Plane Wave Scattering and Absorption by Flat Gratings of Impedance Strips

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***Abstract*—The problem of the plane wave scattering by a flat impedance-strip grating located in free space is considered. The formulation involves a set of generalized boundary conditions relating limiting values of the fields to effective electric and magnetic currents. We develop an accurate numerical solution to this problem using the dual series equations and the method of analytical regularization (MAR) based on the main part inversion. This guarantees a fast convergence and controlled accuracy of computations. Reflected, transmitted, and absorbed power fractions as a function of the frequency and the grating parameters are analyzed. Sharp resonances are revealed in the H-polarized scattering near to the grazing of the higher-order modes; these resonances are absent for the perfect electrically conducting strip grating. Low-frequency asymptotics for the reflectance and transmittance in the single-mode regime are presented and compared with numerical solutions.**

***Index Terms*—Absorption, grating, impedance strip, scattering.**

I. Introduction

METALLIC gratings in the form of flat periodic arrays of thin strips are used today across a wide range of electromagnetic field frequencies and applications. This is due to their high polarization selectivity, first discovered by Hertz in 1889 for a wire grid [1], if the period is smaller than the wavelength. Such gratings can be identified as key elements of polarization dividers, polarization attenuators, polarization converters, diffractometers and interferometers operating in millimeter (mm) and sub-millimeter (sub-mm) wave regions. Besides, at the lower frequencies printed resistive strip gratings offer an attractive design of flat microwave absorbers. In optics, silver and gold strip gratings are used in the substrates for the surface-enhanced Raman spectroscopy, and in the vertical-cavity semiconductor lasers for the mode selection.

Therefore, it is not a surprise that such gratings have been attracting attention from theoreticians since the time of the pioneering paper of Lamb [2] who had demystified the Hertz effect. Several techniques have been reported for building the numerical solutions to the scattering by the gratings of the perfect electric conducting (PEC) strips and later of the penetrable imperfect strips, resistive and dielectric: the spectral Galerkin moment method [3], the inverse Fourier transform method [4], the singular integral equation method with projection to orthogonal polynomials [5], and the method of the Riemann–Hilbert boundary value problem (RHP) [6], [7[1]]. The latter two methods belong to the family of the analytical regularization techniques [8] whose many advantages follow from the Fredholm second kind nature of the resultant infinite-matrix equations. The main of them is the guaranteed and fast convergence and hence the controlled accuracy when truncating the matrix and the right-hand part at progressively larger orders. The goal of the present paper is to modify the approach originally developed in [7] for the penetrable strip gratings to the plane wave scattering by a flat grating of thin impenetrable two-face impedance strips. Analysis of the features of such a grating can help establish the boundaries of good validity of common approximation of thin metal strips as PEC. Besides, it opens a way to study the gratings of strips covered with thin material layers, e.g., painted, oxidized, etc.

The remainder of this paper is organized as follows. In Section II we formulate the boundary value problem, reduce it to the dual series equations, and perform their analytical regularization. Section III contains a numerical study of how a departure from the perfect conductivity affects the strip grating performance, especially in terms of the power absorption. In Section IV, we obtain analytical formulas for reflectance and transmittance in the low-frequency region and test their accuracy with respect to the results of our computations. Conclusions are formulated in Section V. The time dependence is assumed as $e^{j\omega t}$ and omitted.

II. Formulation and Basic Equations

Consider the two-dimensional (2-D) scattering of a plane wave by a grating made of zero-thickness impenetrable strips with two different face impedances $Z^{\pm}$. The geometry of the problem is illustrated in Fig. 1(a). Infinite number of strips, parallel to the z-axis, are located in the plane $x=0$ with period d. Each strip has the width $2w$. The propagation vector of the incident plane wave makes the angle φ with respect to the negative x axis.

We shall assume that, at the strips, generalized two-side boundary conditions (GBC) involving jumps and mean values of the tangential field components are imposed. These GBC are derived on consideration of the fields near a thin infinite material slab and can be cast into the following form [9]:

$$\begin{aligned}\frac{1}{2}\left[\vec{E}_T^{+}(0,y)+\vec{E}_T^{-}(0,y)\right]&=R\vec{J}_T(y)+W\vec{x}\times\vec{M}_T(y)\\ \frac{1}{2}\left[\vec{H}_T^{+}(0,y)+\vec{H}_T^{-}(0,y)\right]&=Q\vec{M}_T(y)+W\vec{x}\times\vec{J}_T(y)\end{aligned}\quad(1)$$

[1]Regarding [7], note that in Fig. 6, the near-grazing resonances are missing because of too coarse resolution.

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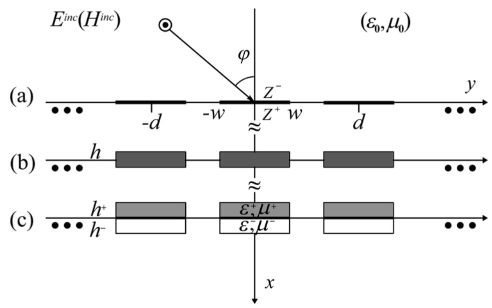

Fig. 1. Geometry of the impedance strip grating scattering problem.

where electric and magnetic surface currents are, respectively

$$\vec{J}_T(y) = \vec{x}\times\left[\vec{H}_T^+(0,y)-\vec{H}_T^-(0,y)\right]$$
$$\vec{M}_T(y) = -\vec{x}\times\left[\vec{E}_T^+(0,y)-\vec{E}_T^-(0,y)\right]. \quad (2)$$

Here, the superscript $\pm$ indicates the limiting value of the function at $x\to\pm 0$, the subscript T denotes vectors tangential to the strips, and $\vec{x}$ is the unit vector normal to the strips. The GBC allow to neglect the fields inside the slab and characterize its properties by means of so-called electrical resistivity R, magnetic resistivity Q, and cross-resistivity W. Additionally, for an impenetrable impedance layer, if only $Z^+ + Z^- \neq 0$, the resistivities are coupled together and relate to the surface impedances as [9]

$$R=\frac{Z^+Z^-}{Z^++Z^-},\quad Q=\frac{1}{Z^++Z^-},\quad W=\frac{1}{2}\frac{Z^+-Z^-}{Z^++Z^-}. \quad (3)$$

For the uniqueness of solution, we complete the formulation, similarly to [7], with the edge condition and the radiation condition at $x\to\pm\infty$. The former condition has the form of demand that the field power contained in a finite domain, including the strip edges, is bounded. The latter requests that the field is expandable in the convergent series of the outgoing (finite number) and exponentially decaying (infinite number) Floquet–Rayleigh–Bloch spatial harmonics.

Note that cross-resistivity W vanishes for a layer with two identical face impedances, and then the effective electric and magnetic currents decouple. This takes place when the impedance slab is a model of the homogeneous well conducting metal slab having the thickness h greater than the skin depth, $h_{\rm skin}$ [Fig. 1(b)]. Then the surface impedances of both faces are the same and given by $Z=(1+j)\sqrt{\omega\mu/2\sigma}$ (see, e.g., [10]), where $\mu=\mu_0\mu_r$ is the permeability and σ is the electron conductance. Note that the exact value of the thickness h does not matter here, as far as $h_{\rm skin}\ll h\ll k_0^{-1}$.

Another interesting for applications case is a PEC slab covered with magnetodielectric coatings, in general different from each other [Fig. 1(c)]. Surface impedance model of a thin grounded material slab is well-known—see, e.g., [10]. It leads to the expression, $Z/\zeta_0 = j\sqrt{\mu_r/\varepsilon_r}\tan(\sqrt{\varepsilon_r\mu_r}k_0h)$, where k_0 is the free space wavenumber, ε_r is the relative permittivity, μ_r is the relative permeability, h is the thickness of the coating, and ζ_0 is the free space impedance. This formula is valid for thin $(k_0h\ll 1)$ and high-contrast $(|\varepsilon_r\mu_r|\gg 1)$ magnetodielectric coatings.

Two alternative cases of the E-polarized (only E_z, H_x, and H_y components are nonzero), and H-polarized wave scattering (only H_z, E_x, and E_y are nonzero) can be considered along similar lines. Denote by $U(x,y)$ the z-components of the field, i.e. either E_z or $\zeta_0 H_z$ depending on polarization. Then the incident wave is

$$U^{\rm inc}(x,y)=exp\left(-j(\alpha_0x+\beta_0y)\right). \quad (4)$$

Due to periodicity of the boundary conditions, the z-components of the scattered fields are quasi-periodic functions of y and can be expanded in the Floquet series

$$U^{\rm scat}(x,y)=\sum_{n=-\infty}^{n=\infty}\left\{\begin{matrix} a_{E(H),n}, & x>0\\ b_{E(H),n}, & x<0\end{matrix}\right\} \times\exp\left(-j\left(\alpha_n|x|+\beta_ny\right)\right) \quad (5)$$

where $\alpha_0 = k_0\cos\varphi$, $\beta_0 = k_0\sin\varphi$, $\alpha_n = (k_0^2-\beta_n^2)^{1/2}$, $\beta_n=\beta_0+2n\pi/d$, and the radiation condition requires that either $\mathrm{Re}\,\alpha_n>0$ or $\mathrm{Im}\,\alpha_n<0$ for each n.

To determine the unknown Floquet-mode amplitudes of the scattered field in the transmission and reflection half-space, i.e., a_n and b_n, respectively, we use two dual sets of conditions that hold on the complementary subintervals of the elementary period. They are GBC on the strip part, M, and conditions of continuity of the total field tangential components on the slot part, S

$$\frac{1}{2}\left[\vec{E}_T^+(0,y)+\vec{E}_T^-(0,y)\right]=R\vec{J}_T(y)+W\vec{x}\times\vec{M}_T(y)\quad y\in M$$
$$\vec{H}_T^+(0,y)=\vec{H}_T^-(0,y),\quad y\in S \quad (6)$$
$$\frac{1}{2}\left[\vec{H}_T^+(0,y)+\vec{H}_T^-(0,y)\right]=Q\vec{M}_T(y)+W\vec{x}\times\vec{J}_T(y)\quad y\in M$$
$$\vec{E}_T^-(0,y)=\vec{E}_T^+(0,y),\quad y\in S. \quad (7)$$

These conditions, together with the Floquet field expansions, lead to two coupled pairs of the dual series equations (DSE). The E-wave case is shown in (8) and (9), and the H-wave case is shown in (10) and (11), all located at the bottom of the following page.

Here, $g_n=(1-(\sin\varphi+n/\kappa)^2)^{1/2}$, $r_n=|n|-jg_n\kappa$, $\kappa=d/\lambda_0$, and $\theta=2\pi w/d$. Besides, $c_{E(H),n}=a_{E(H),n}+b_{E(H),n}$, $d_{E(H),n}=a_{E(H),n}-b_{E(H),n}$, while $t_{E,n}=c_{E,n}g_n-W/(R/\zeta_0)d_{E,n}$, $t_{H,n}=c_{H,n}g_n+W/(Q\zeta_0)d_{H,n}$.

The reflected $(P^{\rm ref})$, transmitted $(P^{\rm tr})$, and absorbed $(P^{\rm abs})$ by the grating fractions of power of the incident wave at a single period are coupled by the power balance equation

$$P^{\rm ref}+P^{\rm tr}+P^{\rm abs}=1. \quad (12)$$

Here the following expressions hold:

$$P^{\mathrm{ref}} = g_0^{-1} \sum_{\mathrm{Re}g_n>0} g_n|b_n|^2, \quad P^{\mathrm{tr}} = g_0^{-1} \sum_{\mathrm{Re}g_n>0} g_n|a_n + \delta_{n0}|^2 \tag{13}$$

with δ_{n0} standing for the Kronecker delta, and summation is taken over the modes that carry power to infinity.

Further, we assume for a moment that R, Q, and W are constants and make analytical inversion of the static parts of (8), (9), and (10), (11). This operation is equivalent to the inversion of the *main* parts of DSEs in terms of the dependences on the summation index n. Analytical inversion needs application of the RHP technique and the inverse Fourier transform (IFT) depending on the equation features (see Appendix). This procedure leads, in each case, to the 2×2 block-type infinite-matrix equations, equivalent to the original boundary-value problem

$$(1 + \mathrm{A})\mathrm{X} = \mathrm{B} \tag{14}$$

where the matrix and right-hand part elements are given by

$$\begin{aligned}
\mathrm{A} &= \left\{ \mathrm{A}^{ij}_{E(H),mn} \right\}^{\infty}_{m,n=-\infty,i,j=1,2} \\
\mathrm{X} &= \left\{ t_{E(H),n}, d_{E(H),n} \right\}^{\infty}_{n=-\infty} \\
\mathrm{B} &= \left\{ \mathrm{B}^{1}_{E(H),m}, \mathrm{B}^{2}_{E(H),m} \right\}^{\infty}_{m=-\infty} \\
\mathrm{I} &= \{\delta_{mn}\delta_{ij}\}^{\infty}_{m,n=-\infty,i,j=1,2} \\
A^{11}_{E,mn} &= S_{mn}(\theta)(R/\zeta_0)^{-1}(2g_n)^{-1} \\
A^{12}_{E,mn} &= WS_{mn}(\theta)(R/\zeta_0)^{-2}(2g_n)^{-1} \\
A^{21}_{E,mn} &= 2j\kappa W T_{mn}(\theta) \\
A^{22}_{E,mn} &= \left[2j\kappa Q\zeta_0 + 2j\kappa W^2(R/\zeta_0)^{-1} - r_n\right] T_{mn}(\theta) \\
B^{1}_{E,m} &= -S_{m0}(\theta)(R/\zeta_0)^{-1}, \quad B^{2}_{E,m} = 2r_0 T_{m0}(\theta)
\end{aligned} \tag{15}$$

$$\begin{aligned}
A^{11}_{H,mn} &= S_{mn}(\theta)(Q\zeta_0)^{-1}(2g_n)^{-1} \\
A^{12}_{H,mn} &= -WS_{mn}(\theta)(Q\zeta_0)^{-2}(2g_n)^{-1} \\
A^{21}_{H,mn} &= -2j\kappa W T_{mn}(\theta) \\
A^{22}_{H,mn} &= \left[2j\kappa R/\zeta_0 + 2j\kappa W^2(Q\zeta_0)^{-1} - r_n\right] T_{mn}(\theta) \\
B^{1}_{H,m} &= -S_{m0}(\theta)(Q\zeta_0)^{-1}, \quad B^{2}_{H,m} = 2r_0 T_{m0}(\theta).
\end{aligned} \tag{16}$$

The expressions for $T_{mn}(\theta)$ and $S_{mn}(\theta)$ can be found in Appendix. Based on the large-index asymptotics of the Legendre polynomials, one can verify that, uniformly for all θ, $T_{mn}(\theta) = \mathrm{O}(|mn|^{-1/2}|m - n + 1|^{-1})$ and $S_{mn}(\theta) = \mathrm{O}(|m - n + 1|^{-1})$. Then one can see that

$$\sum_{m,n=-\infty}^{\infty} \left| A^{ij}_{E(H),mn} \right|^2 < \infty$$
$$\sum_{m=-\infty}^{\infty} \left| B^{i}_{E(H),m} \right|^2 < \infty, \quad i,j = 1,2. \tag{17}$$

Therefore, (14) is a Fredholm second kind matrix equation in the space of sequences $l_2 \times l_2$, and its truncation yields stable and convergent numerical solution. As (15) and (16) can be easily computed with machine precision, the overall accuracy of solution is controlled by the truncation number.

III. Numerical Results

First we study the effect of the imperfect conductivity on the metal strip grating performance. Here, to justify the use of GBC (1), we consider steel, iron, and silver strips having thickness greater than the corresponding skin depth (this is $3.3 \cdot 10^{-4}$mm, $1.08 \cdot 10^{-4}$mm, $4.4 \cdot 10^{-5}$mm, respectively, for $\lambda_0 = 0.143$ mm) however still much smaller than the wavelength (note that actual value of h does not matter here). As high electron conductance values for these metals result in the reflectance and transmittance very close to those for the PEC-strip grating, we focus on the absorption.

$$\begin{cases}
\sum\limits_{n=-\infty}^{\infty} t_{E,n} e^{jn\varphi} = -\frac{1}{R/\zeta_0} - \frac{1}{2R/\zeta_0} \sum\limits_{n=-\infty}^{\infty} \frac{t_{E,n}}{g_n} e^{jn\varphi} - \frac{W}{2(R/\zeta_0)^2} \sum\limits_{n=-\infty}^{\infty} \frac{d_{E,n}}{g_n} e^{jn\varphi}, & |\varphi| < \theta \\
\sum\limits_{n=-\infty}^{\infty} t_{E,n} e^{jn\varphi} = 0, & \theta < |\varphi| \le \pi
\end{cases} \tag{8}$$

$$\begin{cases}
\sum\limits_{n=-\infty}^{\infty} |n| d_{E,n} e^{jn\varphi} = 2r_0 - 2j\kappa W \sum\limits_{n=-\infty}^{\infty} t_{E,n} e^{jn\varphi} + \sum\limits_{n=-\infty}^{\infty} d_{E,n} \left(r_n - 2j\kappa Q\zeta_0 - 2j\kappa \frac{W^2}{R/\zeta_0} \right) e^{jn\varphi}, & |\varphi| < \theta \\
\sum\limits_{n=-\infty}^{\infty} d_{E,n} e^{jn\varphi} = 0, & \theta < |\varphi| \le \pi
\end{cases} \tag{9}$$

$$\begin{cases}
\sum\limits_{n=-\infty}^{\infty} t_{H,n} e^{jn\varphi} = -\frac{1}{Q\zeta_0} - \frac{1}{2Q\zeta_0} \sum\limits_{n=-\infty}^{\infty} \frac{t_{H,n}}{g_n} e^{jn\varphi} + \frac{W}{2(Q\zeta_0)^2} \sum\limits_{n=-\infty}^{\infty} \frac{d_{H,n}}{g_n} e^{jn\varphi}, & |\varphi| < \theta \\
\sum\limits_{n=-\infty}^{\infty} t_{H,n} e^{jn\varphi} = 0, & \theta < |\varphi| \le \pi
\end{cases} \tag{10}$$

$$\begin{cases}
\sum\limits_{n=-\infty}^{\infty} |n| d_{H,n} e^{jn\varphi} = 2r_0 + 2j\kappa W \sum\limits_{n=-\infty}^{\infty} t_{H,n} e^{jn\varphi} + \sum\limits_{n=-\infty}^{\infty} d_{H,n} \left(r_n - 2j\kappa R/\zeta_0 - 2j\kappa \frac{W^2}{Q\zeta_0} \right) e^{jn\varphi}, & |\varphi| < \theta \\
\sum\limits_{n=-\infty}^{\infty} d_{H,n} e^{jn\varphi} = 0 & \theta < |\varphi| \le \pi.
\end{cases} \tag{11}$$

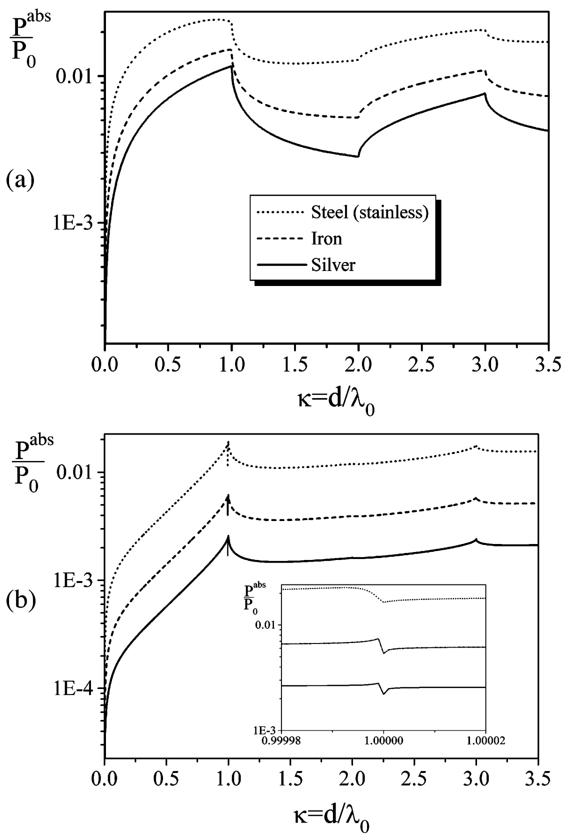

Fig. 2. Absorbed power fractions as functions of normalized frequency for the scattering of the (a) E-wave and the (b) H-wave from metal strip gratings. $\varphi = 0^0$, $2\mathrm{w}/\mathrm{d} = 0.5$, $\mathrm{d} = 0.5$ mm, $\sigma = 1.1 \times 10^6$ S/m (steel), $\sigma = 1.03 \times 10^7$ S/m (iron), $\sigma = 6.173 \times 10^7$ S/m (silver) [8].

Fig. 2 demonstrates the frequency dependences of the absorbed power fractions for the mentioned strip gratings having periods $d = 0.5$ mm and strip width $2w = 0.25$ mm. One can see that in the wavelength band of $\kappa < 3.5$ (i.e., $\lambda_0 > 0.143$ mm) the absorption by a grating of thin steel strips is the greatest among the three gratings and exceeds 0.01 at $\kappa > 0.12$ and $\kappa > 0.77$ in the E-wave and the H-wave cases, respectively. Thus, the Ohmic losses in the scattering by a grating of steel strips become noticeable in the mm and sub-mm wave regions, respectively, i.e., in the situations where metallic strip gratings are most widely used. The absorption by a grating of thin iron strips is several times smaller however climbs up to 0.01 in the vicinity of the grazing of the 1st and the 3rd spatial harmonics in the E-wave case. Unlike this, the silver strip grating absorption amounts to 0.01 only in the vicinity of the grazing of the 1st harmonic in the H-wave case. The inset in Fig. 2(b) shows increase in absorption in the narrow band below the frequency $\kappa = 1$, i.e., where the $\pm$ first Floquet harmonics are just before grazing. Such a resonant behavior is explained by the excitation of the natural oscillations of the grating suppressed by the relatively large losses (here the surface impedance has equal real and imaginary parts).

Fig. 3 demonstrates the frequency dependences of the power fractions for the scattering of the H-wave from the grating of PEC strips having one or both faces covered (e.g., painted) with thin material coatings with large electric losses. These strips are characterized by two equal (curves 1) or different surface impedances (curves 2 and 3), respectively. Almost total transmis-

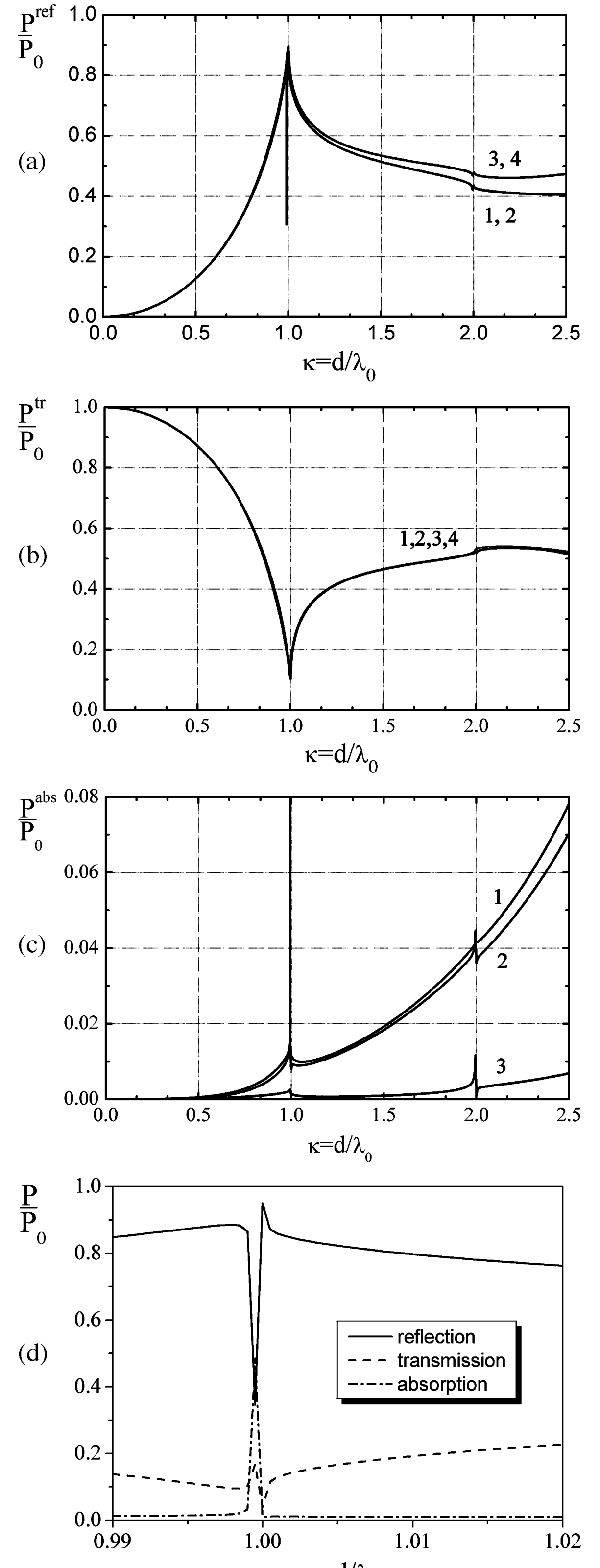

Fig. 3. (a) Reflected, (b) transmitted, and (c) absorbed power fractions as functions of κ for the scattering of the H-wave from a coated grating with electric losses. $\varphi = 0^0$, $h/\mathrm{d} = 0.01$, $2\mathrm{w}/\mathrm{d} = 0.5$, $\mu_r^- = \mu_r^+ = 1$, (curves 1) $\varepsilon_r^- = \varepsilon_r^+ = 1 - 30\mathrm{j}$, (curves 2) $\varepsilon_r^- = 1 - 30\mathrm{j}$, $\mathrm{Z}^+ = 0$, (curves 3) $\mathrm{Z}^- = 0$, $\varepsilon_r^+ = 1 - 30\mathrm{j}$, (curves 4) are the same as for PEC grating. (d) is the same as (a)–(c) for the case (curves 1) in the vicinity of resonance.

sion is observed in the low-frequency limit. Thus, the Hertz effect takes place: small period impedance strip grating $(d \ll \lambda_0)$ is almost transparent for the H-polarized wave. At the same time, Fig. 3 shows a sharp drop in reflection and increase in absorption in the narrow band below the frequency $\kappa = 1$ (i.e., $d = \lambda_0$ for normal incidence), i.e., where the first Floquet harmonic is just before grazing. This resonant behavior is similar to that visible in Fig. 2(b). However, now the resonances are sharper because the real part of the surface impedance is $(\mathrm{Im}\varepsilon_r/\mathrm{Re}\varepsilon_r)^{1/2}(k_0 h)^2$ times smaller than the imaginary part. For $d/\lambda_0 > 1$, the values of reflectance and transmittance are quite comparable to each other. An interesting however not unexpected observation is that the reflection by a dark-face coated grating (curves 3) exceeds the reflection by two other structures if $1.5 < d/\lambda_0 < 2.5$. This is because the absorption by the only dark-face coated grating is small.

Fig. 4 shows frequency characteristics of the power fractions for the H-wave scattering by the impedance grating with a magnetic-lossy coating. Curves 1 correspond to two identical surface impedances, and curves 2 and 3 correspond to different surface impedances. One can see again that in the low frequency limit a magnetic-lossy material coated grating is completely transparent for the H polarization. Reflection by a dark-face magnetic lossy material coated grating exceeds reflection by two other gratings in the whole considered frequency range. This is because absorption by the dark-face coated grating is much smaller than absorption by the two-face coated and illuminated-face coated gratings. However, in general the absorption by a lossy-magnetic material coated grating is always much greater than absorption by a lossy-electric material coated one. It runs up to 0.8 whereas for the electric-lossy coated grating the absorption fraction does not exceed 0.1. This is because in the case of the H-polarization the magnetic field is not small near a PEC strip whereas the electric field tangential component is zero. Here, absorption resonances in the vicinity of the grazing are completely suppressed (compare with Figs. 2 and 3) because real and imaginary parts of the surface impedance relate as $\mathrm{Im}\mu_r/\mathrm{Re}\mu_r$.

Fig. 5 demonstrates frequency dependences of the power fractions for the E-wave scattering from the PEC gratings with one or both faces covered with thin material coatings having electric losses. As for a similar PEC grating without a coating, almost total reflection is observed in the low-frequency limit (both transmission and absorption approach zero). Thus, small-period coated strip grating $(d \ll \lambda_0)$ behaves like a PEC grating [4] in the E-polarization case. If $d/\lambda_0 > 1$, the values of reflection and transmission power fractions are comparable to each other. The absorption power fractions do not exceed 0.1 similarly to the H-wave scattering from the same gratings (Fig. 3). Note that there are no sharp resonances near the grazing.

Finally, Fig. 6 shows frequency dependences similar to those presented in Fig. 5 but for the strip gratings with magnetic-lossy coatings. Comparison of the curves discloses the same effect as in the H-case (see Fig. 4), i.e., absorption by lossy-magnetic material coated gratings is much greater than absorption by lossy-electric material coated ones. In the first case the absorption amounts to 0.76 while in the second case it does not exceed 0.1.

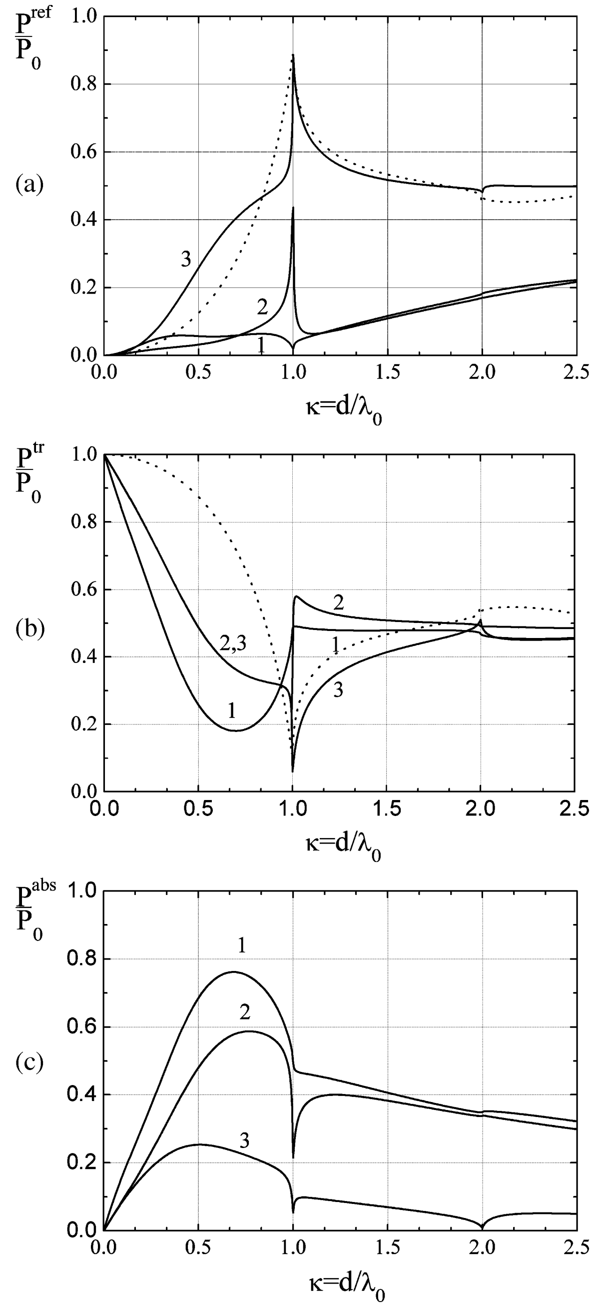

Fig. 4. (a) Reflected, (b) transmitted, and (c) absorbed power fractions as functions of κ for the scattering of the H-wave from impedance grating with magnetic losses. $\varphi = 0^0$, $h/\mathrm{d} = 0.01$, $2\mathrm{w}/\mathrm{d} = 0.5$, $\varepsilon_r^- = \varepsilon_r^+ = 1$, (curves 1) $\mu_r^- = \mu_r^+ = 1 - 30\mathrm{j}$, (curves 2) $\mu_r^- = 1 - 30\mathrm{j}$, $\mathrm{Z}^+ = 0$, (curves 3) $\mathrm{Z}^- = 0$, $\mu_r^+ = 1 - 30\mathrm{j}$. Dotted lines are the same as for PEC grating.

IV. Modified Lamb Formulas

Besides the numerical solution with guaranteed accuracy, regularized matrix (14) can be solved by iterations, provided that the norm of the corresponding matrix operator is less than unity, $\|A\| < 1$. As we have inverted the static parts of the full-wave DSE, assuming that R, Q, and W are fixed, it is not surprising to see that, in the each polarization case, $\|A\| < \kappa \mathrm{O}(1)$. This means that in the low-frequency range the Lamb-type formulas can be obtained after expanding all the quantities in (15) or (16) in terms of the power series of κ and retaining the first-order terms, i.e.,

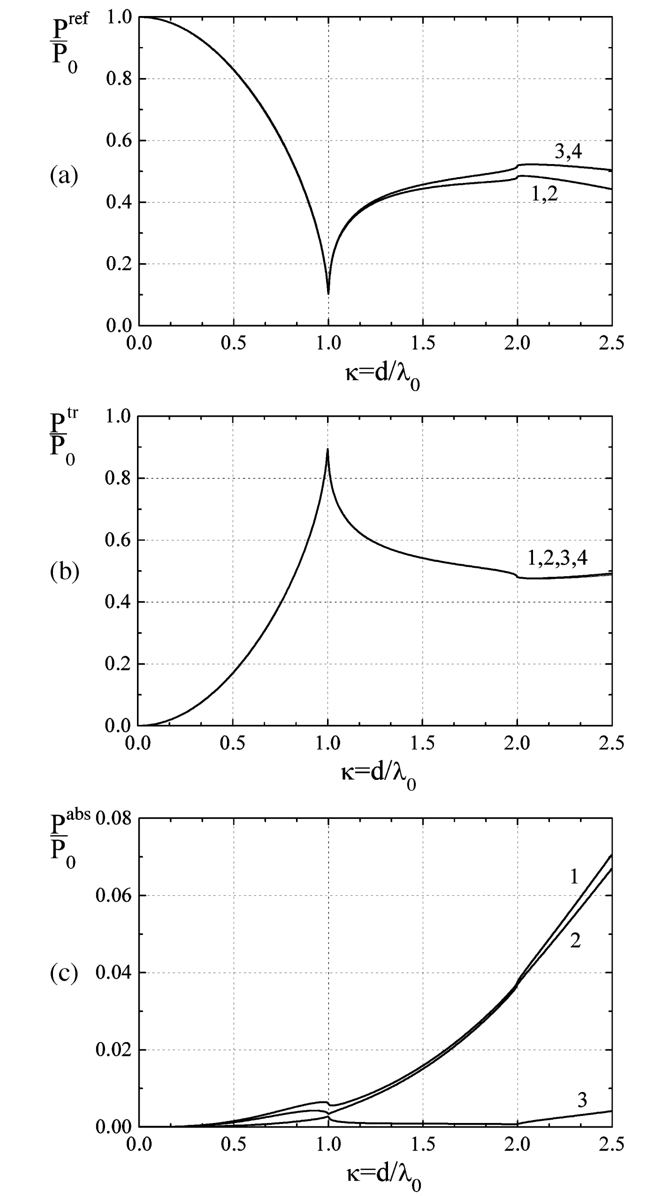

Fig. 5. (a) Transmitted, (b) reflected, and (c) absorbed power fractions as functions of κ for the scattering of the E-wave from impedance grating with electric losses. $\varphi = 0^0$, $h/\mathrm{d} = 0.01$, $2\mathrm{w}/\mathrm{d} = 0.5$, $\mu_{\mathrm{r}}^{-} = \mu_{\mathrm{r}}^{+} = 1$, (curves 1) $\varepsilon_{\mathrm{r}}^{-} = \varepsilon_{\mathrm{r}}^{+} = 1 - 30\mathrm{j}$, (curves 2) $\varepsilon_{\mathrm{r}}^{-} = 1 - 30\mathrm{j}$, $\mathrm{Z}^{+} = 0$, (curves 3) $\mathrm{Z}^{-} = 0$, $\varepsilon_{\mathrm{r}}^{+} = 1 - 30\mathrm{j}$, (curves 4) are the same as for PEC grating.

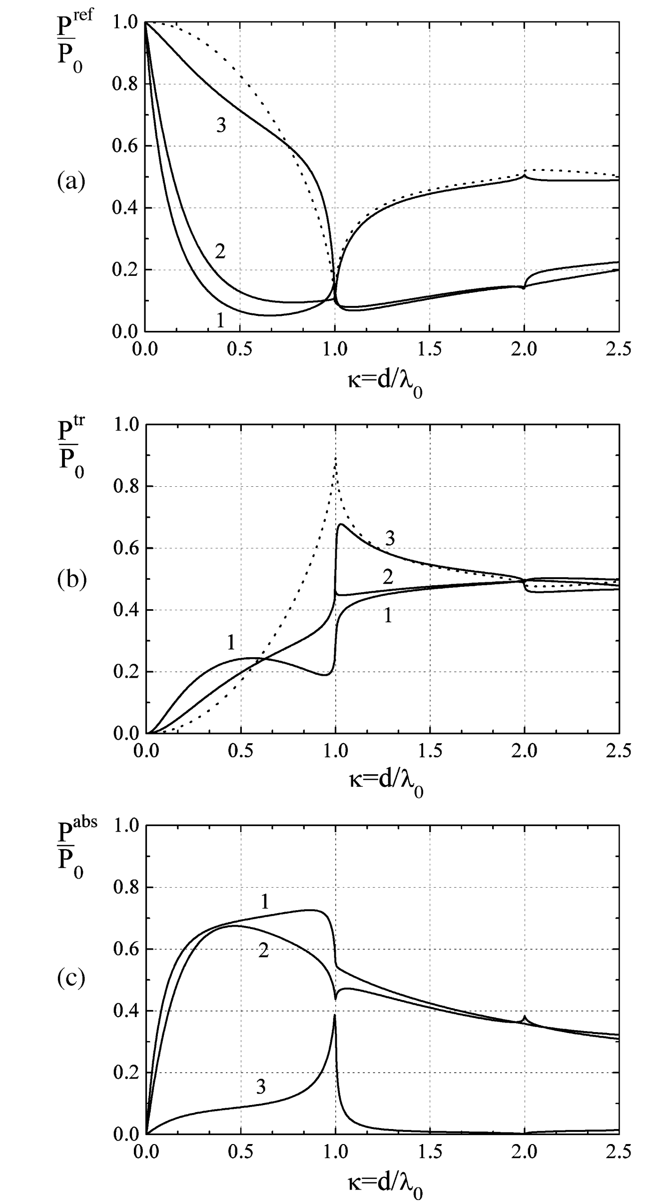

Fig. 6. (a) Transmitted, (b) reflected, and (c) absorbed power fractions as functions of κ for the scattering of the E-wave from impedance grating with magnetic losses. $\varphi = 0^0$, $h/\mathrm{d} = 0.01$, $2\mathrm{w}/\mathrm{d} = 0.5$, $\varepsilon_{\mathrm{r}}^{-} = \varepsilon_{\mathrm{r}}^{+} = 1$, (curves 1) $\mu_{\mathrm{r}}^{-} = \mu_{\mathrm{r}}^{+} = 1 - 30\mathrm{j}$, (curves 2) $\mu_{\mathrm{r}}^{-} = 1 - 30\mathrm{j}$, $\mathrm{Z}^{+} = 0$; (curves 3) $\mathrm{Z}^{-} = 0$, $\mu_{\mathrm{r}}^{+} = 1 - 30\mathrm{j}$. Dotted lines are the same as for a PEC grating.

$a_0^{H(E)} \approx B_0^{H(E)}/(1 + A_{00}^{H(E)}) + \mathrm{O}(\kappa^2)$. By using this approach and matrix (14), Lamb-type formulas for a grating made of impedance strips are found to be

$$\begin{pmatrix} a_0^H \\ b_0^H \end{pmatrix} = \frac{1}{2\Delta^H \cos\varphi} \times \left\{ -\frac{S_{00}(\theta)}{Q\zeta_0} - j\kappa T_{00}(\theta) \times \left[\left(\frac{1+2W}{2Q\zeta_0} + \cos\varphi \right) \frac{S_{00}(\theta)}{Q\zeta_0} - 2\left(\frac{W}{Q\zeta_0} \mp \cos\varphi \right) \times \left(\cos\varphi + \frac{S_{00}(\theta)}{2Q\zeta_0}(1+2W) \right) \right] \right\} \quad (18)$$

$$\begin{pmatrix} a_0^E \\ b_0^E \end{pmatrix} = \frac{1}{2\Delta^E \cos\varphi} \times \left\{ -\frac{S_{00}(\theta)}{R/\zeta_0} - j\kappa T_{00}(\theta) \times \left[-\left(\frac{1-2W}{2R/\zeta_0} + \cos\varphi \right) \frac{S_{00}(\theta)}{R/\zeta_0} - 2\left(\frac{W}{R/\zeta_0} \pm \cos\varphi \right) \times \left(-\cos\varphi + \frac{S_{00}(\theta)}{2R/\zeta_0}(1-2W) \right) \right] \right\} \quad (19)$$

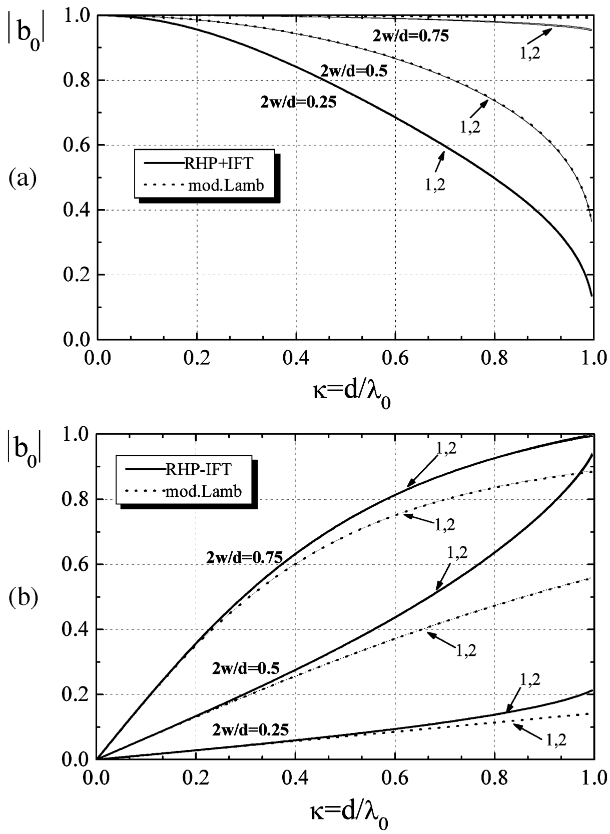

Fig. 7. Comparison of the accurate and approximate solutions for reflection coefficients. $\varphi = 0^0$, $h/\mathrm{d} = 0.01$. (1) $\varepsilon_r^- = 1 - 30\mathrm{j}$, $\mu_r^- = 1$, $\varepsilon_r^+ = 1$, $\mu_r^+ = 1$; (2) $\varepsilon_r^- = 1$, $\mu_r^- = 1$, $\varepsilon_r^+ = 1 - 30\mathrm{j}$, $\mu_r^+ = 1$. (a) E-wave. (b) H-wave.

where

$$\begin{aligned}\Delta^H =& 1 + \frac{S_{00}(\theta)}{2Q\zeta_0 \cos\varphi} + j\kappa T_{00}(\theta) \\ & \times \left[\cos\varphi + \frac{1}{2Q\zeta_0} + \frac{S_{00}(\theta)}{2Q\zeta_0\cos\varphi}\right. \\ & \left.\times \left(\cos\varphi + \frac{1-4W^2}{2Q\zeta_0}\right)\right] \end{aligned} \tag{20}$$

$$\begin{aligned}\Delta^E =& 1 + \frac{S_{00}(\theta)}{2R/\zeta_0 \cos\varphi} + j\kappa T_{00}(\theta) \\ & \times \left[\cos\varphi + \frac{1}{2R/\zeta_0} + \frac{S_{00}(\theta)}{2R/\zeta_0\cos\varphi}\right. \\ & \left.\times \left(\cos\varphi + \frac{1-4W^2}{2R/\zeta_0}\right)\right]. \end{aligned} \tag{21}$$

Fig. 7 shows frequency dependences of the absolute values of the 0th harmonic amplitude (i.e., field reflection coefficient in the single-mode regime) for the grating whose illuminated face is coated with a lossy dielectric layer (curves 1) and for the similar grating whose dark face is coated with the same dielectric layer (curves 2), for three different values of the strip width $2w/d$. The dependences are obtained by means of accurate solutions (14) and by the modified Lamb formulas (18)–(21) for the E-wave and the H-wave cases. One can see that the long-wave asymptotics are in good agreement with those obtained by the accurate solution for both gratings considered. It has been found that the error defined as $|(b_0^{\mathrm{exact}} - b_0^{\mathrm{Lamb}})/b_0^{\mathrm{exact}}|$ does not exceed 10% for $\kappa < 0.3$ and arbitrary values of $2w/d$ at the normal incidence of both E and H-polarized waves.

The Lamb-type formulas greatly simplify engineering design of polarization discriminators based on the small-period imperfect-strip gratings.

V. Conclusion

We have developed a powerful numerical solution to the scattering problem concerning an impedance strip grating in free space illuminated by a plane electromagnetic wave.

The computations have been carried out for the reflected, transmitted, and absorbed power fractions as a function of the electrical and material parameters of the several types of gratings. They show that metallic strip gratings may have a large power fraction lost through the absorption in the thin lossy coatings, especially if the illuminated face of the strip grating is covered with a magnetic type coating. In such a case one can obtain considerable absorption at any frequency.

Besides, the effect of the resonant absorption of the H-wave just below the frequency values of the higher space harmonics grazing has been found. It is caused by a resonant enhancement of the near field and can be used for the narrow band polarization and angle-of-incidence discrimination.

Appendix

Consider a DSE of the following form:

$$\begin{cases} \sum\limits_{n=-\infty}^{\infty} x_n |n| e^{in\varphi} = \sum\limits_{n=-\infty}^{\infty} f_n e^{in\varphi}, & \theta < |\varphi| \le \pi \\ \sum\limits_{n=-\infty}^{\infty} x_n e^{in\varphi} = 0, & |\varphi| < \theta \end{cases} \tag{A1}$$

where the expansion coefficients f_n of the right-hand side are supposed to be known and are decreasing as $\mathrm{O}(|n|^{-1+\alpha})$, $\alpha > 0$ for large $|n|$. Following [6] and [11], an exact analytical solution to this equation can be conveniently written as

$$x_m = \sum_{n=-\infty}^{\infty} f_n T_{mn}(\theta), \quad m = 0, \pm 1, \pm 2, \ldots \tag{A2}$$

where

$$T_{mn}(\theta) = \begin{cases} \frac{P_{m-1}(u)P_n(u) - P_m(u)P_{n-1}(u)}{2(m-n)}, & m \ne n \\ \frac{1}{2|m|} \sum\limits_{k=0}^{|m|} q_{|m|-k}(u) P_{|m|-k-1}(u), & m = n \ne 0 \\ -\ln\frac{1+u}{2}, & m = n = 0 \end{cases} \tag{A3}$$

P_n are the Legendre polynomials, $u = \cos\theta$, and

$$q_k(u) = \begin{cases} 1, & k = 0 \\ -u, & k = 1 \\ P_k(u) - 2uP_{k-1}(u) + P_{k-2}(u), & k = 2, 3, \ldots \end{cases} \tag{A4}$$

For a series equation given as

$$\sum_{n=-\infty}^{\infty} x_n e^{in\varphi} = \begin{cases} \sum_{n=-\infty}^{\infty} f_n e^{in\varphi}, & \theta < |\varphi| \le \pi \\ 0, & |\varphi| < \theta \end{cases} \quad \text{(A5)}$$

we apply the inverse Fourier transform; that is, multiply both sides by $e^{-im\varphi}$ and integrate from 0 to 2π. The result is

$$x_m = \sum_{n=-\infty}^{\infty} f_n S_{mn}(\theta), \quad m = 0, \pm 1, \pm 2, \ldots \quad \text{(A6)}$$

and

$$S_{mn}(\theta) = -\frac{\sin\theta(m-n)}{\pi(m-n)}, m \ne n, \quad S_{mm}(\theta) = 1 - \frac{\theta}{\pi}. \quad \text{(A7)}$$

Note that both (A2) and (A6) form the number sequences of the class l_2, because $\sum_{n=-\infty}^{\infty} |x_n|^2 < \infty$.

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References

[1] H. Hertz, “Ueber strahlen elektrischer kraft,” *Ann. Phys. Chem. Leipzig.*, vol. 36, pp. 769–783, 1889.

[2] H. Lamb, “On the reflection and transmission of electric waves by a metallic grating,” *Proc. London Math. Society*, vol. 29, pp. 523–544, 1898.

[3] R. C. Hall and R. Mittra, “Scattering from a periodic array of resistive strips,” *IEEE Trans. Antennas Propag.*, vol. 33, no. 9, pp. 1009–1011, 1985.

[4] R. Petit and G. Tayeb, “Theoretical and numerical study of gratings consisting of periodic arrays of thin and lossy strips,” *J. Opt. Soc. Amer.*, vol. 7, no. 9, pt. A, pp. 1686–1692, 1990.

[5] A. Matsushima, T. L. Zinenko, H. Minami, and Y. Okuno, “Integral equation analysis of plane wave scattering from multilayered resistive strip gratings,” *J. Electromagn. Waves Applicat.*, vol. 12, pp. 1449–1469, 1998.

[6] Z. S. Agranovich, V. A. Marchenko, and V. P. Shestopalov, “Diffraction of a plane electromagnetic wave from plane metallic lattices,” Transl.: English Transl. *Soviet Phys. Tech. Phys.*, vol. 7, pp. 277–286, 1962.

[7] T. L. Zinenko, A. I. Nosich, and Y. Okuno, “Plane wave scattering and absorption by resistive-strip and dielectric-strip gratings,” *IEEE Trans. Antennas Propag.*, vol. 46, no. 10, pp. 1498–1505, 1998.

[8] A. I. Nosich, “MAR in the wave-scattering and eigenvalue problems: foundations and review of solutions,” *IEEE Antennas Propag. Mag.*, vol. 42, no. 3, pp. 25–49, 1999.

[9] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, “Surface-integral equations for electromagnetic scattering from impenetrable and penetrable sheets,” *IEEE Antennas Propag. Mag.*, vol. 36, no. 6, pp. 14–25, 1993.

[10] D. M. Pozar, *Microwave Engineering*. New York: Wiley, 1989.

[11] A. I. Nosich, “Green’s function-dual series approach in wave scattering from combined resonant scatterers,” in *Analytical and Numerical Methods in Electromagnetic Wave Theory*, M. Hashimoto, M. Idemen, and O. A. Tretyakov, Eds. Tokyo, Japan: Science House, 1993, ch. 9, pp. 419–469.

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