

Studying the Microcavity Lasers as Active Dielectric Resonator Antennas

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1. Introduction

Micron-scale semiconductor, crystalline and polymer lasers exploit emission of light from the cavities made of active dielectric materials, which are able to display inversed population under pumping and thus provide optical gain [1-9]. They are intensively studied both experimentally and theoretically, as coherent sources of the visible, infrared and terahertz waves. Here, well developed etching and epitaxial technologies enable fabrication of thin planar cavities with controlled contour shape. Thus, the laser resonators either stand on small pedestals in free space or lay on a substrate, with in-plane dimensions comparable to and thickness much smaller than the optical wavelength.

From the one hand, if one considers a laser as a source then its main characteristics are the frequencies of emission and the associated thresholds of pumping or, equivalently, of material gain.

From the other hand, as the intrinsic property of laser is the radiation so it can be also viewed as an antenna. Then, the sought far-field characteristics are the angular pattern of emission and the directivity. In this paper, we study the lasing frequencies, thresholds and far-field patterns for a kite cavity using specifically tailored electromagnetic eigenvalue problem [1,8].

2. Modelling of Lasers Using Eigenvalue Problems

As known, useful information can be obtained if all non-electromagnetic effects associated with lasing are neglected, and the optical modes are viewed as solutions of the linear set of source-free time-harmonic Maxwell equations. Until recently, linear modelling of microcavity lasers has implied exclusively the calculation of the natural modes of the *passive* open dielectric resonators. Mathematically this means solving the Maxwell eigenvalue problem for the complex-valued natural frequencies, ω or wavenumbers, $k = \omega/c$. These eigenvalues form a *discrete* set on the half of the complex plane and hence can be numbered using some index s . On finding them, the modes with the largest Q-factors, $Q_s = \text{Re} k_s / 2 |\text{Im} k_s|$, have been associated with the lasing [1-5,7]. The eigenfunctions corresponding to these eigenvalues are the modal fields; they decay in time as $e^{-|\text{Im} k_s| ct}$ however grow in space as $\sim e^{|\text{Im} k_s| R}$ (here R is the distance and c is the light velocity).

As known, 3-D optical-field problem for a thinner-than-wavelength disk can be approximately reduced to the 2-D one, in the disk plane, where the bulk refractive index is replaced with the effective-index value [6,9]. This approximation agrees well with the predominance of the in-plane radiation and leads to the separate analysis of the E_z and H_z -polarized modes (z -axis being directed normally to the cavity plane), with a conclusion that the latter are dominant.

The highest Q-factors are achieved for the whispering-gallery modes of circular cavities; however their directivities are only 2 because of many identical beams in the emission patterns. The quest for better directionality stimulates research into the modes of non-circular cavities such as ellipse, limaçon and kite (Fig. 1). Here, the most widespread early approach has been the “billiards theory” [5,9]. It has led to the discovery of certain promising shapes; however it fails to quantify the field leakage and modal Q-factors, and generally fails to grasp the discreteness of natural modes. On the other side, the popular today FDTD codes are not able to solve eigenvalue problems directly. Instead, they need a pulsed source placed inside a cavity, so that evaluation of the natural frequencies and Q-factors is done via studying Fourier transform of a transient signal [2]. Therefore the use of FDTD critically depends on the choice of the source and observation points. Besides,

FDTD codes are extremely time-consuming that makes a parametric analysis aimed at improvement of directivity or threshold impossible. Therefore, a powerful current trend (see [1,3,4,7]) is the use of integral-equations (IEs), which, if derived properly, are fully equivalent to the Maxwell problem.

It is obvious, however, that the lasing phenomenon is not addressed directly through the Q-factor analysis – the value of threshold gain that is needed to force a mode to become lasing is not included in the formulation. To fill this gap, complicated non-linear theories were proposed [9].

Remarkably, this drawback in the characterisation of lasers can be overcome by a relatively simple modification of the formulation of the linear electromagnetic problem. Namely, introduction of macroscopic gain, say γ , into the cavity material enables one to extract not only the real frequencies but also the threshold values of gain as eigenvalues. A convenient way for doing this is through the *active* imaginary part of the complex refractive index ν : if the time dependence is assumed as $e^{-i\omega t}$, then $\nu = \alpha - i\gamma$, $\alpha, \gamma > 0$. Such a *lasing eigenvalue problem* (LEP) was discussed in [1,6,8] in more details. Similarly to the Q-factor analysis, any LEP can be studied using IEs.

Note that general properties of the LEP eigenvalues have been established for arbitrary-shape open resonators [6,8] and show that (i) all $\gamma_s > 0$: no thresholdless lasing is possible; (ii) eigenvalues form a discrete set on the plane (k, γ) ; (iii) each eigenvalue continuously depends on the resonator geometry and refractive index, and may disappear only at infinity on the plane (k, γ) . To link the LEP with the more traditional Q-factor analysis, we remind that each discrete natural frequency is a function of the gain parameter, γ . Hence, one may look for a specific value, γ_s , that provides $\text{Im}k_s(\gamma_s) = 0$, and consider this as the threshold of lasing at which the radiation losses are balanced exactly with the macroscopic gain of active medium. Note that the LEP modal fields do not decay in time and decay in distance, as cylindrical wave in 2-D. The gain per unit length, the traditional quantity in the Fabry-Perot cavities, is $g = k\gamma$, where $k = \omega/c = 2\pi/\lambda$.

3. Method of Muller Integral Equations

Denote the interior domain of a 2-D model of an active dielectric (non-magnetic) microcavity as D_i , its closed contour as Γ , and the outer domain as D_e . Consider a function $U(x, y)$, which is either the E_z or the H_z field component. When simulating a microlaser, we look for the real-valued pairs of numbers (k, γ) generating non-zero functions U . Off Γ , U must solve homogeneous Helmholtz equation $(\Delta + k^2\nu^2)U = 0$ with a piecewise-constant coefficient. Here effective refractive index ν equals to $\nu_i = \alpha_i - i\gamma$ ($\gamma > 0$) in D_i , and $\nu_e = \alpha_e$ in D_e . Besides, the following two-side boundary conditions are required on Γ : $U^e = U^i$ and $\eta_e \partial U^e / \partial n = \eta_i \partial U^i / \partial n$, where the superscripts “ i, e ” refer to the corresponding domains, $\eta_{i,e} = 1$ (E-polarisation) or $\eta_{i,e} = 1/\nu_{i,e}^2$ (H-polarisation), and \vec{n} is the outward normal vector to Γ . Furthermore, the time-averaged electromagnetic energy must be locally integrable to prevent source-like field singularities, and the Sommerfeld radiation condition must be satisfied at infinity. This is the LEP that we have discussed above to study the modes in active cavities.

Introduce the Green’s functions $G_j(R) = (i/4)H_0^{(1)}(k\nu_j R)$, where $j = i, e$, $R = |\vec{r} - \vec{r}'|$ is the distance between \vec{r} and \vec{r}' , and $H_0^{(1)}(\cdot)$ is the Hankel function. After applying the second Green’s formula to the functions $G_j(\vec{r}, \vec{r}')$ and U_j , using boundary conditions, and taking into account the properties of single-layer and double layer potentials, we obtain two integral equations as

$$\varphi(\vec{r}) + \int_{\Gamma} \varphi(\vec{r}') A(\vec{r}, \vec{r}') dl' - \int_{\Gamma} \psi(\vec{r}') B(\vec{r}, \vec{r}') dl' = 0, \quad \frac{\eta_i + \eta_e}{2\eta_e} \psi(\vec{r}) + \int_{\Gamma} \varphi(\vec{r}') C(\vec{r}, \vec{r}') dl' - \int_{\Gamma} \psi(\vec{r}') D(\vec{r}, \vec{r}') dl' = 0 \quad (1)$$

where dl' is the element of Γ , $\varphi(\vec{r}) = U_i(\vec{r})$ and $\psi(\vec{r}) = \partial U_i(\vec{r}) / \partial n$, and the kernels are continuous or logarithmic-singular, $A(\vec{r}, \vec{r}') = \partial G_i(\vec{r}, \vec{r}') / \partial n' - \partial G_e(\vec{r}, \vec{r}') / \partial n'$, $B(\vec{r}, \vec{r}') = G_i(\vec{r}, \vec{r}') - \eta_i / \eta_e G_e(\vec{r}, \vec{r}')$, $C(\vec{r}, \vec{r}') = \partial^2 G_i(\vec{r}, \vec{r}') / \partial n \partial n' - \partial^2 G_e(\vec{r}, \vec{r}') / \partial n \partial n'$, and $D(\vec{r}, \vec{r}') = \partial G_i(\vec{r}, \vec{r}') / \partial n - (\eta_i / \eta_e) \partial G_e(\vec{r}, \vec{r}') / \partial n$.

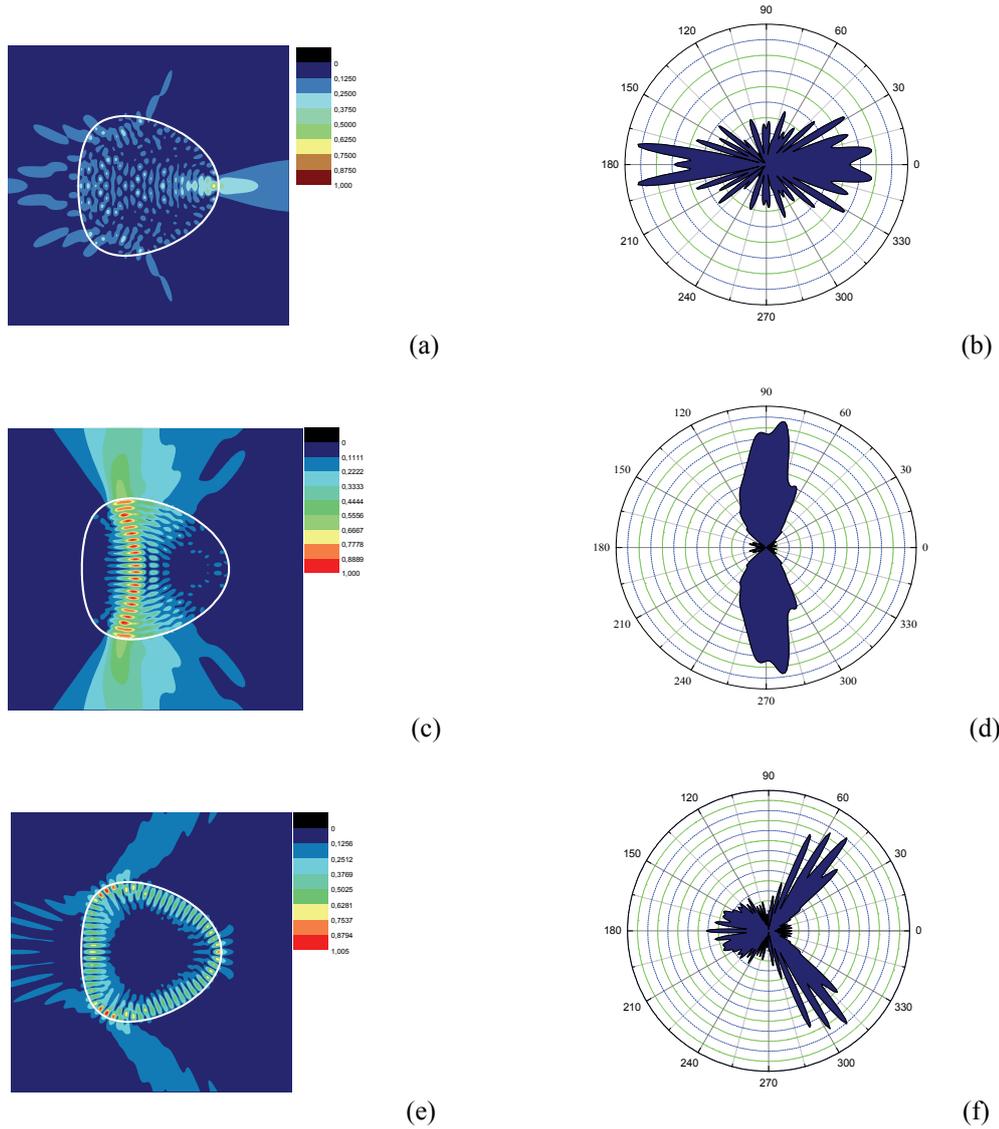


Fig 1. Normalized near- and far-field $|H_z|$ patterns for the lasing modes in a kite cavity (a), (b): Fabry-Perot mode: $ka=23.49$; $\gamma = 3.46 \cdot 10^{-2}$; $D = 5.5$ (c), (d): Fabry-Perot mode: $ka = 23.49$; $\gamma = 3.5 \cdot 10^{-2}$; $D = 5.64$ (e), (f) perturbed whispering gallery mode: $ka = 23.64$; $\gamma = 9.8 \cdot 10^{-3}$; $D = 6.78$. Refractive index is $\alpha = 1.5$, kite deformation parameter is $\delta = 0.165$, interpolation order is $N = 50$.

One of the most efficient discretization techniques used for numerical solution of IEs is the method of quadratures. It is also known as the Nystrom method and is based on the replacement of the integrals with approximate sums with the aid of appropriate quadrature formulas. In our case, it is convenient to represent all of the kernels in (1) in such a way that the logarithmic singularities are extracted. Then we apply two different quadrature rules, one for the regular and another for the singular parts with the same equidistant set of points. Namely, we use a trigonometric quadrature rule for the parts with logarithmic singularities and a trapezoidal rule for the regular parts. Using the quadrature rules, we obtain a determinantal equation for the eigenvalues. A secant-type iterative method is further used to find the eigenvalues numerically from this equation.

The directionality of light emission for each lasing mode can be conveniently quantified using the value of directivity from the theory of antennas, $D = 2\pi P^{-1} |\Phi(\varphi_{\max})|^2$, $P = \int_0^{2\pi} |\Phi(\varphi)|^2 d\varphi$, where φ_{\max} is the angle of the main beam radiation, $\Phi(\varphi)$ is far-field emission pattern, and P is, within a constant, the total power radiated by the lasing mode.

4. Results and Discussion

In this paper, we consider a kite-shaped microcavity using the LEP formalism. The kite contour Γ is represented using the following smooth (i.e. infinitely continuously differentiable) function: $\vec{r}(t) = \{x(t), y(t)\}$, where $x(t) = a(\cos t + \delta \cos 2t - \delta)$, $y(t) = a \sin t$, $t \in [0, 2\pi]$. Here, δ is the normalized contour shape deformation parameter; if $\delta = 0$ then the curve turns to circle of radius a .

Varying kite-contour parameter, e.g. deforming a circle to a kite, we have been able to trace the variations in the values of the mode lasing thresholds, frequencies, and also the directivities of light emission. In Fig. 1, presented are three sets of the near and far-field patterns for the lasing modes of kite-shaped microlaser with the normalized frequencies around $ka \approx 23.5$. The cavity has the refractive index $\alpha = 1.5$ as typical for polymer lasers and its contour is convex and characterized by the deformation parameter $\delta = 0.165$. One can see that the first two modes are essentially Fabry-Perot like modes although this is more obvious for the second of them. Their fields are concentrated along a certain line. Unlike them, the third mode is a whispering-gallery mode perturbed by the contour deformation as certified by the field confinement along the cavity boundary. This mode, as expected, shows the lowest threshold gain value. The values of the directivity for all three modes are a few times larger than for the whispering-gallery modes in the circular resonator.

A series of experiments have been performed for the kite-shaped polymer lasers. There is a good agreement between the measurements and the modeling both in spectra and in far-field directional emission that consolidates the predictions of the computations for the near-field.

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