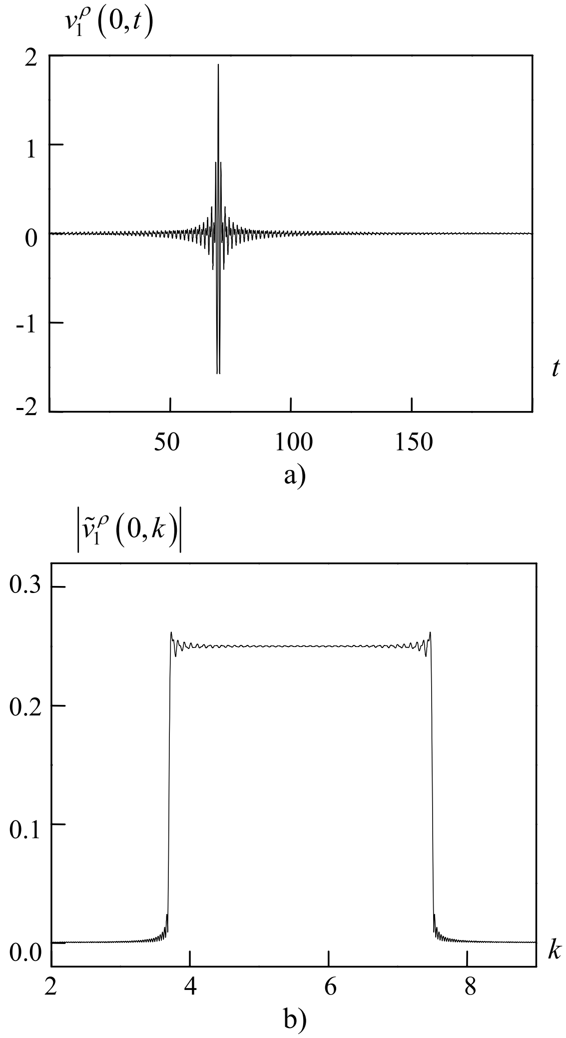
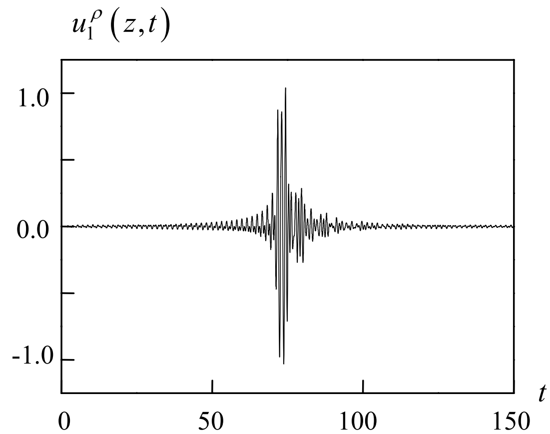
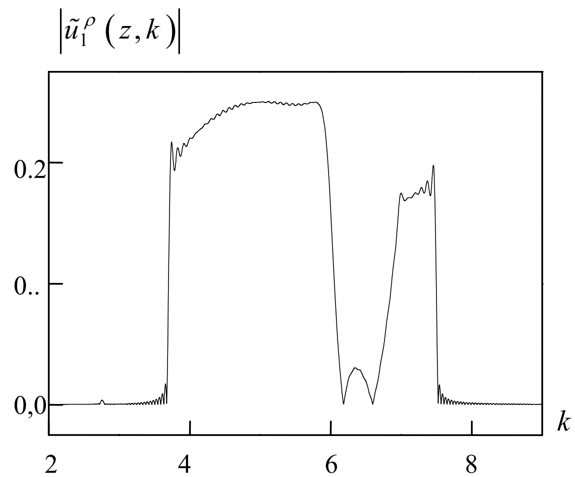


FIGURE3. The time (a) and spectral (b) amplitudes of the *Eρ*- field component of TM_{01}- wave of excitation $U^i(z,\rho,t)$ at the boundary L_1 in the plane z = 0.

In Fig.4 are shown the spatial-time and spectral amplitudes of the transmitted pulse wave. The total reflection occurs at frequencies $k_1 \approx 6.18$ and $k_2 \approx 6.6$, corresponding to the regime of the second mode, closed in the extension, or to the regime $\{N, M, P\}$ with $N = P = 1$ and $M = 2$; N and P is the number of waves propagating in the left and the right waveguide; M is the number of waves propagating in the extension. The experimental values of k_1 and k_2 correspond to the values of κ_1 and κ_2 of relative frequencies given in the book [3].



a)



b)

FIGURE 4. The time (a) and spectral (b) amplitudes of the transmitted impulse TM_{01}-wave (E_ρ-component) at the boundary L_2 in the plane $z = 2$

MODE CONVERSATION OF PULSE WAVES

Below we give the results of the study on the synthesis of a transmission resonator (see Fig.5) transforming the TE_{02}-waves $\{H\rho, E\phi, Hz\}$ of the circular waveguide in TE_{01}-waves. The left (L_1) and the right (L_2) parts of the virtual boundary $L = L_1 \bigcup L_2$ in the computation domain Q_L are situated in the planes z=0 and z = 0.8. N.P. Yashina and V.P. Tkachenko solved the synthesis problem using the frequency domain methods. We are basing on their results by simulating the corresponding conditions in the time domain.

In the first experiment the structure is excited by the impulse TE_{02}-wave with the space-time amplitude $v_\rho^2(0,\ t) = f(t)$ (see formula (6); $\tilde{T} = 10$, $A = 1$, $\Delta k = 6$, $\tilde{k} = 66$).

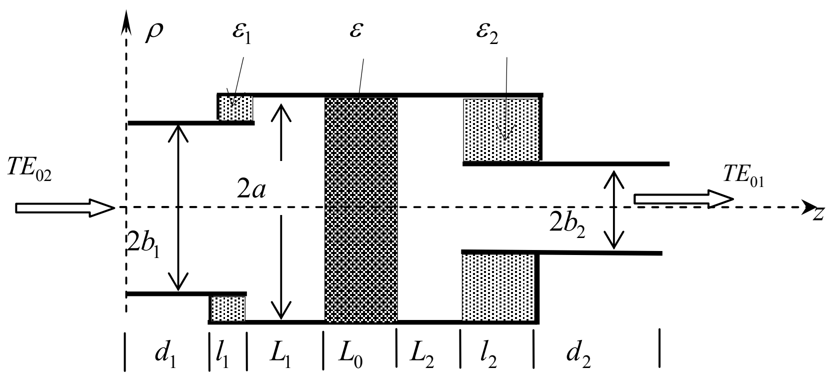


FIGURE 5. Geometry of the transmission resonator a = 0.160; b_1 = 0.121; b_2 = 0.078; l_1 = 0.012; l_2 = 0.082; L_1 = 0.022; L_2 = 0.029; L_0 = 0.177; d_1 = 0.178; d_2 = 0.30; ε_1 = 1.07; ε = 1.57; ε_2 = 1.52

In Fig.6 presented are the spectral amplitudes of the excitation signal and impilse waves propagating in the left and right waveguides. It is seen that in the band 66.5 < k < 68 (mode $\{N, M, P\}$ with N = 2, P = 1 and M = 3) the TE_{02}-wave of the left waveguide transforms into the TE_{01}-wave of the right waveguide almost fully.

In Fig.7 one can see the spatial distribution of the complete electric field strength (E_ϕ-field component) at the instant of observation time t = 3.5 in the case of quasimonochromatic excitation of the transformer by the TE_{02}-wave with the amplitude $v_\rho^2(0,\ t) = \sin(\widetilde{k}\ t)\ \chi\ (10 - t)$. Here χ is the Heavisade step function and $\widetilde{k}$ = 67.6 is the central part of the signal. A rather wide band of the almost full transformation is formed due to the excitation in the transmission resonator of quasi-eigen modes. To them corresponding are near eigen frequencies and $\bar{k}_1$ and $\bar{k}_2$ with $\mathrm{Re}\,\bar{k}_1 \approx 66.075$ and $\mathrm{Re}\,\bar{k}_2 \approx 68.64$.

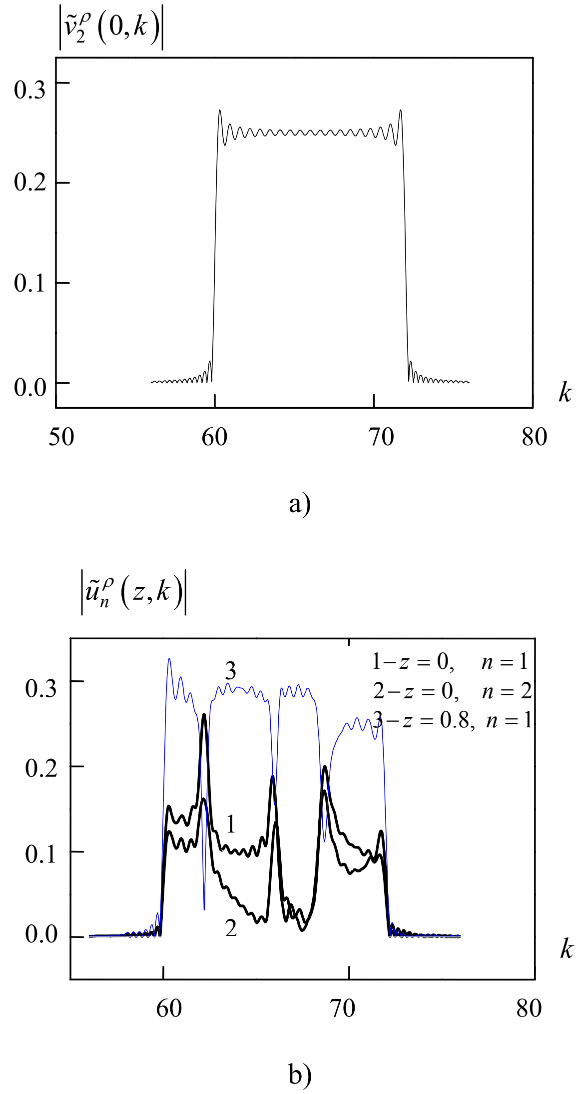


FIGURE 6. Spectral amplitudes of the excitation angle (a) and propagating pulse modes in the left (z = 0) and right (z = 0.8) waveguides (b)

We obtain the characteristics of "natural" resonances by the methods developed in [9,14]; the quantities $\operatorname{Re}\bar{k}_1$ and $\operatorname{Re}\bar{k}_2$ are determined by the behavior of the spectral amplitudes $\tilde{H}_\rho(g,k) \leftrightarrow H_\rho(g,t)$ in the series of points $g \in Q_L$ (see Fig.8), and the configuration of free oscillation fields - by the "long-lived" constituents in the structure response to the supernarrow-band (with the central frequency $\tilde{k} = \mathrm{Re}\,\bar{k}_1$ or $\tilde{k} = \mathrm{Re}\,\bar{k}_2$) signal of a finite duration. One can judge on the power of the transmission resonator to accumulate the energy in the mode of quasimonochromatic excitation at frequencies close to their eigen ones by the time dependencies $H_\rho\,(g,\ t)$ or $H_z\,(g,\ t)$ taken off in the points g of the domain Q, coinciding (or almost coinciding) with the local maxima in the spatial distribution of the strength of free oscillation fields (see Fig.9).

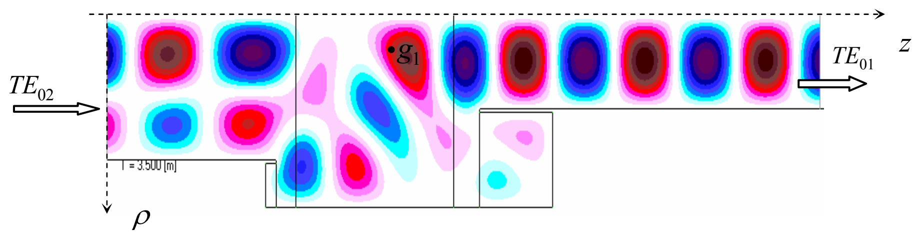


FIGURE 7. Spatial distribution of the electric field strength in the domain Q_L in the case of quasimonochromatic excitation of the transmission resonator by the pulse TE_{02}-wave*:* $t = 3.5$

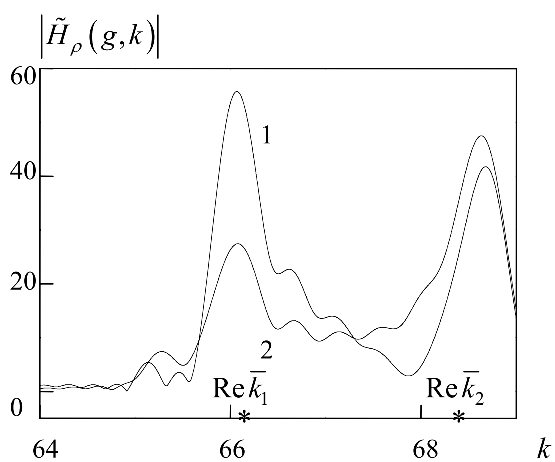
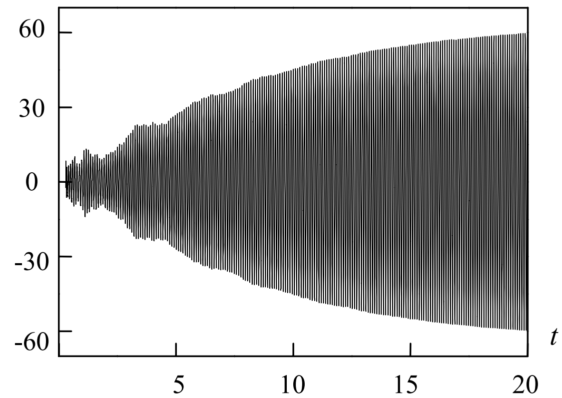
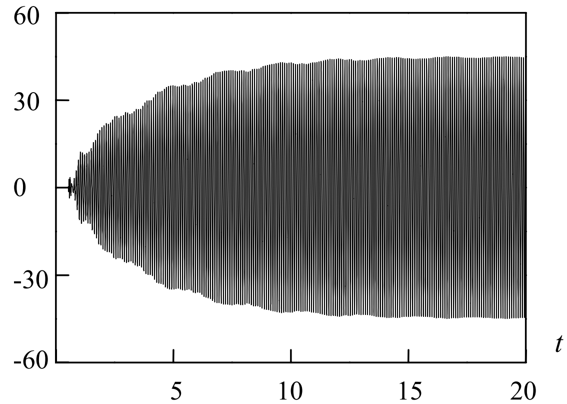


FIGURE 8. Spectral amplitudes $\tilde{H}_\rho(g,k) \leftrightarrow H_\rho(g,t)$ of the pulsed field generated by the source with $v_2^\rho(0,t) = f(t)$ ($\tilde{T} = 10$, $A = 4$; $\Delta k = 2$, $\tilde{k} = 67$) in the points g_1 = {0.035, 0.29} (curve 2) of the domain Q_L.



a)



b)

FIGURE 9. H_ρ (g, t)-component of the complete filed in the points g_1 (a) and g_2 (b) of the domain Q_L in the case of excitation of the transmission resonator by the quasimonochromatic signal with $v_2^\rho(0,t) = \sin(\tilde{k}t)\chi(20-t)$; $\tilde{k}_1 = 66.075$ (a) and $\tilde{k}_2 = 68.64$ (b).

APPLIED PROBLEMS

Statement and solving of initial-boundary problems (1) were stimulated, first of all, by the actual needs in reliable model processing of a number of functional units of powerful superwide-band electromagnetic pulse radiators. This topic will receive full coverage in the following papers. Here we give only one example demonstrating the potentialities of the developed algorithm and concerning the electrodynamic analysis of the sharpening and cut-off spark section of a picosecond pulsed oscillator operating in the “cold’ (spakless) regime. Figure 10 presents the spatial distribution of the complete electric filed

(E_ρ-component) distribution in the domain Q_L at the instant of observation time t = 0.48 in the case of the section fragment excitation (proportions are conserved) by the videopulse *TEM*-wave. In Fig.11 shown are the space-time and spectral amplitudes of the $E\rho$-components of the videopulse *TEM*-wave incoming at the boundary L_1 and of the *TEM*-wave outgoing from the boundary $L2$ in the direction of z increasing. The parts L_1 and L_2 of the virtual boundary L of the analysis domain Q lie in he planes z = 0 and z = 0.28 (see Fig.10).

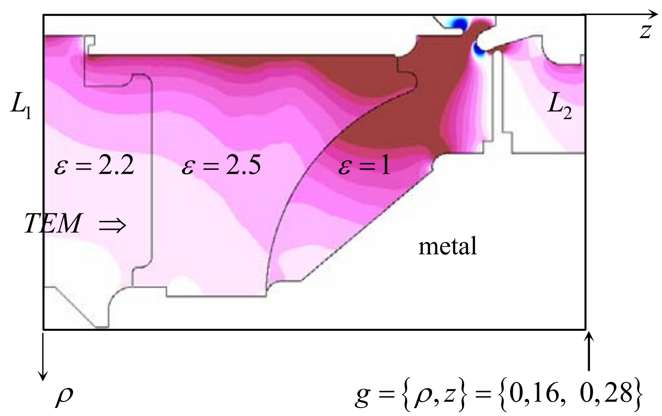


FIGURE 10. Toward the electrodynamical analysis of a fragment of the sharpening and cut-off sections for high-power superwide-band systems.

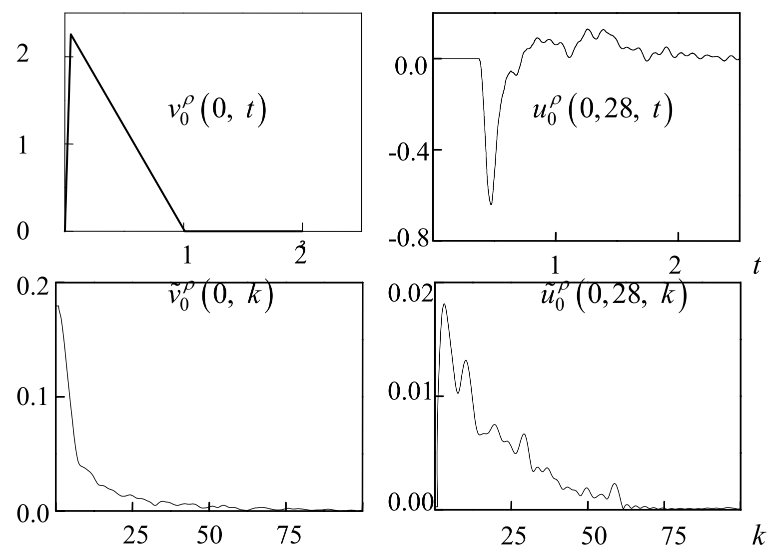


FIGURE 11. Space-time and spectral amplitudes of $E\rho$-components of the incoming (a) and outgoing (b) *TEM*-waves

CONCLUSIONS

The computational experiments, we have carried out, confirmed the effectiveness and reliability of the defined solution of model initial-boundary value problems (1). The exact “absorbing” conditions considerably reduce the space (and, consequently, the time) of the computation in comparison with other approaches making it possible to reach a required accuracy. They do not increase the error of the standard finite-difference approximation and do not sophisticate the physics of simulated processes as in the case when known classic conditions are used. It is important that this solution involves a very wide-ranging set of problems and can be used for the study of many electromagnetic wave scattering modes being of interest for theory and practice.

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