## REDUCTION OF LASING THRESHOLDS IN CIRCULAR PHOTONIC MOLECULE MICRODISK LASERS

Elena I. Smotrova<sup>1</sup>, Alexander I. Nosich<sup>1</sup>, Trevor M. Benson<sup>2</sup>, and Phillip Sewell<sup>2</sup>

<sup>1</sup> Institute of Radio-Physics and Electronics NASU, Kharkov, 61085 Ukraine <sup>2</sup> George Green Institute for Electromagnetics Research, University of Nottingham, Nottingham NG7 2RD, UK

Microdisk lasers have been attracting much attention since the 1990's as extremely compact sources of light [1-3]. Lasing in disks of 1-10 µm diameter and 100-200 nm thickness, containing one or several quantum wells or lavers of quantum dots has been achieved both with photopump and injection of current. The main features of such lasers are (i) periodically spaced frequencies of lasing, (ii) ultralow thresholds, and (iii) predominantly in-plane light emission. The one disk lasing modes are identified as whispering-gallery (WG) ones confined at the rim due to almost total internal reflection. Description of the WG modes in the isolated disk has been done with many techniques ranging from a WKB analytical study to the FDTD numerical codes [4,5]. Note that the 3-D electromagnetic problem for a thinner than the wavelength disk can be approximately reduced to the 2-D one in the disk plane, with the effective-index approach.



Fig. 1. 3-D geometry of optically coupled circular disk semiconductor microresonators forming a circular PM.

The traditional way of cold-cavity modeling of microlasers calculates of the natural modes of *passive* open dielectric resonators. However this approach does not affect the lasing phenomenon directly since the specific value of threshold gain needed to force a mode to become lasing is not included in the formulation.

Therefore in [6] we proposed a *Lasing Eigenvalue Problem* (LEP), specifically tailored to extract not only frequencies but also threshold gains from the field equations. Refining this analysis, in [7] we accurately accounted for the effective index dispersion and studied the effect of the gain non-uniformity on the thresholds in single microdisk.

Presently great attention is paid to the manufacturing and studying of microcavity laser arrays and photonic molecules (PMs) [8,9]. Experiments with such resonators show the splitting of the coupled WG modes into symmetry classes that may have quite different lasing properties. This was confirmed by our recent analysis of the lasing in a twin-microdisk PM [10]. Here, we study the LEP for circular PMs made of several identical microdisks, which support optically coupled WG modes.

Fig. 1 shows the geometry of circular PM of M identical microdisk cavities located in the same plane in the free space. Suppose that each disk has thickness d, radius a, and real-valued refractive index  $\alpha$ . Separation between the adjacent disks is denoted as w. Time dependence is implied as  $e^{-i\omega t}$ , and free-space wavenumber is  $k = \omega/c = 2\pi/\lambda$ , where  $\lambda$  is the wavelength. Assume that we have already reduced the problem to the 2-D model by using the effective-index model [7]. Then we can consider M identical circular resonators on the plane (x,y), characterized by the same effective refractive index  $\alpha_{eff}$  and air gaps w.

In 2-D, one can treat either of the two polarization states separately, with the aid of one function U, which is either the  $E_z$  or  $H_z$ field component. The LEP statement implies (see [6,7]) that U must satisfy 2-D Helmholtz equation, where, if r < a, the refractive index  $\alpha_{\rm eff}$  is replaced with the complex-valued parameter  $v = \alpha_{eff(q)}^{H,E} - i\gamma$ , otherwise  $\alpha_{eff} = 1$ . Here, the effective index is associated with the q-th guided wave of a slab of the same thickness as the disk  $(1 < \alpha_{eff(q)}^{H,E} < \alpha)$  [7]. We shall assume that the material gain is uniform across the disks. At the disk rims, L, field continuity conditions are demanded. Moreover, the condition of local power finiteness is to be satisfied. Considering the LEP, we look for two real numbers,  $\kappa = ka$ and  $\gamma$ , which are normalized lasing frequency and threshold material gain. Thanks to the real k, U must obey the Sommerfeld radiation condition (no divergence at infinity).

From the symmetry considerations it is clear that all possible field functions split to 2M independent classes. By using series expansions

in local coordinates and addition theorems for cylindrical functions, we reduce the problem, for each class, to a determinant equation with favorable features. Here, numerical convergence is guaranteed if the truncation number is taken sufficiently large. Practical accuracy of 4-5 digits is achieved with a few more equations than resonator's optical size,  $ka\alpha$ .

We computed lasing spectra and thresholds with a two-parameter secant-type iterative method [7] taking  $\alpha_{eff} = const$  when computing a specific optical mode. If, e.g.,  $\lambda_0 = 1.55 \ \mu m$  and d = 100nm, then  $\alpha_{eff(0)}^{H} = 2.63 \ (E_z$ -polarized modes can be neglected in view of very high thresholds [6,7]).

Lasing modes of an *isolated circular cavity* split, thanks to the symmetry, into independent families according to the azimuth index, *m*, and those with m > 0 are twice degenerate [6]. One can clearly distinguish between the non-WG modes, which have very high thresholds,  $\gamma \approx const/\kappa$ , and the true WG modes. The latter have  $m/\alpha_{eff} < \kappa < m$  and display drastically smaller thresholds,  $\gamma \approx const e^{-\kappa}$  [7].



Fig. 2. Threshold gains for the H<sub>z</sub>-polarized modes of the family  $(H_z)_{7,1}$  of the maximally symmetric field class in a GaAs disk,  $\lambda = 1.55$  nm,  $\alpha = 3.374$  and d/a = 0.1. Straight line is the threshold of the corresponding mode in single microcavity.

In Figs. 2 and 3, we present the dependences of thresholds on the normalized separation parameter, w/a, for the coupled WG modes of the family  $(H_z)_{7,1}$  of the maximally symmetric and anti-symmetric field classes, respectively. If separation is smaller than a certain value, then the thresholds of most of the modes get drastically higher than for the isolated disk. However, if w is comparable to  $\lambda$ , one can achieve a threshold that is much lower than the limit one for  $w \rightarrow \infty$ .

Note that threshold reduction with respect to the single-disk case is relatively small and needs precise tuning of the separation between resonators.



Fig. 3. The same as in Fig. 2 for the modes of the maximally anti-symmetric field class.

It may occur for the modes of any class of symmetry. If the separation gets smaller, then the WG modes obtain frequency shifts – half of them are redshifted and another half blueshifted (not shown here). Normalized emission patterns of the WG modes of a circular PM made of M microdisks display either M or 2M identical main beams (due to the M-fold symmetry), along the symmetry axes.

This work was supported by EPSRC-UK via grant GR/S60693/01P, Royal Society via project IJP-2004/R1-FS, and INTAS-EU via grant 04-83-3340.

## REFERENCES

- S.L. McCall, et al., "Whispering-gallery mode microdisk lasers," Appl. Physics Lett., vol. 60, no 3, pp. 289-29, 1992.
- [2] T. Baba, et al., "Lasing characteristics of GaInAsP-InP strained quantum-well microdisk injection lasers with diameter of 2-10 m," *IEEE Photonics Technol. Lett.*, vol. 9, no 7, pp. 878-890, 1997.
- [3] B. Gayral, et al., "High-Q wet-etched GaAs microdisks containing InAs quantum boxes," *Appl. Physics Lett.*, vol. 75, pp. 1908-1910, 1999.
- [4] N.C. Frateschi, A.F.J. Levi, "The spectrum of microdisk lasers," J. Appl. Physics, vol. 80, no 2, pp. 644-653, 1996.
- [5] M. Fujita, A. Sakai, T. Baba, "Ultrasmall and ultralow threshold GaInAsP-InP microdisk injection lasers," *IEEE J. Selected Topics Quantum Electronics*, vol. 5, no 3, pp. 673-681, 1999.
- [6] E.I. Smotrova, A.I. Nosich, "Mathematical analysis of the lasing eigenvalue problem for the WG modes in a 2-D circular microcavity", *Optical and Quantum Electronics*, vol. 36, no 1-3, pp. 213-221, 2004.
- [7] E.I. Smotrova, A.I. Nosich, T.M. Benson, P. Sewell, "Coldcavity thresholds of microdisks with uniform and nonuniform gain", *IEEE J. Selected Topics in Quantum Electronics*, vol. 11, no 4, 2005.
- [8] P.W. Evans, N. Holonyak, "Room temperature photopumped laser operation of native-oxide-defined coupled GaAs-AlAs superlattice microrings", *Appl. Phys. Lett.*, vol. 69, no 16, pp. 2391-2393, 1996.
- [9] A. Nakagawa, S. Ishii, T. Baba, "Photonic molecule laser composed of GaInAsP microdisks", *Appl. Phys. Lett.*, vol. 86, 041112, 2005.
- [10] E.I. Smotrova, A.I. Nosich, T.M. Benson, P. Sewell, "Optical coupling of whispering-gallery modes in two identical microdisks and its effect on photonic molecule lasing", *IEEE J. Selected Topics in Quantum Electronics*, vol. 11, no 6, 2005.