Spectra and Thresholds of the Whispering-Gallery Modes in Microdisk Laser with Radially Non-Uniform Gain Area

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ABSTRACT
Quasi-3D analysis of lasing spectra and thresholds of microdisk lasers with radially non-uniform gain is presented. This analysis is based on the approximate reduction of a 3-D problem for a finite thickness disk to a 2-D one, with effective index method. New point is full account of the effective index dispersion. 2-D study is based on the set of the Maxwell equations with transmission conditions at the disk boundaries and radiation condition at infinity.
Keywords: microdisk laser, eigenvalue problem, lasing spectrum, threshold gain, whispering gallery modes.

1. INTRODUCTION
Semiconductor microdisk lasers with photopump and injection of current exploiting whispering-gallery (WG) modes around the disk rim have been intensively investigated in resent years as promising sources of light for dense photonic integrated circuits. A number of papers with experimental and theoretical study have been published [1-6]. Still surprisingly, it appears that accurate study of the circular-cavity lasing modes is absent, as the simplified analyses of [3-5] were based on rough assumptions, and FDTD simulations like that of [6] did not address the modes directly. In [7], a new Lasing Eigenvalue Problem (LEP) was formulated to quantify both frequency spectra and threshold gains of microdisk lasers. In that study, optically pumped lasers were simulated as microdisks with radially uniform gain and effective index dispersion was neglected. In contrast, here we shall build an adaptive algorithm taking into account the index dispersion through the accurate solution of a corresponding natural-mode problem, for the slab of the same thickness as the disk. Further, lasers with injection of current are frequently designed as stacked structures where a microdisk cavity is sandwiched between the substrate and metal contacts. If the contact is located in the disk center, the density of injected carries and hence the material gain has obviously grater value in the center of the cavity, and vise versa. In this paper we shall simulate such lasers with non-uniform gain to show that ring-shape contacts provide much smaller thresholds than central ones.

2. EFFECTIVE REFRACTION INDEX METHOD
Suppose that a disk of the thickness d and radius a is nonmagnetic, has real-valued refraction index α, and is placed in vacuum. The electromagnetic field is assumed to have the time dependence as $e^{-i\omega t}$, where ω is the angular frequency. Then, free-space wavenumber is $k=\omega/c=2\pi/\lambda$, where c is the free-space light velocity.

So-called effective-index approach starts from the assumption that the dependences of the field $\{\boldsymbol{E},\boldsymbol{H}\}$ on the vertical coordinate z and in-plane coordinates $\boldsymbol{r}=(r,\varphi)$ can be separated everywhere, e.g., $E_z(\boldsymbol{R})=V_E(z)U_E(r,\varphi)$, $H_z(\boldsymbol{R})=V_H(z)U_H(r,\varphi)$. In fact, this is incorrect because neither boundary conditions on disk surface, nor radiation condition at $R\to\infty$ is separable. However, such an assumption enables one to write independent differential equations for the functions of z and $\boldsymbol{r}$. They are, respectively,

$$\left(\frac{d^2}{dz^2}+k^2\alpha^2-k^2\alpha_{eff}^2\right)V_{H,E}(z)=0,\quad \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+k^2\alpha_{eff}^2\right]U_{H,E}(r,\varphi)=0\,, \tag{1}$$

where α turns 1 off the interval $|z|<d/2$, and effective refraction index $\alpha_{eff}=\alpha_{eff}^{H,E}$ inside cavity $(r<a)$ and 1 outside. The transmission-type boundary conditions for $V_{H,E}(z)$ depend on polarization, namely

$$V_{H,E}(\pm d/2\mp 0)=V_{H,E}(\pm d/2\pm 0),\quad \left.\frac{dV_{H,E}}{dz}\right|_{z=\pm d/2\mp 0}=\beta^{H,E}\left.\frac{dV_{H,E}}{dz}\right|_{z=\pm d/2\pm 0}, \tag{2}$$

where $\beta^H=\alpha^{-2}$ and $\beta^E=1$. To complete the formulation in the open domain, one needs a condition at infinity.

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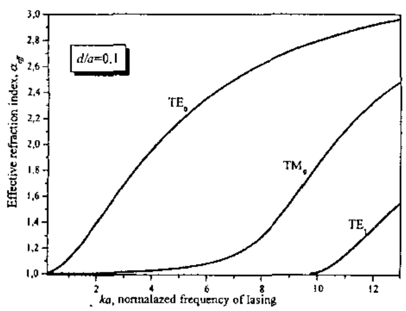

Figure 1. Dispersion characteristics of the guided modes of infinite dielectric slab made of GaAs with the bulk refraction index α=3.374.

Unfortunately, there is no "continuous" way to derive 1-D condition ($z \to \pm\infty$) from the 3-D condition for $\{E, H\}$. In order to reproduce the outgoing wave propagation off the disk plane, one has to request that

$$V_{E,H}(z) \sim e^{ik(1-\alpha^2)^{1/2}|z|}, \quad z \to \pm\infty, \tag{3}$$

The first of equations (1), together with (2) and (3), forms a familiar 1-D eigenvalue problem for the parameter α_{eff}^H or α_{eff}^E, which is a normalized propagation constant of a TE (or H_z) or TM (or E_z) type guided wave of infinite dielectric slab They are reduced to the following four transcendental equations:

$$\tan pkd = \beta^{E,H} gp^{-1}, \ \cot pkd = -\beta^{E,H} gp^{-1} \tag{4}$$

For the each type of waves, there exists finite number $Q^{H,E} \geq 1$ of real-valued solutions, $\alpha_{eff(q)}^{H,E}$: $1 < \alpha_{eff(q)}^{H,E} < \alpha$, $q = 0,...,Q^{H,E}+1$. Two largest of them correspond to the TM_0 and TE_0 waves, respectively. The even (odd) value of the wave index indicates to the symmetry (anti-symmetry) of the wave field E_z or H_z with respect to the middle plane of the slab. Plots in Fig.1 demonstrate the dependences of the effective indices on the frequency normalized by the disk radius, i.e., on $ka = kd(d/a)^{-1}$.

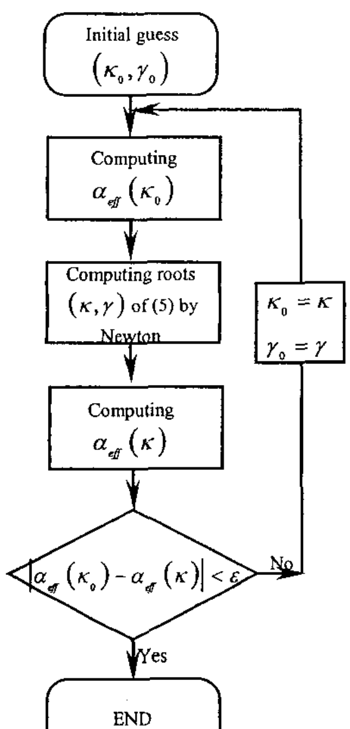

Figure 2. Algorithm chart.

3. ACCOUNT OF DISPERSION

Consider now a cavity with non-uniform gain area shaped as a circle of smaller radius $b<a$ in the center of the cavity. To find the in-plane field patterns of the lasing modes in a disk cavity, we shall suppose that U is either E_z or H_z field component, depending on polarization, and introduce material gain $\gamma > 0$ in the cavity. According to [7], off the disk contour this function must satisfy the Helmholtz equation, $[\Delta + k^2 \nu^2(r,\varphi)]U(r,\varphi) = 0$, where step-wise function $\nu(r,\varphi)$ is assumed 1 outside the cavity and a complex value inside circle of radius b in the center of the cavity: $\nu = \alpha_{eff} - i\gamma$ and real value $\nu = \alpha_{eff}$ outside the circle in the center. We shall assume that the gain is uniform within the smaller circle and zero outside. The field must also satisfy the transmission conditions across the boundary of the resonator and the Sommerfeld radiation condition at infinity that selects outgoing field solutions. Following [7], we look for the eigenvalues as discrete pairs of real-valued parameters, (κ, γ), that yield normalized frequencies of lasing, $\kappa = ka$, and the associated threshold material gains.

By using the separation of variables, this problem is reduced to the search of the roots of the following determinant equation:

$$\det \begin{bmatrix} H_m^{(1)}(\kappa) & -J_m(\kappa\nu) & -N_m(\kappa\nu) & 0 \\ 0 & J_m(\kappa\nu\delta) & N_m(\kappa\nu\delta) & -J_m(\kappa\delta) \\ H_m^{\prime(1)}(\kappa) & -\beta^{H,E} J_m'(\kappa\nu) & -\beta^{H,E} N_m'(\kappa\nu) & 0 \\ 0 & \beta^{H,E} J_m'(\kappa\nu\delta) & \beta^{H,E} N_m'(\kappa\nu\delta) & -J_m'(\kappa\delta) \end{bmatrix} = 0, \tag{5}$$

where the functions involved are the cylindrical functions. Here, $\beta^{H,E}$ equals ν^{-2} in the case of H_z polarization or 1 in the case of E_z polarization. Note that $\alpha_{eff(q)}(\kappa)$ can be associated with any guided wave of the relevant slab. Therefore each pair should be denoted as $(\kappa_{mnq}, \gamma_{mnq})$, where azimuth and radial indices are $m = 0,1,2,...$, $n = 1,2,...$, respectively, and effective refraction index brings the third number, q=0,1,2, (see Fig.1). Further we use two-parametric iterative Newton method to solve (5) numerically. Unlike [7], we combine it here with accurate account of the dispersion of corresponding effective index through the equations (4). Fig. 2 shows the chart of the adaptive algorithm used to calculate the eigenpairs of LEP.

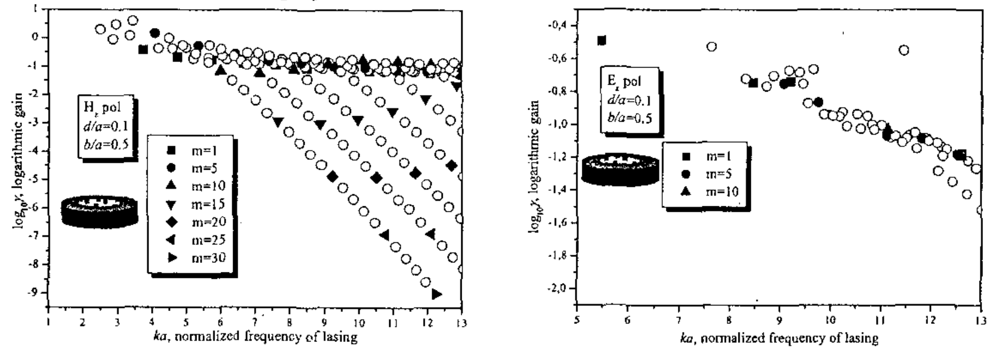

Figure 3. Lasing spectra and threshold gains for the H_z-polarized and E_z-polarized modes of the families $(H_z)_{mn0}$ and $(E_z)_{mn0}$ correspondently in a GaAs/InAs disk with circular gain extending to the half of disk radius, $\alpha = 3.374$ and $d/a = 0.1$.

4. NUMERICAL RESULTS

Analysis shows that the plane (κ, γ) is inhabited by the eigenvalues in non-uniform manner. One can clearly see a hyperbola, $\gamma \approx const/\kappa$, "saturated" with modes of all the m-th families. These modes have very high thresholds characterized with dimensionless material gain $\gamma > 0.1$. Above that curve, there are no lasing modes. In contrast, in each family with $m > \alpha$ the modes, which have $\kappa < m$ but still $\kappa > m/\alpha$, keep the same distance in κ however display drastically smaller values of γ. These values are getting even smaller for larger m. Below the mentioned hyperbola the modes (i.e., the eigenvalues) form inclined layers, each layer corresponding to a certain value of the radial index n. Thus, not automatically all the modes in a circular cavity show "whispering" property although all show the "gallery" property, i.e., periodicity in κ. Only the modes having the frequencies within the strip $m/\alpha < \kappa < m$ are "whispering", i.e., have exponentially low thresholds. This is because only these modes experience quasi-total internal reflection when propagating along the rim of the cavity. The lowest-threshold layer is formed by the WG modes having a single variation in radius $(n = 1)$. Below that layer, in the domain $\kappa < m/\alpha$, no eigenvalues are found in the m-th family. Still besides, Fig. 3 demonstrates the effect of the effective index dispersion. For the same disk, the E_z-polarized modes of the same index q have smaller effective index, than the H_z-polarized ones, in wide band of frequencies. Therefore the E_z-modes have smaller chances to go lasing unless m and hence κ is not large enough (see Fig. 1). If gain area is changed the modes keep their location in frequency, however the lasing thresholds

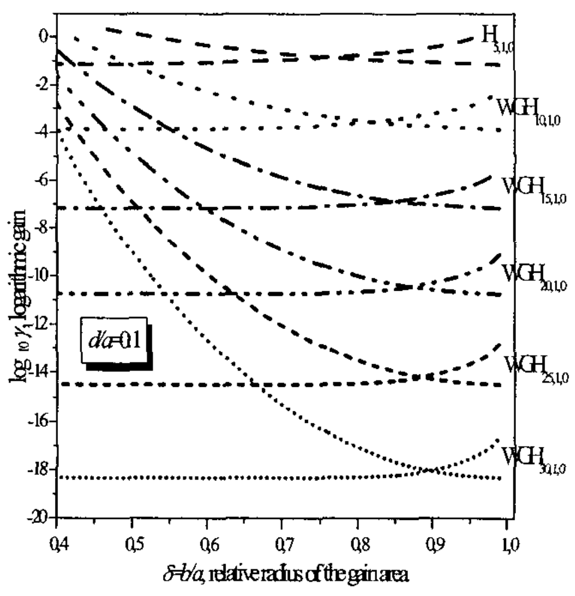

Figure 4. Dependences of threshold material gains of the modes $(H_z)_{m10}$ in a GaAs/InAs disk with a step-like gain on the relative radius of the gain area.

drastically depend on the gain area. To characterize this effect, the dependences of the lasing thresholds on the relative radius of gain area for the $(H_z)_{mn0}$ modes of the lowest-threshold layer on the plane (κ, γ) are presented in Fig. 4.

Alternatively, if the gain area is shaped as a ring along the disk rim, then the ultra-low thresholds are observed up to the values of δ=0.9 (Fig. 4). This is a bright demonstration of the role of a good overlap between the gain area and the lasing mode E-field pattern. In fact, this intuitive consideration has been recently used in experiments with microdisk lasers. First, in [8], more efficient optical pumping was arranged with an axicon lens placed at the axis of the pump beam. Then, papers [9,10] presented the data on the performance of microdisk lasers with current injected from the ring shaped contacts. In each case, extremely low thresholds were recorded.

5. CONCLUSIONS

We have studied the lasing spectra and thresholds for the disk lasers with the radially non-uniform gain areas. Quasi-3D analysis enables us to conclude that $(H_z)_{mn0}$ modes have always lower thresholds than their $(E_z)_{mn0}$ counterparts, thanks to the greater value of the effective refraction index $\alpha_{(0)}^{H}$ than $\alpha_{(0)}^{E}$, for the same disk thickness. The WG modes with single variation in radius have the lowest threshold in each mode family. We have also shown that if the active zone is concentrated in the center, that is typical to injection lasers, then the ultra-low-threshold feature of the WG modes of microdisk laser is effectively lost. However, if the gain area is shaped as a ring along the disk rim, then the WG modes keep the ultra-low thresholds. Thus, our study serves as theoretical validation of better efficiency of lasers with doughnut-shape optical-beam pumping or with ring-shape contacts.

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