# Plasmon-Assisted Scattering of Light by a Circular Silver Nanowire with Concentric Dielectric Coating

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# ABSTRACT

We study the scattering and absorption of an H-polarized plane electromagnetic wave by a silver nanowire, in the visible range of wavelengths. The wire is assumed to have circular cross-section and coated with a concentric circular dielectric layer. The analytical solution of the wave-scattering problem is obtained in classical manner, using the separation of variables and the transfer-matrix method in polar coordinates. In computations, we use the accurate data of Johnson and Christy for the complex refractive index of silver. If the coating thickness and its dielectric permittivity become larger, the computed spectra of the total scattering cross-section (TSCS) and the absorption cross-section (ACS) display, at first, a shift of broad surface-plasmon resonances to the red side and then their hybridization with the modes of the coating. This data can be useful for the design of nanowire-based sensors where the analyzed substance makes a finite-thickness layer on the wire.

Keywords: silver wire, scattering cross-section, absorption cross-section, dielectric coating, near field, localized surface plasmon resonance, coating resonance.

# **1. INTRODUCTION**

Gold and silver nanowires are widely used in today's nanotechnologies as building blocks of biosensors, lasers, and photovoltaic devices. This is because, if illuminated by the orthogonally polarized visible light, they display bright resonances accompanied by the enhanced scattering and absorption. These effects are caused by the collective oscillations of electron gas and are called localized surface plasmon resonances (plasmons, for brevity) [1,2]. Here, the circular cross-section is the simplest albeit important shape, which enables one to obtain full understanding of all wave phenomena [3-5]. For example, good approximation for the wavelength  $\lambda_P$  of the plasmon resonance on a circular metal wire is found from the condition  $\text{Re } \varepsilon_{met}(\lambda_P) \approx -\text{Re } \varepsilon$ , where  $\varepsilon_{met}$  and  $\varepsilon$  are the dielectric permittivities of the metal and the infinite outer host medium, respectively ( $\text{Re } \varepsilon > 0$ ). Such wavelength is specific for every metal but weakly depends on the wire radius insofar as it is considerably smaller than the wavelength – see Fig. 1. This property serves as physical basis for the "sensing" of the host medium.

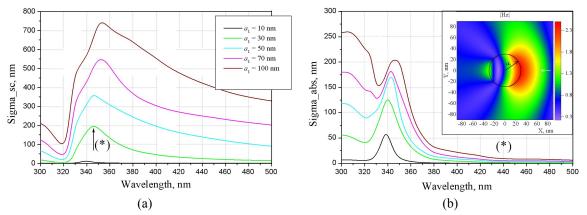


Figure 1. TSCSs (a) and ACSs (b) as a function of the wavelength for bare Ag wire of radius  $a_1 = 10$  to 100 nm in free space, the case of normal incidence. Insert: near field, a = 30 nm  $\lambda = 346$  nm,  $\varepsilon_{Ag} = -1.5$ -i $\cdot 0.31$ .

However in certain practical applications the material that is analyzed by the sensor forms not a thick background layer but only a finite-thickness coating around each wire of a sensor. Then, the plasmon resonance is still present however its wavelength is a function of two parameters – the coating permittivity  $\varepsilon$  and its thickness *h*. This makes the task of determining  $\varepsilon$  more difficult and needs more detail analysis.

## 2. SHIFT OF PLASMON RESONANCE DUE TO A SMALL-CONTRAST COATING

We consider an H-polarized plane wave normally incident on a circular silver wire of the radius *a* coated with a concentric circular layer of dielectric with the thickness *h* and dielectric permittivity  $\varepsilon$ . The electromagnetic field in the presence of such a scatterer must satisfy the Helmholtz equation, the tangential components continuity conditions, and the condition of the local power finiteness; its scattered-filed part must also satisfy the radiation condition at infinity. Using the separation of variables and the boundary conditions on two circular contours, the field is found analytically in the form of infinite Fourier series with the cylindrical functions in coefficients. This procedure was first derived by Rayleigh in 1881 [6].

In Fig. 2, we present the visible-range wavelength dependences of TSCS and ACS computed for a silver cylinder of a = 30 nm coated with rather small-contrast dielectric layer of  $\varepsilon = 2.25$  and several values of thickness *h*. In computations, we have used the bulk refractive index of silver taken from the paper [7].

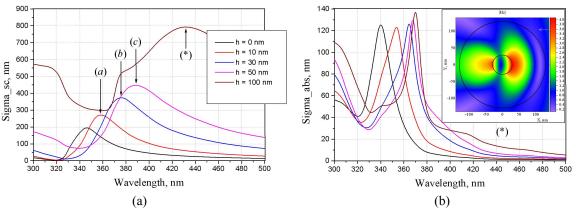


Figure 2. TSCSs (a) and ACSs (b) versus the wavelength, for Ag wire of radius a = 30 nm coated with a concentric layer of dielectric of  $\varepsilon = 2.25$  and thickness h. Insert: near field, h = 100 nm,  $\lambda = 432$  nm,  $\varepsilon_{Ag} = -6.09 - i \cdot 0.2$ .

As one can see, the fundamental plasmon resonance  $P_1$  is red-shifted in this configuration, as expected. What is not expected is that if a > 10 nm, this shift is larger than the value of  $\lambda = 359$  nm predicted for a subwavelength silver cylinder placed into the homogeneous medium of  $\varepsilon = 2.25$ . Besides, if the coating is as thick as 100 nm, the other resonance appears near 430 nm – this is the first resonance  $H_{11}$  of the coating as suggested by the fact that it is visible only in the scattering but not in the absorption. Visualization of the near field patterns in the mentioned resonances is presented in Fig. 3. Here, we show the magnitude of the total magnetic field near and inside the wire and the coating. The patterns demonstrate that a thin coating of low-contrast dielectric material has little effect on the resonance field dominated by a dipole-like contribution. If the thickness of coating reaches 50 nm, the bright spots of the near field get into the coating entirely. To understand the absorption, one should also keep in mind that the electric field has the magnitude reaching maxima where the magnetic field has minima. The near field of the coating resonance (see inset in Fig. 2b) shows bright spots inside the coating and also at the silver wire boundary. This points out to the hybridization of the modes of coated wire as an open resonator composed of two distinctively different parts – thus these modes are in fact supermodes in the same sense as it was described in [8] for a circular dielectric microcavity with an external concentric ring.

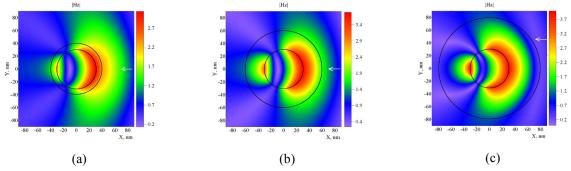


Figure 3. Near field, coated wire illuminated by a plane wave: a = 30 nm,  $\varepsilon = 2.25$ , (a)  $\lambda = 358 \text{ nm}$ ,  $\varepsilon_{Ag} = -2.22 \text{-i} \cdot 0.27$ , h = 10 nm, (b)  $\lambda = 376 \text{ nm}$ ,  $\varepsilon_{Ag} = -3.18 \text{-i} \cdot 0.2$ , h = 30 nm, (c)  $\lambda = 367 \text{ nm}$ ,  $\varepsilon_{Ag} = -2.73 \text{-i} \cdot 0.23$ , h = 50 nm.

#### 3. HYBRIDIZATION OF RESONANCES DUE TO A LARGE-CONTRAST COATING

An optically denser coating makes hybridization of modes and formation of supermodes well visible even for coatings as thin as 10 nm (see Fig. 4). Here, the ACS spectra serve as a much finer instrument for the detection of various modes of coated wire that the TSCS spectra, as follows from the curves in (a) and (b) panels.

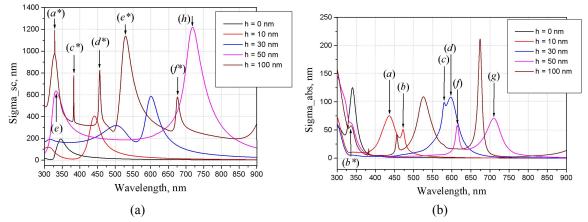


Figure 4. TSCSs (a) and ACSs (b) as a function of the wavelength, for an Ag wire of radius a = 30 nm coated with a dielectric layer with  $\varepsilon = 12$  and thickness h = 10 nm to 100 nm.

Thus, even a 30-nm thin coating shifts the plasmon resonance far beyond the  $\approx 530$  nm value where Re  $\varepsilon_{Ag} = -12$  predicted by the infinite-host-medium assumption. Even more interesting is that such a coating brings the first of the coating modes (of the dipole type) into a close proximity of the plasmon dipole mode. Although not visible in the scattering, this hybridization is revealed by the double peak in absorption around  $\lambda = 590$  nm. The near field of such a supermode is clearly seen in Figs. 5c and 5d.

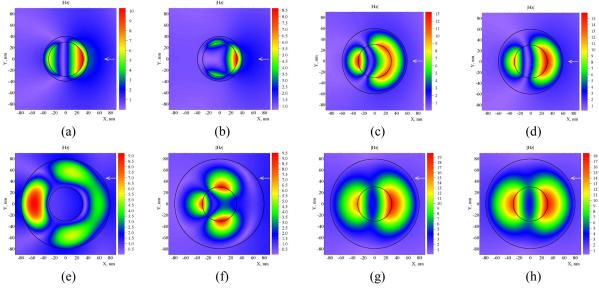


Figure 5. Near field, coated wire: a = 30 nm,  $\varepsilon = 12$ ,  $(a) \lambda = 436 \text{ nm}$ ,  $\varepsilon_{Ag} = -6.35 \cdot i \cdot 0.2$ , h = 10 nm,  $(b) \lambda = 472 \text{ nm}$ ,  $\varepsilon_{Ag} = -8.29 \cdot i \cdot 0.29$ , h = 10 nm,  $(c) \lambda = 580 \text{ nm}$ ,  $\varepsilon_{Ag} = -14.75 \cdot i \cdot 0.39$ , h = 30 nm,  $(d) \lambda = 597 \text{ nm}$ ,  $\varepsilon_{Ag} = -15.89 \cdot i \cdot 0.44$ , h = 30 nm,  $(e) \lambda = 332 \text{ nm}$ ,  $\varepsilon_{Ag} = -0.69 \cdot i \cdot 0.28$ , h = 50 nm,  $(f) \lambda = 614 \text{ nm}$ ,  $\varepsilon_{Ag} = -17.07 \cdot i \cdot 0.5$ , h = 50 nm,  $(g) \lambda = 710 \text{ nm}$ ,  $\varepsilon_{Ag} = -23.84 \cdot i \cdot 0.38$ , h = 50 nm,  $(h) \lambda = 718 \text{ nm}$ ,  $\varepsilon_{Ag} = -24.46 \cdot i \cdot 0.37$ , h = 50 nm.

If the high-contrast coating gets even thicker, say 100 nm then such a scatterer displays properties of a circular microcavity with a small plasmonic inclusion. As a result, the visible range of wavelengths becomes inhabited by a considerable number of natural modes, with associated peaks in the TSCS and ACS spectra (see brown curves in Figs. 4a and 4b, respectively). The in-resonance patterns shown in Figs. 6 reveal that some of the modes have clear plasmon-type near fields with bright spots that stick to the silver wire boundary like in panel (f\*) where the quadrupole plasmon mode  $P_2$  can be identified. It is shining in the red at 676 nm however 30% of the extinction at that wavelength goes to the absorption. The other modes in the violet part of the visible range show all features characteristic to the whispering-gallery modes  $H_{31}$ ,  $H_{41}$  and  $H_{51}$  of a dielectric cavity – see panels (d\*), (c\*) and (a\*). They shine brightly in the scattering spectrum with very little absorption because their

modal fields have deep minima in the center where the lossy silver wire is located. Still other pattern is displayed in the panel (e<sup>\*</sup>): here one can see a hybrid pattern of the supermode formed by the two optically coupled modes: the coating mode  $H_{21}$  and the plasmon mode  $P_1$  of the silver wire in the medium with  $\varepsilon = 12$ . Note also that the high-quality whispering-gallery mode  $H_{51}$  shows a narrow scattering spike standing on top of the broad peak, which is also present in the absorption and can be associated with higher-order coating mode  $H_{12}$ . This is certified by the near-field pattern in the corresponding maximum of ACS shown in panel (b<sup>\*</sup>).

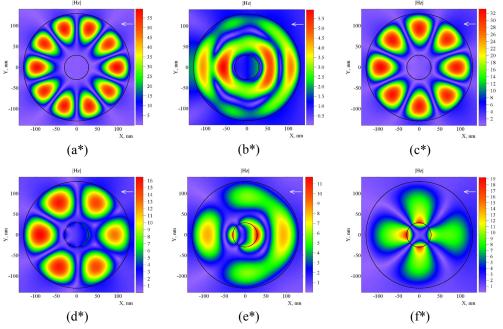


Figure 6. Near field, coated wire: a = 30 nm,  $\varepsilon = 12$ , h = 100 nm,  $(a^*) \lambda = 329.2 \text{ nm}$ ,  $\varepsilon_{Ag} = -0.47 \text{-}i \cdot 0.3$ ,  $(b^*) \lambda = 336 \text{ nm}$ ,  $\varepsilon_{Ag} = -0.932 \text{-}i \cdot 0.3$ ,  $(c^*) \lambda = 382.7 \text{ nm}$ ,  $\varepsilon_{Ag} = -3.54 \text{-}i \cdot 0.19$ ,  $(d^*) \lambda = 456 \text{ nm}$ ,  $\varepsilon_{Ag} = -7.36 \text{-}i \cdot 0.23$ ,  $(e^*) \lambda = 529 \text{ nm}$ ,  $\varepsilon_{Ag} = -11.57 \text{-}i \cdot 0.35$ ,  $(f^*) \lambda = 676 \text{ nm}$ ,  $\varepsilon_{Ag} = -21.26 \text{-}i \cdot 0.43$ .

# 4. CONCLUSIONS

The analysis of the scattering and absorption of light by a concentrically coated silver wire has been done using accurate data for the silver refractive index from Johnson's and Christy's paper. It has revealed that the wavelength shift of the fundamental dipole-type plasmon resonance strongly depends on the thickness and the permittivity of the coating. Besides of this, thicker and optically denser lossless coatings bring additional resonances to the visible range which display high-quality scattering peaks however little in-resonance absorption. Variation in the coating thickness results in the tuning of the overall spectral lineshape, which opens a novel direction for the engineering of the plasmonic response of sub-wavelength metal wires.

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