LASING OF CIRCULAR PHOTONIC MOLECULES COMPOSED OF THREE AND FOUR MICRODISKS

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Abstract – Photonic molecules (PMs) composed of several identical active microdisks placed in a circle are investigated in this paper. We study the lasing eigenvalue problems for such PMs as electromagnetic field boundary value problems formulated in special manner, which enables one to extract not only natural frequencies but also thresholds of optical modes. The problem is reduced to an infinite matrix equation of the Fredholm second kind. Then we use a secant method to compute the roots of truncated determinant equation that converge to the eigenvalues of original problem when the determinant size is taken progressively larger. Here, all modes in a circular PM composed of M microdisks split into $2M$ independent symmetry classes. Coupling of microcavities leads to the shift in frequencies and modification of thresholds in comparison to the single microdisk. With carefully tuned separation it is possible to achieve a considerable reduction of thresholds in PMs composed of three and four microdisks.
Keywords: microdisk, photonic molecule, threshold, whispering-gallery mode

INTRODUCTION

Whispering-gallery (WG) mode microdisk semiconductor lasers are intensively investigated today as promising light sources for photonic integrated circuits. This is due to their small cavity volumes, ultra-low thresholds, and the fact that the electromagnetic field is emitted mainly in the disk plane. At the present time much experimental attention is paid to fabricating and studying microdisk arrays [1]. The reason is the expectation to enhance light output due to optical field coupling. Closely spaced microresonators are called photonic molecules (PMs) [2]. If they contain active regions, PMs start lasing when the pump balances the radiation losses. The first microring PM lasers were apparently reported in [3] where it was demonstrated that the disk modes split to multiplets and have different thresholds. Then PM lasers composed of two microspheres were measured [4] and showed similar features. Today, microdisks remain very attractive objects of research due to well-developed planar etching and deposition technologies. This has enabled the authors of [5] to design PM lasers composed of linear and square arrays of microdisks with fixed distance between them.

The previous analysis of modes in optically coupled passive microdisks has been made with the FDTD numerical approximation to the fields excited by a pulsed source [5]. In contrast, here we accurately study the frequencies and thresholds of modes of circular PMs composed of two, three and four microdisks. We consider boundary-value problems for the PMs with the Maxwell equations and exact boundary and radiation conditions, as specific "cold cavity with gain" eigenvalue problems. Such a formulation was introduced in [6,7] and called lasing eigenvalue problem (LEP). To characterize the influence of the field coupling on lasing characteristics we investigate them as a function of the separation between microdisks.

LASING EIGENVALUE PROBLEM IN 2D CASE FOR PMS

If the disk thicknesses are only a fraction of the lasing wavelength then the 3-D field problem can be reduced to a 2-D one with the aid of the effective index method [7]. Then we can consider a microcavity with effective refractive index α_{eff} and look for the optical field in the disk plane. Fig. 1 shows the geometries of the "circular" PMs in the 2-D case. Two different polarizations can be treated. However, the effective refractive index is close to 1 in the case of the E_z-polarized modes. Therefore, in thin disks, only the H_z-polarized modes can be studied with the aid of one function U, which is the H_z field component. The LEP statement implies that U must satisfy the 2-D Helmholtz equation where, if $r_j < a,\ j = 1,...,M$, the index α_{eff} is replaced with the complex-valued parameter $\nu = \alpha_{eff} - i\gamma$, otherwise $\alpha_{eff} = 1$. We shall assume that the material gain $\gamma > 0$ is constant, i.e., uniform across the disks. At the disk rims, $L: \bigcup_{j=\overline{1,M}} (r_j = a)$, the optical transparency conditions hold. Thanks to the real-valued k, U must obey the 2-D Sommerfeld radiation condition and does not diverge at infinity.

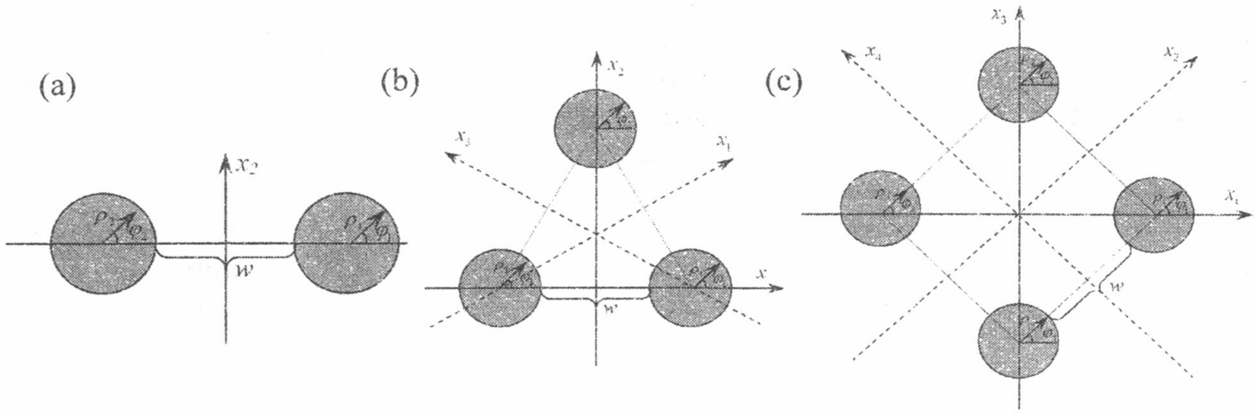

Fig. 1 – 2D geometries of circular PMs composed of two (a), three (b) and four (c) circular microcavities.

A circular PM of M identical microdisks has M-fold symmetry. Therefore, all natural modes of the circular PMs split into M independent classes according to the parity with respect to the symmetry axes. Our previous analysis of a PM composed of two identical microdisks has shown that the class of modes that is most interesting is thatwhose fields are anti-symmetric with respect to all existing symmetry lines [7]. Therefore we will consider here only such maximally anti-symmetric modes in circular PMs of three and four microdisks. The simple geometry of PMs enables one to use efficiently the modified method of separation of variables and reduce boundary-value problems to infinite matrix equations. After certain algebraic operations, these matrix equations are cast to the following forms, respectively, for PMs of three and four microdisks:

$$x_m + \sum_{p=1}^{\infty} x_p K_{mp}(\kappa,\gamma)\left[H^{(1)}_{p-m}(\kappa l)(e^{i(m+p)\pi/3} + e^{ip2\pi/3}) - (-1)^{m+p} H^{(1)}_{m+p}(\kappa l)(e^{i(m-p)\pi/3} + 1) \right] = 0 \quad (1)$$

$$\begin{aligned} & x_m + \sum_{p=1}^{\infty} (-1)^p x_p K_{mp}(\kappa,\gamma)[H^{(1)}_{p-m}(\kappa l)(e^{i(m+p)\pi/4} + e^{-i(m+p)\pi/4}) - \\ & (-1)^m H^{(1)}_{m+p}(\kappa l)(e^{i(m-p)\pi/4} + e^{-i(m-p)\pi/4}) + H^{(1)}_{p-m}(\kappa l\sqrt{2}) - (-1)^m H^{(1)}_{m+p}(\kappa l\sqrt{2})] = 0 \end{aligned} \quad (2)$$

where

$$K_{mp}(\kappa,\gamma) = J_m(\kappa) V_p(\kappa,\gamma)[F_p(\kappa,\gamma) J_p(\kappa)]^{-1},$$
$$F_m(\kappa,\gamma) = J_m(\kappa\nu) H'^{(1)}_m(\kappa) - \nu\beta^H J'_m(\kappa\nu) H^{(1)}_m(\kappa)$$
$$V_m(\kappa,\gamma) = J_m(\kappa\nu) J'_m(\kappa) - \nu\beta^H J'_m(\kappa\nu) J_m(\kappa),$$

Here, $\kappa = ka$, $l = 2 + w/a$ is the normalized distance between the centers of the resonators, and the prime denotes differentiation with respect to the argument.

In operator notation, each of equations (1) and (2) can be written as $[I+G(\kappa,\gamma)]X=0$, where $X=\{x_p\}_{p=1}^{\infty}$, $I=\{\delta_{mp}\}_{m,p=0(1)}^{\infty}$ is identity operator, and $G=\{G_{mp}\}_{m,p=1}^{\infty}$ are compact operators if only $F_p(\kappa,\gamma)J_p(\kappa)\neq 0$, $p=0,1,\ldots$. Therefore the equations obtained are the Fredholm second kind equations. Then the search for the LEP eigenvalues is reduced to finding zeros of the determinants of truncated equations, $Det[I+G(\kappa,\gamma)]=0$, and convergence to the exact eigenvalues of infinite matrices is guaranteed if the truncation number is increased. We computed lasing spectra and thresholds for coupled semiconductor microdisks with a two-parameter secant-type iterative method published in [7].

NUMERICAL RESULTS

In Figs. 2, we present the dependences of lasing frequencies and thresholds on the normalized separation parameter, *w/a*, for the WG modes of the families $(H_z)_{7,1}$ of maximally antisymmetric class in circular PMs of two, three and four microdisks. The first index corresponds to the number of field variations along the elementary disk rims and the second corresponds to the number of field variations along the radii. They show that, if the separation is very small ($w \ll 1$), then the maximally antisymmetric WG modes are blueshifted and their thresholds grow up. However, if the separation is comparable to disk radius, the thresholds can get significantly lower values than for isolated disk. Increasing the number of microdisks in the PM leads to a lowering of minimum values of thresholds. If the separation is very large ($w \gg 1$), thresholds and frequencies oscillate close to their values for an isolated cavity.

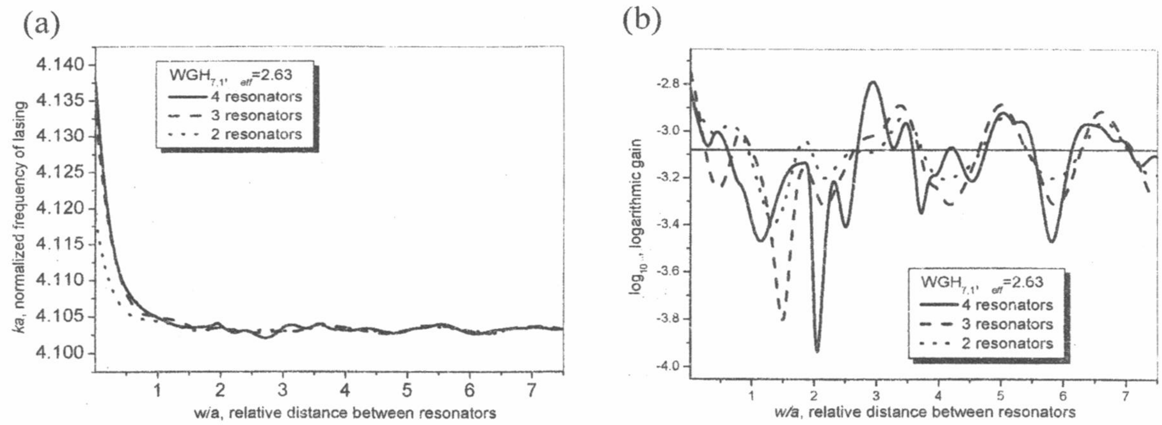

Fig. 2 – Frequencies (a) and threshold gains (b) of the WG modes $(H_z)_{7,1}$ of maximally anti-symmetric class in circular PMs composed of two, three, and four GaAs disk resonators. $\lambda = 1.55$ µm, $\alpha_{eff} = 2,63$. Straight line is the threshold of the similar mode in one microdisk.

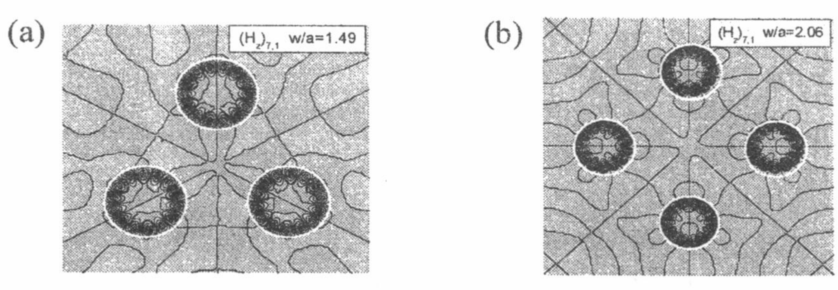

Fig.3 – Near-field portraits of $(H_z)_{7,1}$ modes of maximally anti-symmetric class in circular PMs of three (a) and four (b) resonators. Distances correspond to the lowest threshold values in Fig. 2-b.

CONCLUSIONS

A specialized eigenvalue problem, i.e. LEP, enables one to study PM lasing. Instead of two degenerate WG modes of each azimuth-index family in a single circular resonator, the modes of M coupled resonators arranged in a circle fall into one of $2M$ symmetry classes, each class having a unique set of parities across the symmetry axes. We have demonstrated that the "cold-cavity" thresholds of the coupled modes in circular PM can be lowered with respect to the corresponding mode thresholds of a photonic atom, i.e. one disk, provided that the separation between resonators is properly tuned. The minimum thresholds obtain even lower values with increasing the number of atoms in PM.

ACKNOWLEDGEMENTS

We are grateful to S.V. Boriskina for valuable discussions. This work was supported by EPSRC-UK via grant GR/S60693/01P, Royal Society via project IJP-2004/R1-FS, and INTAS-EU via Ph.D. fellowship 04-83-3340.

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