Excitation of a partially shielded round dielectric rod by lumped sources

J. No14

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(Submitted December 9, 1982; resubmitted March 4, 1983)

Zh. Tekh. Fiz. 53, 2312–2321 (December 1983)

Excitation of a round, partially shielded, dielectric rod by elementary electric and magnetic sources is considered in a vector formulation. The Riemann-Hilbert method reduces the problem to finding the spectral densities for the fields by solving two coupled infinite systems of linear equations. These systems can be solved to any specified accuracy for arbitrary parameter values, and in this sense the solution is rigorous. Excitation of a cylindrical slit line by a longitudinal magnetic dipole is considered in detail [the line consists of a rod shielded by a metal layer with a narrow longitudinal slit and the radius of the rod is assumed small compared with the wavelength]. The structure, regions of generation, and distribution of energy among wave fields of various types are studied.

PACS numbers: 84.40.Sr, 77.90.+k

Dipole excitation of an open waveguide (a round dielectric rod) was first considered rigorously in Ref. 1. Partially shielded round dielectric rods (Fig. 1) form a new class of open model waveguide structures in which the properties of a metal waveguide are combined with those of a dielectric rod located in empty space. Analysis of the natural oscillations[2-4] has revealed that the case when the shielding metal layer contains a narrow slit is of particular interest; in this case, the structure forms a cylindrical slit line (CSL). Because of the distinctive "slit-like" character of the fundamental wave, CSLs have many advantages.[5]

In this paper we rigorously examine the excitation of an arbitrary partially shielded round rod by lumped sources and discuss in detail the case when a CSL is excited at low frequencies by a longitudinal magnetic dipole located at the center of the slit. We use techniques related to the methods in Refs. 1, 3, and 6 to study the structures, regions of generation, and energy distribution among wave fields of various types.

1. FORMAL SOLUTION

We assume that the infinitely long, round dielectric rod (Fig. 1) with radius a and dielectric constant ε is coated by an ideally conducting thin metal layer. A longitudinal slit of angular dimension 2θ extends along the metal layer. The rod is excited by an elementary electromagnetic (EM) field source lying outside the rod. We denote the Hertz electric and magnetic vectors for the primary field by V_0 and W_0. We need to calculate the secondary field, which must satisfy the Maxwell equations, obey the boundary conditions, decay at infinity, and have the property that the energy is finite near the sharp edges of the metal layer.

We introduce a cylindrical system of coordinates (r, φ, z) coaxial with the rod. If, for example, the source is a longitudinal dipole of length $b \ll \lambda$ (where λ is the wavelength) with an electric or magnetic current of magnitude I located at the point $(l, \varphi_0, 0)$, all of the Hertz vectors for the primary field vanish except for the axial components

$$(V_z^0,\ W_z^0) = c_0 \int_{-\infty}^{\infty} g^{-2} \sum_{n=-\infty}^{\infty} (d_n,\ D_n)\, e^{in\varphi} e^{ihz} dh, \tag{1}$$

where

$$c_0 = -\frac{Ib}{2\omega}, \quad d_n = D_n = g^2 e^{-in\varphi_0} \begin{Bmatrix} H_n^{(1)}(gl)\, J_n(gr) \\ J_n(gl)\, H_n^{(1)}(gr) \end{Bmatrix} \begin{matrix} r < l, \\ r > l, \end{matrix} \tag{2}$$

J_n, $H_n^{(1)}$ are the Bessel and Hankel functions, $g = \sqrt{k^2 - h^2}$, $k = 2\pi/\lambda = \omega/c$, and the radiation condition implies that we must take the branch of the square root for which $\operatorname{Im} g > 0$ as $h \to \pm\infty$, and the path of integration goes around the branch points $h = k$ and $h = -k$ from below and from above, respectively.

It is also helpful to express the primary field in the form (1) when the rod is excited by an arbitrary elementary source. In general, none of the six components of the field will vanish when the rod is present. They can be described in terms of the axial components $(E_z, H_z) = (\partial^2/\partial z^2 + k^2\varepsilon)(V_z, W_z)$, which we express in a form analogous to (1):

$$\{E_z,\ H_z\} = c_0 \int_{-\infty}^{\infty} \begin{Bmatrix} E^{(+)}(h),\ H^{(+)}(h) \\ E^{(-)}(h) + E^0,\ H^{(-)}(h) + H^0 \end{Bmatrix} e^{ihz} dh,$$

$$\{E^{(+)}(h),\ H^{(+)}(h)\} = \sum_n (A_n,\ A_n')\, J_n(pr)\, e^{in\varphi}, \quad r < a,$$

$$\{E^{(-)}(h),\ H^{(-)}(h)\} = \sum_n (B_n,\ B_n')\, H_n^{(1)}(gr)\, e^{in\varphi}, \quad r > a. \tag{3}$$

Here and below, $p = \sqrt{k^2\varepsilon - h^2}$ and the sums are over n from $-\infty$ to $+\infty$.

The original problem reduces to a system of Dirichlet and Neumann boundary-value problems for the spectral functions E(h), H(h), which are coupled by the matching

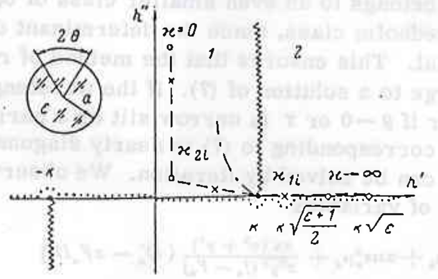

FIG. 1. 1) fast waves; 2) slow waves.

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conditions on the slit. If we impose the boundary conditions on the fields (3), we find after some manipulation the following system of coupled equations which involve sums of trigonometric functions:

$$\sum_n \rho^n e^{in\varphi}=0,\ \theta<|\varphi|\leqslant\pi,$$

$$\sum_n \rho_n|n|e^{in\varphi}=\sum_n\Big\{\Delta_n^E\rho_n+\alpha n\varepsilon_n^E\mu_n+\frac{an(x^2+y^2)}{x^2y^2(f_n-F_n)}(D_n'-xF_nD_n)+\frac{x^2+y^2}{2x(\varepsilon+1)}(d_n'-xF_nd_n)\Big\}e^{in\varphi},\ |\varphi|<\theta,$$

$$\sum_n \mu_n e^{in\varphi}=0,\ |\varphi|<\theta, \tag{4}$$

$$\sum_n \mu_n|n|e^{in\varphi}=\sum_n\Big\{\Delta_n^H\mu_n+\beta n\varepsilon_n^H\rho_n+\frac{(x^2+y^2)f_n}{x^2(f_n-F_n)}(D_n'-xF_nD_n)\Big\}e^{in\varphi}, \tag{5}$$

where

$$\theta<|\varphi|\leqslant\pi,$$

$$\rho_n=A_nJ_n,\ \mu_n=A_n'J_n'\frac{f_n-F_n}{xyf_nF_n}+\frac{\beta nA_nJ_n}{x^2F_n}+\frac{1}{x}\Big(D_n-\frac{D_n'}{xF_n}\Big),$$

$$f_n=\frac{J_n'}{yJ_n},\ F_n=\frac{H_n'}{xH_n},\ \alpha=\frac{ihka^2(\varepsilon-1)}{2x(\varepsilon+1)},$$

$$\beta=\frac{ihka^2(\varepsilon-1)}{xy^2},\ \varepsilon_n^E=1+\frac{(x^2+y^2)F_n}{y^2(f_n-F_n)},\ \varepsilon_n^H=1-\frac{(x^2+y^2)f_n}{x^2(f_n-F_n)},$$

$$\Delta_n^E=|n|-\frac{x^2+y^2}{2(\varepsilon+1)}\Big\{\varepsilon f_n-F_n-\frac{n^2h^2k^2a^2(\varepsilon-1)^2}{x^4y^4(f_n-F_n)}\Big\},$$

$$\Delta_n^H=|n|+\frac{(x^2+y^2)f_nF_n}{f_n-F_n} \tag{6}$$

Here and below, $x=ga$, $y=pa$, $J_n=J_n(y)$, $H_n=H_n^{(1)}(x)$, and primes denote derivatives with respect to the argument.

We use the Riemann–Hilbert method[6] to invert the static part of the operators (4), (5). We then get two infinite coupled Fredholm systems of linear equations of the second kind:

$$\rho_m-\sum_n\Big[\Delta_n^E\rho_n+\alpha n\varepsilon_n^E\mu_n+\frac{an(x^2+y^2)}{x^2y^2(f_n-F_n)}(D_n'-xF_nD_n)+\frac{x^2+y^2}{2x(\varepsilon+1)}(d_n'-xF_nd_n)\Big]T_{mn}(u)=0,$$

$$\mu_n-\sum_n\Big[\Delta_n^H\mu_n+\beta n\varepsilon_n^H\rho_n+\frac{(x^2+y^2)f_n}{x^2(f_n-F_n)}(D_n'-xF_nD_n)\Big](-1)^{m+n}T_{mn}(-u)=0, \tag{7}$$

where $T_{00}(\pm u)=-\ln[(1\pm u)/2]$, $T_{0n}=V_{n-1}^{-1}/n$, $T_{mn}=V_{m-1}^{n-1}/m$ and the functions $V_{m-1}^{n-1}(u)$ are defined in Ref. 6 (we write $u=\cos\theta$). We remark that the operator determined by system (7) belongs to an even smaller class of operators than the Fredholm class, since the determinant of system (7) is normal. This ensures that the method of reduction will converge to a solution of (7). If the wavelength is long, $ka\sqrt{\varepsilon}\to 0$, or if $\theta\to 0$ or π (a narrow slit or a narrow ribbon), the matrix corresponding to (7) is nearly diagonal and system (7) can be solved by iteration. We observe also that the change of variables

$$\gamma_n^E=\Delta_n^E\rho_n+\alpha n\varepsilon_n^E\mu_n+\frac{an(x^2+y^2)}{x^2y^2(f_n-F_n)}(D_n'-xF_nD_n)+\frac{x^2+y^2}{2x(\varepsilon+1)}(d_n'-xF_nd_n),$$

$$\gamma_n^H=\Delta_n^H\mu_n+\beta n\varepsilon_n^H\rho_n+\frac{(x^2+y^2)f_n}{x^2(f_n-F_n)}(D_n'-xF_nD_n) \tag{8}$$

reduces Eqs. (7) to a form that does not contain any infinit sums in the right-hand side.

2. DETERMINATION OF THE SPECTRAL DENSITIES OF THE EM FIELD COMPONENTS

The spectral density functions for the axial field components can be expressed in terms of the coefficients ρ_n, μ_n as Fourier series outside the rod,

$$E^{(-)}(h)=\sum_n\frac{(\rho_n-d_n)}{H_n}H_n(gr)e^{in\varphi},$$

$$H^{(-)}(h)=\sum_n\Big(\mu_n-\frac{\beta n\rho_n-D_n+xF_nD_n'}{x^2f_n}\Big)\frac{x^2f_nF_n}{(f_n-F_n)H_n'}H_n(gr)e^{in\varphi} \tag{9}$$

with similar expressions holding inside the rod.

The series (9) and the analogous series for $E^{(+)}$ and $H^{(+)}$ converge very slowly for $r\simeq a$, i.e., near the surface of the rod, since the terms decay no faster than $O(n^{-3/2})$ a $n\to\infty$. After substituting the coefficients ρ_n, μ_n given by system (7) into (9) and changing the order of summation, we get the following functional series:

$$E^{(\pm)}(h)=\sum_n\gamma_n^E S_n^{(\pm)E}+P^{(\pm)},$$

$$H^{(\pm)}(h)=\sum_n\{\gamma_n^H S_n^{(\pm)H}+\beta\gamma_n^E S_n^{(\pm)HE}\}+Q^{(\pm)}, \tag{10}$$

where

$$P^{(+)}=0,\ P^{(-)}=-\sum_n\frac{d_n}{H_n}H_n(gr)e^{in\varphi},$$

$$Q^{(+)}=\frac{y}{x}\sum_n\frac{(D_n'-xF_nD_n)f_n}{(f_n-F_n)J_n'}J_n(pr)e^{in\varphi},$$

$$Q^{(-)}=\sum_n\frac{(D_n'-xF_nD_n)F_n}{(f_n-F_n)H_n'}H_n(gr)e^{in\varphi}, \tag{11}$$

and

$$S_n^{(\pm)E}=\sum_m T_{mn}(u)\Big\{\frac{J_m(pr)}{J_m},\ \frac{H_m(gr)}{H_m}\Big\}e^{im\varphi},$$

$$S_n^{(\pm)H}=x\sum_m(-1)^{m+n}T_{mn}(-u)\frac{f_mF_m}{f_m-F_m}\Big\{\frac{yJ_m(pr)}{J_m'},\ \frac{xH_m(gr)}{H_m'}\Big\}e^{im\varphi},$$

$$S_n^{(\pm)HE}=\frac{1}{x}\sum_m T_{mn}(u)\frac{mf_mF_m}{f_m-F_m}\Big\{\frac{yJ_m(pr)}{J_m'F_m},\ \frac{xH_m(gr)}{H_m'f_m}\Big\}e^{im\varphi}. \tag{12}$$

Here the superscripts + and − correspond to the first and second expressions in the curly brackets, respectivel

The above series (10) now converge as $O(n^{-5-2})$ or better owing to the rapid convergence of system (7). In addition, it can be shown that each of the functions $S_n^{(\pm)E}$, $S_n^{(\pm)H}$, $S_n^{(\pm)HE}$ separately satisfies the boundary condition at the edge of the slit. In other words, each ter of series (10) satisfies this condition, regardless of the error in calculating γ_n^E and γ_n^H. It is also important to note that the slowly converging infinite series defining th functions in (12) can be rearranged and combined into a principal part which also satisfies the boundary condition at the edge of the slit.

3. PROPERTIES OF THE SPECTRAL FUNCTIONS

The evaluation of the field amplitudes inside and outside the rod requires that the integration (3) be carried out. In order to derive appropriate asymptotic estimates as $z \to \infty$ it is necessary to examine the properties of the spectral functions E(h), H(h) for the total field in the complex h-plane. It can be shown that E(h) and H(h) are analytic functions on a double-sheeted Riemann surface; they have only the two branch points $h = \pm k$ and a certain number of coincident poles. The latter are determined by the same dispersion equation for a partially shielded rod,

$$\mathrm{Det}\,(h) = \left| \begin{matrix} \{\Delta_n^E T_{mn}(u) - \delta_{mn}\}, & \alpha\{n\varepsilon_n^E T_{mn}(u)\} \\ \beta\{n\varepsilon_n^H (-1)^{m+n} T_{mn}(-u)\}, & \{(-1)^{m+n}\Delta_n^H T_{mn}(-u) - \delta_{mn}\} \end{matrix} \right|_{m,\,n=-\infty}^{\infty} = 0, \tag{13}$$

where Det (h) is the determinant of the block matrix of system (7). We note that any roots of (13) must be distributed symmetrically about $h = 0$, and it is convenient to take the cuts from the branch points as shown in Fig. 1.

The analysis of the existence and location of the roots of (13) is a much more complicated problem than the study of the dispersion equation for a round unshielded dielectric rod,[1] not to mention the even simpler case of a closed round waveguide. The method of Ref. 7 shows only that the spectrum is discrete in the h-plane and that the roots of the reduced equation (13) converge to the eigenvalues of the original problem as the order of the reduction increases. However, several properties of the solutions of (13) can be found by studying Eqs. (13) iteratively as $\theta \to 0$ and using some physical intuition.

1) The real roots of (13) lie in the interval $k \le h < k\sqrt{\varepsilon}$. There are finitely many such roots. 2) There are no purely imaginary or real roots outside the interval $k \le h < k\sqrt{\varepsilon}$, nor are there any complex roots in the region where Im g > 0. 3) There is a region between the cuts from $h = \pm k$ and the imaginary axis where Im g < 0, and here (13) may have complex roots. 4) When the frequency, or more precisely, the frequency parameter $\kappa = ka\sqrt{\varepsilon - 1}$ increases and becomes equal to some characteristic (transitional) value κ_{1i}, one of the complex roots h_i of (13) becomes equal to k [$h_i(\kappa_{1i}) = k$], after which h_i approaches the value $k\sqrt{\varepsilon}$. The quasi-TE_{00} wave is an exception, as was shown in Refs. 2, 3; the TE_{00} wave is the fundamental mode as $\theta \to 0$, and for this mode we have $h_0 \to k\sqrt{(\varepsilon + 1)/2.5)}$. If the slit is sufficiently narrow ($\theta \to 0$), each complex root h_i also has another characteristic frequency κ_{2i}. We can regard κ_{2i} as a critical frequency with the property that when κ decreases and becomes equal to κ_{2i}, $|h_i|$ assumes its minimum value (close to zero) and $\arg h_i\,(\kappa_{2i}) \simeq \pi/4$.

The location of the zero h_i is shown schematically in Fig. 1 by the dashed line. For comparison, we recall that each eigenvalue h_i for a closed waveguide containing a dielectric must either lie on the real axis in the interval $0 \le h < k\sqrt{\varepsilon}$ or (at sub-critical frequencies) on the imaginary axis. Only properties 1-4 above hold for an unshielded dielectric rod[1], and for frequencies less than the transition frequency κ_{1i} (which is usually called the "critical" frequency), Imh_i increases rapidly (cf. the dashed and dotted curve in Fig. 1).

In order to evaluate the fields, we deform the integration contour so that we get integrals along a cut in the h-plane plus a sum of residues corresponding to the poles of the integrands.[1] The residues at the poles on the real axis correspond to slow surface-wave modes. The residues a the poles where Im g < 0 correspond to fast, outward-propagating quasimodes which are damped only weakly wh $\kappa > \kappa_{2i}$. The loop integrals around the cut correspond to a additional field (a "lateral wave" in the terminology of Re 8).

4. EXCITATION OF A SLIT QUASI-TE_{00} WAVE IN A CYLINDRICAL SLIT LINE

We take the excitation source to be a magnetic dipole oriented along the rod axis and located at the center of the slit. The primary field is then given by Eqs. (1), (2) with $\varphi_0 = 0$, $l = a$, $d_{n_9} = 0$.

We use Van der Waerden's method to estimate the functions (3) that describe the secondary field far away from the dipole. Making the change of variable $ih = -s + ik$ we get

$$\{E_z^{(\pm)},\ H_z^{(\pm)}\} = ic_0 e^{ikz} \times \int_L \{E^{(\pm)}(s),\ H^{(\pm)}(s)\}\, e^{-sz} ds, \tag{14}$$

where the contour L goes down the imaginary axis (Fig. 2) The slits extend to the right and left of the points $s = 0$ and $s = 2ik$, respectively, and are parallel to the real axis.

We use Cauchy's theorem to deform the contour L to the right; the integral in (14) is then replaced by the sum c the residues of the integrands plus an integral over the loc L' along the slit extending from the point $s = 0$ (Fig. 2). It can be shown that Watson's lemma applies after the integrands are expanded as power series near $s = 0$.

In what follows we confine ourselves to the case of greatest practical importance, when the slit is "exponentially" narrow, i.e., $|\ln^{-1}\theta| \ll 1$, so that the rod forms a CS. CSL, and the wavelength is long enough so that $ka\sqrt{\varepsilon} < 1$. The last condition implies that

$$\kappa < \kappa_{21} = \sqrt{\frac{\varepsilon - 1}{\varepsilon}}\,[1.84 + O(\theta^2)]; \tag{15}$$

i.e., the line is supercritical for all of the higher modes. There is only one pole in the s-plane; it lies either on the imaginary axis or close to it, and it corresponds to the fundamental quasi-TE_{00} slit mode.[2,3]

We use this terminology because the slit wave is rigorously a TE-wave in a line without any dielectric,[10] and the eigenfrequency (critical frequency) is equal to

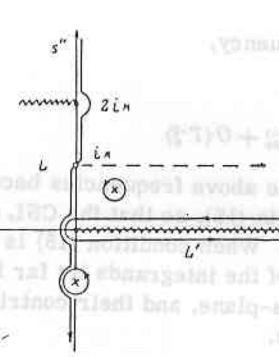

FIG. 2

$$x_n = \left(-2\ln\sin\frac{\theta}{2}\right)^{-1/2}\left(1 + \frac{i\pi}{16}\ln^{-1}\sin\frac{\theta}{2}\right)$$

which tends to zero as $\theta \to 0$. Here we must recall that for any internal Neumann problem for the Helmholtz equation, the spectrum begins with a zero eigenvalue which corresponds to an eigenfunction which is constant over the entire cross section of the waveguide. Maxwell's equations imply that the value of this constant is identically zero. Thus, in a closed waveguide a TE_{00} mode with zero cutoff frequency exists only by convention (its amplitude is zero), so that the TE_{11} mode is usually regarded as the fundamental mode. However, the presence of the slit causes the amplitude of the TE_{00} mode to be nonzero (it is an "outflowing" mode[10]), and the TE_{00} mode becomes the fundamental mode of the slit structure. If the waveguide is filled with a dielectric, the slit wave becomes a hybrid wave with a dominant H_z component, i.e., it becomes a quasi-TE_{00} wave (analogous to a quasi-TEM wave in strip lines).

The location of the pole corresponding to the slit wave is given by

$$s_0 = i(k - h_0),\ h_0 = h_0' + ih_0'' = k\left[1 - x_0^2\varkappa^{-2}(\varepsilon - 1)\right]^{1/2}, \tag{16}$$

where x_0^2 is a solution of the equation to which (13) reduces if terms $O(\varkappa^2 x^4 \ln x)$ are neglected,

$$4\varkappa^2 + \varkappa^4 - 8T_{00}^{-1}(-u) + x^2(8 + 3\varkappa^2 + x^2) + x^2(\varkappa^2 + 2x^2) \tag{17}$$
$$\times \ln\left(-{}^1/_4\gamma^2 x^2\right) = 0,\quad \gamma = 1.781.$$

It is not difficult to show that to first order the approximation (17) gives[2]

$$h_0 = k\left[\frac{\varepsilon + 1}{2} + \frac{1}{2}(ka)^{-2}\ln^{-1}\sin\frac{\theta}{2}\right]^{1/2}. \tag{18}$$

Depending on the values of the parameters $\varkappa$, ε, and θ, we find that the slit wave may resemble either a fast outflowing wave or an undamped surface wave with a retarded phase velocity. The transition frequency separating these two types of behavior is equal to [cf. (17)]

$$\varkappa_{10} = (2/T_{00})^{1/2} + O(T_{00}^{-1}). \tag{19}$$

Below the transition frequency radiation damping occurs:

$$h_0'' = \pi(k^2 - h_0'^2)(8h_0' T_{00})^{-1} + O(T_{00}^{-2}),\quad 0 \leqslant \varkappa_{10} - \varkappa \ll 1. \tag{20}$$

The critical frequency above which a weakly damped outflowing wave can occur is equal to

$$\varkappa_{20} = \left[\frac{2(\varepsilon - 1)}{(\varepsilon + 1)T_{00}}\right]^{1/2} + O(T_{00}^{-1}), \tag{21}$$

and at the critical frequency,

$$h_0(\varkappa_{20}) = e^{i\pi/4}k(\pi/2)^{1/2}T_{00}^{-1} + O(T_{0}^{-2}). \tag{22}$$

As $\theta \to 0$, both of the above frequencies become very small compared to $\varkappa_{21}$ in (15), so that the CSL emits over a wide frequency band. When condition (15) is satisfied, all of the other poles of the integrands lie far from the imaginary axis in the s-plane, and their contribution to the total field is negligible.

We evaluate the residue at the point s_0 corresponding to generation of the quasi-TE_{00} wave. Limiting ourselves to the approximation of an "exponentially" narrow slit, we see that

$$E^{(\pm)}(h) = O(\theta^2),\ H^{(\pm)}(h) = c_0\gamma_0^H e^{ihz} S_0^{(\pm)H}(h, r, \varphi)\left[1 + O(\ln^{-1}\theta)\right] + Q^{(\pm)} + O(\theta^2). \tag{23}$$

Solution of system (7) to first order then gives

$$\gamma_0^H = (D_0' - xF_0D_0)\Delta_0^H\left[x^2F_0(1 - \Delta_0^H T_{00})\right]^{-1} + O(T_{00}^{-2}). \tag{24}$$

Only the first of the two terms in (23) for the magnetic field is singular at the point h_0; this singularity is due to the denominator in γ_0^H, which reduces to Eq. (17). Evaluation of the residue therefore gives the result

$$E_z^{(\pm)} = O(\theta^2),\ H_z^{(\pm)} = K_0 S_0^{(\pm)H}(h_0, r, \varphi)e^{ih_0 z}\left[1 + O(\ln^{-1}\theta)\right], \tag{25}$$

where

$$K_0 = c_0 x_0 \varkappa_{10}^{-1}\ln(\gamma x_0/2i)(2h_0 a^4)^{-1} \tag{26}$$

can be regarded as the excitation coefficient of the slit wave; K_0 vanishes at the transition frequency but increases rapidly away from $\varkappa_{10}$.

The function $S_0^{(\pm)H}(h_0, r, \varphi)$ describes the field configuration of a slit wave traveling down the CSL at a constant velocity h_0 away from the dipole; the configuration is the same as for the corresponding natural mode.[3] The electric field is for the most part transverse ($E_z \ll E_r, E_\varphi$) and concentrated near the slit, while the axis component (25) dominates the magnetic field and reaches a maximum inside the rod at a point opposite to the slit.

5. EVALUATION OF THE ADDITIONAL FIELD INSIDE AND NEAR THE ROD

In addition to the fundamental mode (25), there is an unlocalized field given by the integral around the cut extending from the point s = 0. This additional field decays algebraically provided that the fundamental wave (as a function of the parameter $\varkappa$) either does not decay as $z \to \infty$ or else decays exponentially. Our problem is to find the asymptotic behavior of the additional wave.

Up to terms of order $[1 + O(\ln^{-1}\theta)]$ we can reduce the integrals along the cut to

$$H^{(\pm)} = -2ic_0 e^{ikz}\int_0^\infty \mathrm{Re}\left[\gamma_0^H S_0^{(\pm)H} + Q^{(\pm)}\right]e^{-sz}ds.$$

by using the circuit relations for the cylinder functions.

We first calculate the additional field inside the rod. The factor e^{-sz} causes the integrand to decay rapidly, and only s values close to 0 contribute significantly to the integral. If we introduce the quantity $s_1 = (kza^2)^{-1/2}$, we can show as in Ref. 1 that the functions in the integrand may be replaced for $s \leq s_1$ by their approximations for $|x| \ll 1$ provided the conditions $kz \gg 1$, $kz \gg (ka)^2$ are satisfied. We may therefore substitute the expressions

$$\gamma_0^H = -\frac{i\varkappa^2\varkappa_{10}^2}{\pi a^2(\varkappa_{10}^2 - \varkappa^2)}x\ln\frac{\gamma x}{2i} + O(\varkappa^2 x^3 \ln x), \tag{28}$$

$$S_0^{(+)H} = -\frac{x}{\varkappa_{10}^2}\left[\varkappa_{10}^2 L\left(\frac{r}{a}e^{i\varphi}\right) + 2 - \frac{\varkappa^2 r^2}{2a^2}\right] + O(x^3). \tag{29}$$

Here

$$L(t) = 2\ln\frac{1}{2}\left|1 - t + (t^2 - 2t\cos\theta + 1)^{1/2}\right|.$$

Since $x^2 \simeq -2iks\,a^2$, we get the result

$$H_z^{(+)} = \frac{4ic_0 k \varkappa_{10}^2}{\pi(\varkappa_{10}^2 - \varkappa^2)}\left[1 - \frac{\varkappa^2 r^2}{4a^2} + \frac{\varkappa^2}{2} L\left(\frac{r}{a} e^{i\varphi}\right)\right] \ln\frac{z}{A} \frac{e^{ikz}}{z^2} + O(z^{-3}\ln z), \tag{30}$$

where $A = 1/2 \cdot e\gamma k a^2$. A similar method can be used to estimate the additional field in the region $kz \gg 1$, $kz \gg (kr)^2$ outside the rod. Here $|gr| \ll 1$, so that we can write

$$S_0^{(-)\,H} = \frac{x^3}{\varkappa_{10}^2}\left[\frac{\varkappa_{10}^2}{\varkappa^2} L\left(\frac{a}{r} e^{i\varphi}\right) + \ln\frac{\gamma g r}{2i}\right] + O(x^5 \ln x). \tag{31}$$

Evaluating the integral using the above procedure, we conclude that

$$H_z^{(-)} = -\frac{1}{2} ic_0 (ka)^2 \varkappa^2 \left(1 + \frac{2}{\varkappa_{10}^2 - \varkappa^2}\right) \ln\frac{z}{A_1} \frac{e^{ikz}}{z^3} + O(z^{-3}), \tag{32}$$

where $A_1 = 1/2 \cdot e^{3/2}\gamma kar$. It can be shown that near the frequency κ_{10}, the additional field remains appreciable at significantly greater distances from the dipole. Since $\Delta_0^H = 1/2 \cdot (\kappa^2 + 2x^2)[1 + O(x^2 \ln x)]$, we must replace $k_{10}^2 - \kappa^2$ in Eq. (28) by $-2x^2$ close to the transition frequency, after which the asymptotic behavior of the resulting integrals must again be found. We obtain

$$H_z^{(+)} = -\frac{ic_0 \varkappa_{10}^2}{2a^2}\left[1 - \frac{\varkappa_{10}^2 r^2}{4a^2} + \frac{\varkappa_{10}^2}{2} L\left(\frac{r}{a} e^{i\varphi}\right)\right] \frac{e^{ikz}}{z} + O(z^{-2}), \tag{33}$$

$$H_z^{(-)} = -\frac{ic_0 k \varkappa_{10}^2}{4\pi} \ln^2 \frac{z}{A_1} \frac{e^{ikz}}{z} + O(z^{-1}). \tag{34}$$

We note that the additional field decays more slowly as $z \to \infty$ than the field of the outflowing wave, which decays exponentially. Nevertheless, at moderate distances from the dipole satisfying $\ln(z/a) \gg h''z$, the outflowing slit wave, i.e., the residue at the pole (16) for $\kappa_{20} < \kappa < \kappa_{10}$, may contribute appreciably or even dominate the total field near and inside the rod.

6. SPHERICAL WAVE

We can use the standard method of steepest descent to calculate the field in the far zone outside the region considered in Sec. 5. In this case, the field is regarded as a spherical wave whose amplitude-phase distribution is described by the components E_φ and H_α, where the angle α in the spherical coordinate system (R, φ, α) is measured from the z axis. It suffices to evaluate the potential $W_z^{(-)}$ by setting $kR \sin^2 \times \alpha \gg 1$ and using the fact that in the far region $E_\varphi = -H_\alpha = k^2 \sin\alpha\, W_z^{(-)}$.

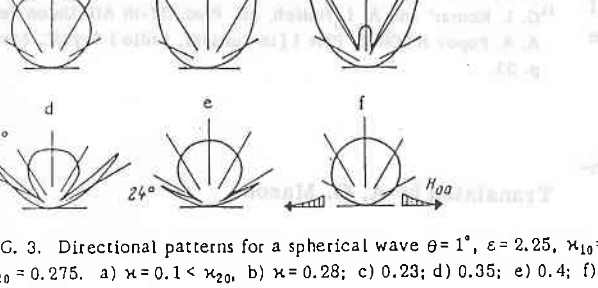

FIG. 3. Directional patterns for a spherical wave $\theta = 1°$, $\varepsilon = 2.25$, $\varkappa_{10} = 0.46$, $\varkappa_{20} = 0.275$. a) $\varkappa = 0.1 < \varkappa_{20}$, b) $\varkappa = 0.28$; c) 0.23; d) 0.35; e) 0.4; f) $\varkappa = 0.6 > \varkappa_{10}$.

The potential $W_z^{(-)}$ is given by an integral similar to (3), and analysis reveals that there is a saddle point at $h_s = k\cos\alpha$. In the limit $kR \to \infty$ we can neglect any poles h_i that might be present near h_s and can even neglect the residues from these poles when we deform the contour of integration into the path of steepest descent, since their contribution is exponentially small. However, at the transition frequency the pole $h_0 = k$ coincides with the saddle point h_s when $\alpha = 0$ and the method of steepest descent does not apply in its usual form.

The method of steepest descent gives the following expressions for the field:

$$E_\varphi = -H_\alpha = c_0 k^2 \sin\alpha\,[1 + \Phi_1(\alpha, \varphi) + \Phi_2(\alpha, \varphi)]\,(e^{ikR}/R), \tag{35}$$

where

$$\Phi_1(\alpha, \varphi) = -x_s^2 \ln\frac{\gamma x_s}{2i} \frac{\varkappa^2 + 2x_s^2 - \varkappa_{10}^2 x_s (4i\cos\varphi + x_s\cos 2\varphi)}{2\varkappa_{10}^2 - (\varkappa^2 + 2x_s^2)(2 - x_s^2 \ln(\gamma x_s/(2i)))},$$

$$\Phi_2(\alpha, \varphi) = -\frac{i\varkappa^2 x_s \cos\varphi}{\varkappa^2 + 2x_s^2} - \frac{1}{8}\varkappa^2 x_s^2 \left(\frac{2\cos 2\varphi}{\varkappa^2 + 2x_s^2} + \ln\frac{\gamma x_s}{2i}\right), \quad x_s = ka\sin\alpha.$$

Here we have recalled that for $ka\sqrt{\varepsilon} < 1$ the cylindrical functions can be expanded in power series, and that for a narrow slit $T_{n0}(-u) = [(-1)^{n+1}/|n|\,] + 0(\theta^2)$.

Analysis of (35) reveals that the amplitude and phase of of the spherical wave depend only weakly on the coordinate φ. Away from the transition frequency, we have $\Phi_1(\alpha, \varphi) = O(\alpha^2 \ln\alpha)$, $\Phi_2(\alpha, \varphi) = O(\alpha)$ as $\alpha \to 0$ and the spherical wave vanishes as it approaches the axis of the rod. However, the poles near the saddle point cannot be neglected if $\kappa = \kappa_{10}$ and $\alpha \to 0$ or if kR is not sufficiently large. In this case, the modified method of steepest descent[8,9] must be used to estimate the integrals, according to which the contribution from a pole is described by an additional probability integral of complex argument.

Calculation of the directional pattern (35) reveals that for $\kappa_{20} \lessgtr \alpha < \alpha_{10}$ there exists an angle α_0,

$$\sin^2\alpha_0 = (\varepsilon - 1)\frac{\varkappa_{10}^2 - \varkappa^2}{2\varkappa^2}\left[1 + \frac{1}{8}\varkappa_{10}^2 \ln\frac{\gamma^2(\varkappa_{10}^2 - \varkappa^2)}{8}\right], \tag{36}$$

for which the real part of the denominator of $\Phi_1(\alpha, \varphi)$ vanishes, so that $\Phi(\alpha, \varphi) = \sin\alpha_0\, O(\ln ka \sin\alpha_0)$. This implies that the radiation pattern of the spherical wave contains a narrow lobe along the direction α_0 (Fig. 3). If α is not within the above interval, the rod distorts the wave emitted by the dipole only slightly, since $a \ll \lambda$.

7. RADIATION RESISTANCE AND ENERGY DISTRIBUTION BETWEEN A SPHERICAL AND A SURFACE WAVE

The radiation resistance R, which is related to the energy by $P = 1/2 \cdot I^2 R$, is a measure of the output power efficiency when the CSL is present. Since a spherical wave is present in the structure for all frequencies κ, while for $\kappa > \kappa_{10}$ two additional undamped slit waves are excited and carry off energy from the dipole in the directions $z \to \pm\infty$, we have

$$R = R_{\rm sph} + \begin{cases} 0, & \varkappa \leqslant \varkappa_{10}, \\ R_{\rm slit}, & \varkappa > \varkappa_{10}, \end{cases} \tag{37}$$

$$R_{\rm sph} = \frac{c}{4\pi I^2}\int_0^{2\pi}\int_0^{\pi} E_\varphi H_\alpha^* R^2 \sin\alpha\, d\alpha\, d\varphi,$$

$$R_{\rm slit} = \frac{c}{4\pi I^2}\int_0^{2\pi}\int_0^{\infty} (E_r H_\varphi^* + E_\varphi H_r^*)\, r\, dr\, d\varphi. \tag{38}$$

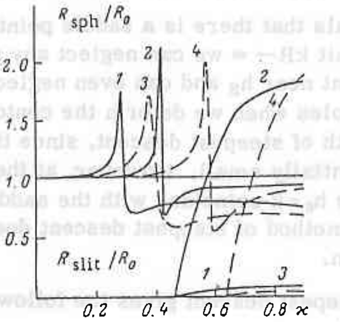

FIG. 4. Radiation resistance vs frequency parameter $\varkappa$; θ (deg), ε, $\varkappa_{10}$, $\varkappa_{20}$; 1) 1, 2.25, 0.46, 0.275; 2) 1, 10, 0.46, 0.42; 3) 10, 2.25, 0.64, 04; 4) 10, 10, 0.64, 0.575.

We can express R alternatively in terms of the field amplitude at the dipole

$$R = -b/l \cdot \mathrm{Re}\, H_z^{(-)}(a, 0, 0), \tag{39}$$

by using a standard theorem concerning the complex power; here $H_z^{(-)}$ is given by the integral in (3). As pointed out in Ref. 1, in order to separate out the real part it is convenient to introduce a cut in the complex plane which extends from the branch point h = k to 0 along the real axis and then ascends the imaginary axis. By deforming the contour as in Sec. 4, we reduce the integral to a sum of the residues at the poles on the real axis plus a loop integral along the cut. Only the portion of the integral taken along the interval [0, k] of the real axis is real and remains finite for $\varphi = 0$, $r \to a$, $z \to 0$. We note that the complex poles (in particular, the pole h_0) do not contribute to (39) when $\varkappa < \varkappa_{10}$, since for our choice of cut they lie on the other sheet of the Riemann surface. In other words, the leaky waves do not participate in energy transport because they decay exponentially where they are defined. After a good deal of calculation, we thus obtain

$$\frac{R_{sph}}{R_0} = 1 + \frac{3}{4k}\int_0^k \mathrm{Re}\left[\gamma_0^H S_0^{(-)H}(h, a, 0) + Q^{(-)}\right] dh, \tag{40}$$

$$\frac{R_{slit}}{R_0} = \frac{3\sqrt{2}}{128} \frac{(\varepsilon - 1)^2 (\varkappa_{10}^2 - \varkappa^2)^2 \ln(\gamma^2/8)(\varkappa_{10}^2 - \varkappa^2)}{\varkappa^3 \sqrt{\varkappa^2(\varepsilon + 1) - \varkappa_{10}^2(\varepsilon - 1)}} \times \left[\varkappa_{10}^2 \ln \frac{\gamma^2}{8}(\varkappa_{10}^2 - \varkappa^2) - 2\right], \tag{41}$$

where $R_0 = 2(kb)^2/3c$ is the radiation resistance of a dipole in vacuum.

Figure 4 shows the calculated frequency-dependence of the radiation resistance. The curves show that the energy carried off by the spherical wave is a maximum at the critical frequency $\varkappa_{20}$ and has a minimum near the beginning of the interval $\varkappa_{20} < \varkappa < \varkappa_{10}$. For $\varkappa > \varkappa_{10}$ undamped surface slit waves are generated, and the energy that they carry off is comparable to the energy removed by the spherical wave and may even exceed it as the frequency increases.

CONCLUSION

We have thus found that at high frequencies, each eigenmode of a partially shielded rod behaves as an undamped slow wave; however, as the frequency decreases, there is a transition frequency $\varkappa_{1i}$ for which outflowing modes are generated. In contrast to what is found for an unshielded dielectric rod, in our case the outflowing waves remain largely undamped not only for frequencies $\approx \varkappa_{1i}$ but throughout a range extending up to the critical frequency $\varkappa_{2i}$. Below $\varkappa_{2i}$ the damping increases rapidly, so that the partially shielded rod behaves like a closed waveguide.

The quasi-TE_{00} slit wave is the fundamental mode if the rod is a cylindrical slit line ($\theta \ll 1$), and both of the characteristic frequencies (transition and critical) for this mode are extremely low. This means in practice that the frequency band for single-mode operation is wide (two octaves or more), and the geometric dimensions of the line are an order of magnitude smaller than for a corresponding TE_{11}-mode closed waveguide. The field of the slit wave is localized near the slit, which ensures that the effective dimensions of the line are small, and the retardation is less than $\sqrt{(\varepsilon + 1)/2}$.

Because of their bandwidth and the fact that outflowing waves are generated for a wide range of parameters, cylindrical slit lines can be employed as antennas as well as microwave circuits. For example, it has been suggested that CSLs be used in miniature scanning millimeter-range slit emitters[11] with a diameter at least 4-6 times smaller than for comparable waveguide emitters.

When a CSL is excited by an elementary source, the EM field in the region $z \gg kr^2$ has the form of a spherical wave whose directional pattern contains a lobe due to radiation from the outflowing slit wave when $\varkappa_{20} < \varkappa < \varkappa_{10}$. The near field (for $z \ll kr^2$) is a superposition of the leaky or surface slit wave and the additional wave, which decays algebraically. The latter appears in addition to the eigenmodes because of the open waveguide structure.

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Translated by A. G. Mason