EFFECTIVE INDEX DISPERSION ACCOUNT IN THE COLD MODEL OF DISK RESONATOR WITH UNIFORM GAIN

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Introduction

Semiconductor microdisk lasers with whispering-gallery (WG) modes attract much attention since the early 1990's, as sources of infrared and optical radiation having ultra-low thresholds [1-3]. Such lasers can be optically pumped or injection ones. Several authors have worked on building theoretical description of the optical modes of microdisks. This was done either with rough analytical assumptions [4,5] or through FDTD simulations [6]. In these papers, 3-D problem for a finite thickness disk was approximately reduced to the 2-D one based on the effective index method [7-9] developed earlier for layered media, however dependence of the effective index on frequency was neglected. Our paper presents a study of 2-D lasing eigenvalue problem for a microdisk, based on the complete Maxwellian formulation with transmission condition at the disk boundary and radiation condition at infinity. Unlike previous papers, we take account of the presence of many guided waves able to propagate in the disk and of the frequency dispersion of each associated effective index.

Effective index approach

Suppose that disk of the thickness d and radius a is nonmagnetic, has real-valued refraction index α, and is placed in vacuum. The electromagnetic field is assumed to have the time dependence as $e^{-i\omega t}$, where ω is the angular frequency. Then, free-space wavenumber is $k=\omega/c=2\pi/\lambda$, where c is the free-space light velocity.

So-called effective-index approach starts from the assumption that the dependences of the field $\{\boldsymbol{E},\boldsymbol{H}\}$ on the vertical coordinate z and in-plane coordinates $\boldsymbol{r}=(r,\varphi)$ can be separated everywhere, e.g., $E_z(\boldsymbol{R})=V_E(z)U_E(r,\varphi)$, $H_z(\boldsymbol{R})=V_H(z)U_H(r,\varphi)$. In fact, this is incorrect because neither boundary conditions on disk surface, nor radiation condition at $R\to\infty$ is separable. However, such an assumption enables one to write independent differential equations for the functions of z and $\boldsymbol{r}$. They are, respectively,

$$\left(\frac{d^2}{dz^2}+k^2\alpha^2-k^2\alpha_{eff}^2\right)V_{H,E}(z)=0,\quad \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+k^2\alpha_{eff}^2\right]U_{H,E}(r,\varphi)=0\,, \tag{1}$$

where α turns 1 off the interval $|z|<d/2$, and effective refraction index $\alpha_{eff}=\alpha_{eff}^{H,E}$ inside cavity $(r<a)$ and 1 outside. The transmission-type boundary conditions for $V_{H,E}(z)$ depend on polarization, namely

$$V_{H,E}(\pm d/2\mp 0)=V_{H,E}(\pm d/2\pm 0),\quad \left.\frac{dV_{H,E}}{dz}\right|_{z=\pm d/2\mp 0}=\beta^{H,E}\left.\frac{dV_{H,E}}{dz}\right|_{z=\pm d/2\pm 0}, \tag{2}$$

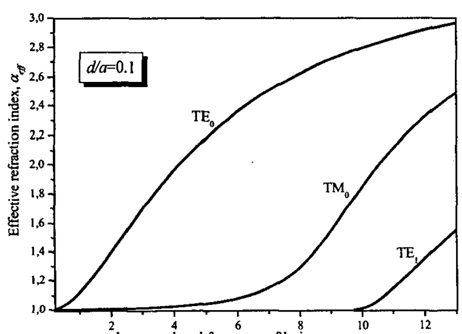

Figure 1. Dispersion characteristics of the guided modes of infinite dielectric slab made of GaAs with the bulk refraction index α=3.374.

where $\beta^H=\alpha^{-2}$ and $\beta^E=1$. To complete the formulation in the open domain, one needs a condition at infinity. Unfortunately, there is no "continuous" way to derive 1-D condition $(z\to\pm\infty)$ from the 3-D condition for $\{\boldsymbol{E},\boldsymbol{H}\}$. In order to reproduce the outgoing wave propagation off the disk plane, one has to request that

$$V_{E,H}(z)\sim e^{ik(1-\alpha^2)^{1/2}|z|},\quad z\to\pm\infty\,, \tag{3}$$

The first of equations (1), together with (2) and (3), forms a familiar 1-D eigenvalue problem for the

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parameter α_{eff}^{H} or α_{eff}^{E}, which is a normalized propagation constant of a TE (or H_z) or TM (or E_z) type guided wave of infinite dielectric slab. They are reduced to the transcendental equations

$$\tan pkd = \beta^{E,H} gp^{-1}, \ \cot pkd = -\beta^{E,H} gp^{-1} \tag{4}$$

For the each type of waves, there exists finite number $Q^{H,E} \geq 1$ of real-valued solutions, $\alpha_{eff(q)}^{H,E}$: $1 < \alpha_{eff(q)}^{H,E} < \alpha$, $q = 0,...,Q^{H,E}+1$. The largest of them correspond to the TM_0 and TE_0 waves, respectively. The even (odd) value of the wave index indicates to the symmetry (anti-symmetry) of the wave field E_z or H_z with respect to the middle plane of the slab. Plots in Fig.1 demonstrate the dependences of the effective indices on the frequency normalized by the disk radius, i.e., on $ka = kd(d/a)^{-1}$.

Lasing eigenvalue problem (LEP) and numerical results

To find the in-plane field patterns of the lasing modes in a disk cavity, we shall suppose that U is either E_z or H_z field component, depending on polarization, and introduce material gain $\gamma > 0$ in the cavity. According to (1), off the disk contour this function must satisfy the Helmholtz equation, $[\Delta + k^2 \nu^2(r,\varphi)]U(r,\varphi) = 0$, where step-wise function $\nu(r,\varphi)$ is assumed 1 outside the cavity and a complex value inside: $\nu = \alpha_{eff} - i\gamma$. The field must also satisfy the transmission conditions across the boundary of resonator and the Sommerfeld radiation condition at infinity that selects outgoing field solutions. Following [10], we look for the eigenvalues as discrete pairs of real-valued parameters, (κ,γ), that yield normalized frequencies of lasing, $\kappa = ka$, and the associated threshold material gains.

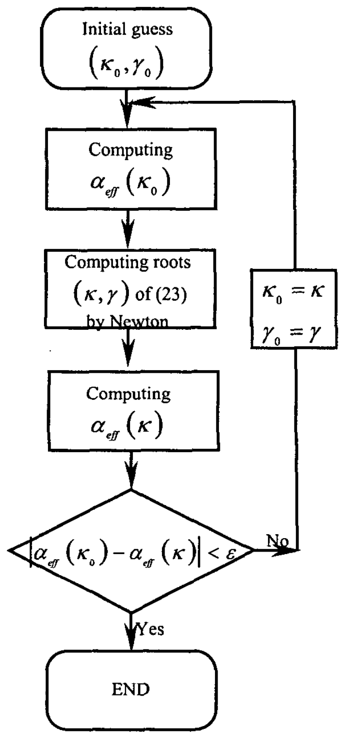

Figure 2. Algorithm chart.

By using the separation of variables, this problem is reduced to the set of independent transcendental equations:

$$J_m(\kappa\nu)H_m'^{(1)}(\kappa) - \beta\nu J_m'(\kappa\nu)H_m^{(1)}(\kappa) = 0 , \tag{5}$$

where the functions involved are the cylindrical functions. Here, β equals ν^{-2} in the case of H_z polarization or 1 in the case of E_z polarization. Note that $\alpha_{eff(q)}(\kappa)$ can be associated with any guided wave of the relevant slab. Therefore each pair should be denoted as $(\kappa_{mnq},\gamma_{mnq})$, where azimuth and radial indices are $m = 0,1,2,...,n = 1,2,...$, respectively, and effective refraction index brings the third number, q=0,1,2,... (see Fig.1). Further we use two-parametric iterative Newton method to solve (5) numerically. Unlike [10], we combine it here with accurate account of the dispersion of corresponding effective index. Fig. 2 shows the chart of the adaptive algorithm used to calculate the eigenpairs of LEP.

The plane (κ,γ) happens to be inhabited by the eigenvalues in non-uniform manner. First of all, one can clearly see a hyperbola, $\gamma \approx const/\kappa$, "saturated" with modes of all the m-th families. These modes have very high thresholds characterized with dimensionless material gain $\gamma > 0.1$. Above that curve, there are no lasing modes. In contrast, in each family with $m > \alpha$ the modes, which have $\kappa < m$ but still $\kappa > m/\alpha$, keep the same distance in κ however display drastically smaller values of γ. These values are getting even smaller for larger m. Below the mentioned hyperbola the modes (i.e., the eigenvalues) form inclined layers, each layer corresponding to a certain value of the radial index n. Thus, not automatically all the modes in a circular cavity show "whispering" property although all show the "gallery" property, i.e., periodicity in κ. Only the modes having the frequencies within the strip $m/\alpha < \kappa < m$ are "whispering", i.e., have exponentially low thresholds. This is because only these modes experience quasi-total internal reflection when propagating along the rim of the cavity. The lowest-threshold layer is formed by the WG modes having a single variation in radius $(n = 1)$. Below that layer, in the domain $\kappa < m/\alpha$, no eigenvalues are found in the m-

th family. Still besides, Fig. 3 demonstrates the effect of the effective index dispersion. For the same disk, the E_z-polarized modes of the same index q have smaller effective index, than the H_z-polarized ones, in wide band of frequencies. Therefore the E_z-modes have smaller chances to go lasing unless m and hence κ is not large enough. Similar data were computed for the other effective indices of the higher-order slab waves. They show considerable blueshifts of the lasing frequencies of the corresponding disk modes and very high thresholds – due to the smaller effective indices (see Fig. 1).

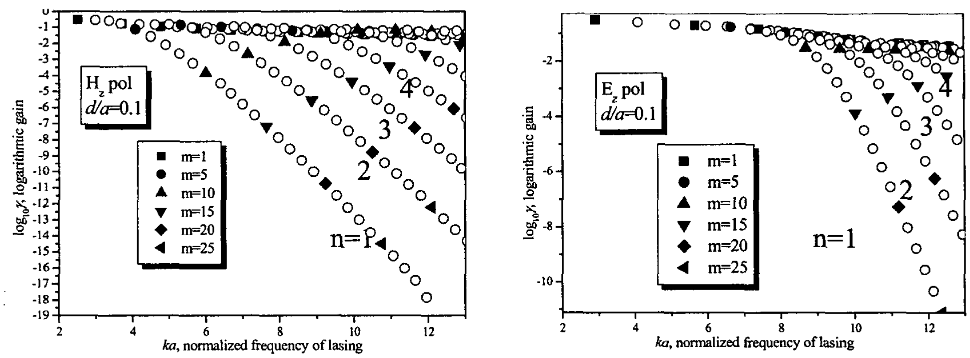

Figure. 3. Lasing spectra and threshold gains for the cold-model modes of the families $(E_z)_{mn0}$ and $(H_z)_{mn0}$ in a GaAs/InAs disk, $\alpha = 3.374$ and $d/a = 0.1$.

Conclusions

We have demonstrated that semiconductor microcavity laser can be efficiently analyzed with specialized "cold model with gain" problem, i.e. LEP. Even analysis of approximate, effective-index-based, 2-D LEPs for a circular microdisk – if done accurately – brings valuable information. For each polarization, in the plane (κ, γ) there exist "no-lasing" domains. As expected, the thresholds of the $(H_z)_{mn0}$ modes are lower than of their $(E_z)_{mn0}$ counterparts, thanks to the greater value of the effective index $\alpha_{(0)}^{H}$ than $\alpha_{(0)}^{E}$, for the same disk thickness and radius. However, in general, in the larger and thicker disks these modes are able to compete for the lasing, and the nearest higher-order, in q, modes $(H_z)_{mn1}$ are also to be considered.

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