COMPARISON OF RESONANCE OPTICAL SCATTERING OF PLANE WAVES BY INFINITE GRATINGS OF SILVER CYLINDERS AND STRIPS

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Abstract – **Periodic structures possess a specific feature, there is a set of high quality resonances near to the wavelength equal to the grating period and their Q-factors rise higher if the grating becomes sparser. Making a grating from silver elements brings other resonances – surface-plasmon ones appearing in the visible-light band. We investigate interaction between the grating and the plasmon resonances in the scattering and absorption by infinite gratings of silver circular cylinders and silver thin strips, for the E and H-polarization cases.**

I. INTRODUCTION

We consider the two-dimensional scattering of normally incident plane waves by two types of infinite gratings consisting of (i) circular silver cylinders and (ii) thin silver strips located in free space. The geometries of the problems are illustrated in Fig.1. Infinite number of cylinders or strips, parallel to the z-axis, are located in the plane $x = 0$ with period d. The radius of each cylinder is equal to a while each strip has the width $2w$ and thickness h.

The scattering problem for a wire grating is solved using accurate method of partial separation of variables. We seek the scattered field as a solution of the Helmholtz equation and Sveshnikov condition of radiation plus the continuity of the tangential field components on the cylinders contours. Based on the Floquet theorem, the sought solution is a periodic function and calculations are reduced to one period. Firstly, we expand the normally incident plane wave and the scattered and internal fields in terms of the Fourier series in the azimuth variable [1], with the Hankel or Bessel functions of the radial coordinate in coefficients. To take into account all cylinders of the grating, the scattered field contains an infinite sum of the Hankel functions that converges slowly. Here, the lattice sums help dramatically accelerate the speed of their convergence [2]. The matching of the fields on the contour of cylinder brings us to a matrix equation with unknowns as the coefficients of the scattered-field expansion. After required normalization, this matrix equation becomes the Fredholm second kind equation. Since expansions in the Hankel series are not convenient, we follow [1] and rewrite them in terms of the Floquet harmonics whose amplitudes allow us to obtain the reflectance, transmittance, and absorbance. Details of the derivations can be found in [3].

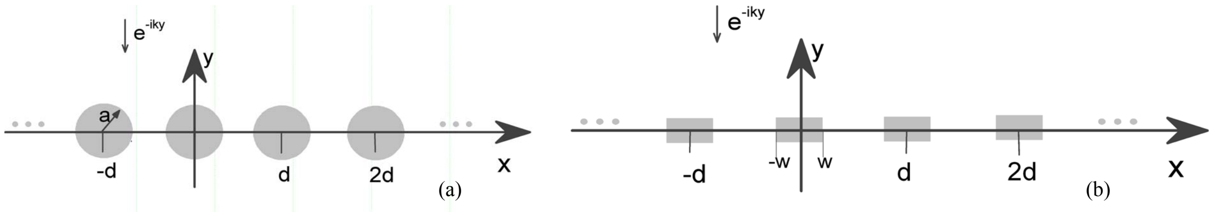

Fig. 1. Geometries of wire-grating and strip-grating scattering

For the solution related to the thin-strip grating made of silver we impose the generalized boundary conditions (GBC) in the form presented in [4] on the strip median lines. GBC have been already used when modeling the properties of the gratings made of thin ($k_0 h << 1$) resistive, dielectric, and impedance strips [5-8]. GBC relates limiting values of the fields across the strip to effective electric and magnetic currents. The complex parameters R and Q in GBC have dimensions of the surface resistance and the surface conductivity, respectively, and are called the electric and magnetic resistivities. In the case of grating made of silver, electric and magnetic resistivities of the strips are expressed trough the parameters of high-contrast ($|\varepsilon|=|\nu^2|>>1$) layer with the refractive index ν as

$$R = -(1/2)i\nu^{-1}\mathrm{ctg}(\nu kh/2), \qquad Q = -(1/2)i\nu\,\mathrm{ctg}(\nu kh/2)\,, \tag{1}$$

where $k = 2\pi/\lambda$ is the wavenumber in free space, λ is the wavelength and $\varepsilon(\lambda)$ is the dielectric function of silver.

To determine the unknown amplitudes of the Floquet spatial harmonics a_n and b_n of the scattered field in the transmission and reflection half-space, respectively, we use the set of dual boundary conditions formed by the GBC at the strip and the total field continuity at the slot. From them, we obtain two decoupled systems of the dual series equations (DSEs) for the unknown values $x_n^+ = a_n + b_n$ and $x_n^- = a_n - b_n$ for each polarization. The static part of each of these systems can be inverted using either the Riemann-Hilbert Problem solution (RHP) [9] or the inverse Fourier transformation (IFT) according to the form of left-hand side of DSE [6]. As a result, this yields two decoupled infinite matrix equations of the Fredholm 2-nd kind, $\sum_{m=-\infty}^{+\infty}(\delta_{mn} + A^{\pm}_{E(H),mn})x^{\pm}_m = B^{\pm}_{E(H),m}$, where δ_{mn} is Kronecker's delta [5]. This guarantees fast convergence and controlled accuracy of computations.

The values for refractive index of silver in the visible wavelength range have been taken from [10] and interpolated using cubic splines.

II. Results

Fig. 2 shows the dependences of the reflectance, transmittance and absorbance of a wire-grating and a strip-grating made of silver with p = 800 nm on the wavelength. The cylinder radius is $a = 48.86\,\mathrm{nm}$ and the width of the strip is 150 nm and its thickness is 50 nm. These parameters are chosen to keep the cross-sectional area of each element equal the same value for both gratings. We can notice two kinds of resonances: grating-type resonance (G) and plasmon resonance (P). The grating-type resonances appear close to the wavelengths equal to the grating period or its half, where the first and the second order harmonics appear at the normal incidence. A very sharp G-resonance close to ±1 Rayleigh anomaly is located distantly from the P resonance and thus does not experience any influence from it. The second G-resonance near to ±2 Rayleigh anomaly splits the broad peak of P-resonance. For a single sub-wavelength circular silver wire, the plasmon peak lies around the wavelength $\lambda \approx 340$ nm [11]. A closer study of the P-resonance brings us to the characteristic equations of a sub-wavelength cylinder in the H-case,

$$F_m(ka,\nu) = \nu^{-1}H_m(ka)J'_m(\nu ka) - H'_m(ka)J_m(\nu ka) = 0 \tag{2}$$

or, if $ka \ll 1$,

$$F_0(ka,\nu) \sim -\frac{2i}{\pi ka}, \qquad F_1(ka,\nu) \sim -i\frac{1+\varepsilon}{\pi\sqrt{\varepsilon}ka}, \qquad F_m(ka,\nu) \sim -i\varepsilon^{m/2}\frac{1+\varepsilon}{\pi\varepsilon ka}\; m \geq 2 \tag{3}$$

The left-hand parts of these asymptotic equations contain the $(1+\varepsilon)$ factor for all $m \geq 1$. Thus the P-peak is caused by the fact that the dielectric function of silver near the mentioned wavelength has the value $\mathrm{Re}\,\varepsilon(\lambda) \approx -1$. Although the terms with $|m| = 1$ have the largest impact, the higher terms also have their plasmon zeros. As all of them lie in some vicinity of $\lambda \approx 340$ nm they merge together in one broad P-resonance because at that wavelength the silver is lossy, $\mathrm{Im}\,\varepsilon(\lambda) \approx 0.31$. The strip scattering analysis needs more elaborated techniques such as volume or boundary integral equations [11]. It has been discovered that the first P-resonance for a silver strip of the $150 \times 50\,\mathrm{nm}^2$ cross-section has wavelength near $\lambda \approx 410$ nm at the normal incidence. Although the plasmon resonances of different geometries have different positions they behave similarly. Fig. 2 (a) shows that the appearance of the second harmonic in the grating resonance at $\lambda \approx 400$ nm pierces the plasmon resonance of the strip grating with a narrow-band induced transparency however displays a double-extremum Fano shape in the case of the grating of cylinders.

More detailed information about the G-resonances origin and behavior can be found in [3] by the example of dielectric and silver wire gratings. In the E-polarization, for both kinds of gratings the reflectance is suppressed at the wavelengths of both ±1 and ±2 Rayleigh harmonics.

Fig. 3 clearly demonstrates the presence of the P-resonances in the H-polarization scattering for both gratings. To discover their impact on the G-resonances we have studied the dependences of reflectance, transmittance and absorbance of the H-polarized plane wave by the infinite cylinder or strip gratings with different periods but the same as above geometrical parameters. The gratings have been investigated for the six period values, $d = 350, 400, 500, 600, 700, 800$ nm . Fig. 3 shows that for the last three values of the period the behaviour of the reflectance in the range of the P-resonance is almost identical. Slight increase of the reflectance for smaller periods can be explained by the fact that these gratings are denser and, consequently, tend to show larger reflectance. Opposite behaviour is seen for the transmittance in Fig. 3 (c). However, the grating with $d = 500$ nm gives us an example of significant increase of the reflectance at the P-resonance due to its

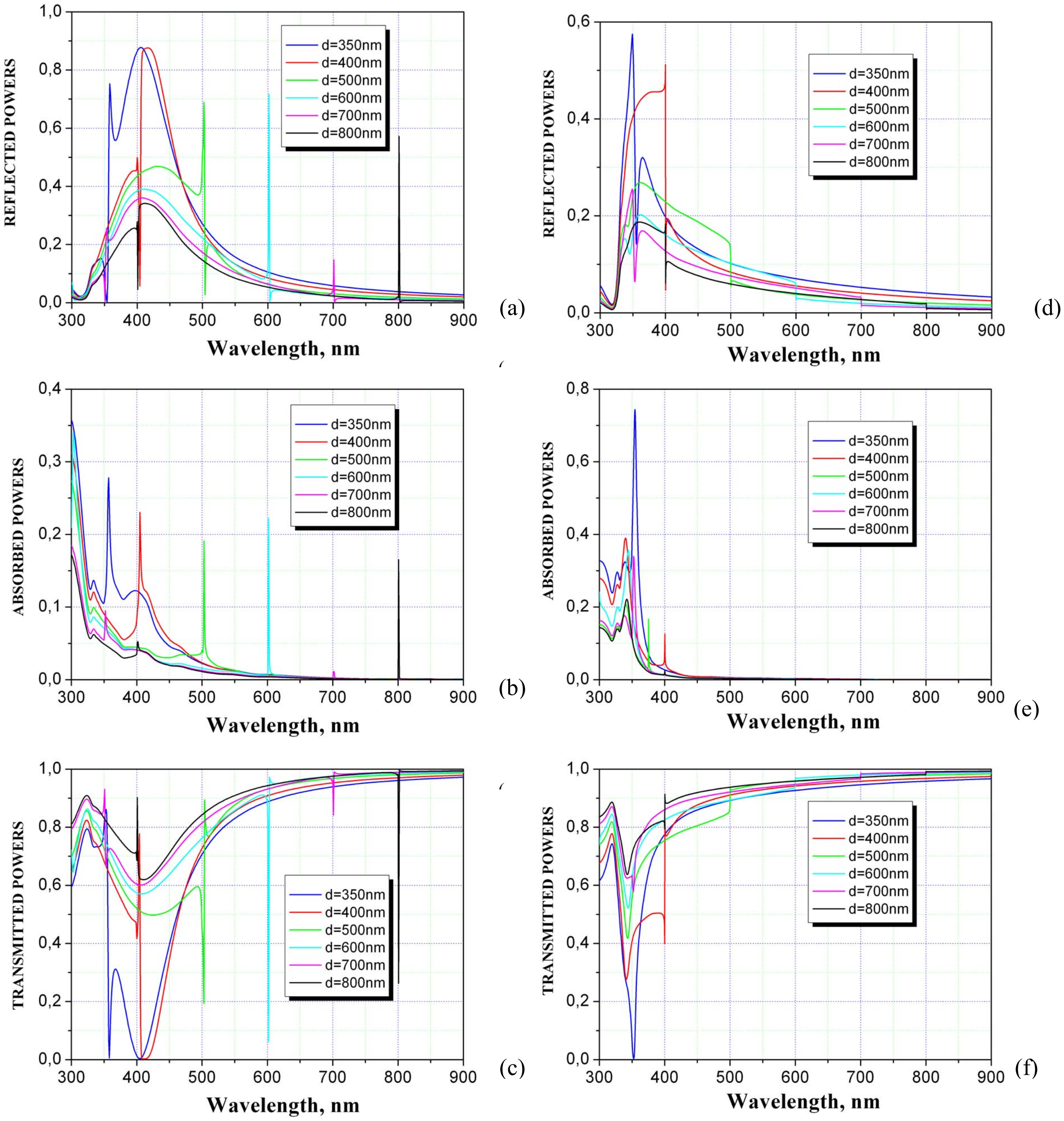

Fig. 3 Reflectance, transmittance, and absorbance of grating of different periods of thin strips (a),(b),(c) with dimensions $50\times150\,\text{nm}^2$ and silver cylinders with radius $a = 48.85$ nm (d),(e),(f). H-polarization.

interaction with the grating resonance. For the periods of $d = 350$ nm and $d = 400$ nm we observe much larger increase of the reflected powers that cannot be explained by the density of the grating. As we suggest the main reason of this is the plasmon resonance gets power from the grating one. The absorbance for this case for any value of the period behaves almost in the same manner, see Fig. 3 (b).

The electromagnetic properties of the grating made of circular cylinders are similar to the above in the coexistence of two kinds of resonances. The reflectance of sparse gratings Fig. 3 (d),(e),(f) in the visible wavelengths shows a broad plasmon peak pierced by the grating resonances. However for a denser grating where the G-resonance overlaps with the plasmon peak, the latter is enhanced. The transmittance shows the behaviour opposite to the reflectance. The absorbance behaves roughly similar for all values of the period.

III. Conclusion

We have studied two kinds of silver-element gratings suspended in free space with equal area of the element cross-section, one made from circular cylinders and the other from thin strips. Both scattering problems have been analyzed using accurate mathematical approaches. We have discovered that although the geometries of the gratings are different, their electromagnetic properties have similar features. For instance, the G-resonances appear and behave regardless of the structure of the grating elements. It has been shown that in the H-polarization case the effect of the grating resonance on the plasmon resonances can be both positive (reflectance enhancement) and negative (reflectance inhibition). In the E-polarization case, where no P-resonances exist, the effect of periodicity is displayed as suppression of reflectance at the wavelengths of Rayleigh anomalies.

Acknowledgement

This work was supported, in part, by the National Academy of Sciences of Ukraine (NASU) via the State Target Program “Nanotechnologies and Nanomaterials” and by the European Science Foundation via the Networking Programme “Newfocus.”

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