2a). However, at $\theta_0 = 20°$ the dimensions of the focal region at k_0b also comprise an angle of 20°, so that practically all the energy of the focused wave is reflected in the reverse direction (Fig. 2b). Calculation of the modulus of the vector $\boldsymbol{P}$ shows that at $k_0b = 5$, $\varepsilon = 2.57$, $\theta_0 = 20°$ in the focus ahead of the screen $|\boldsymbol{P}| = 3$, while beyond the screen $|\boldsymbol{P}| = 0.03$. It is obvious that the amplitude of the wave reflected from the spherical screen will be maximum in the case where the screen overlaps the "focal" region. Increase in screen size may lead to decrease in the reflected wave amplitude. This will occur in the case where the screen overlaps regions in which the flux of the vector $\boldsymbol{P}$ is directed within the sphere.

Comparison of the results presented above with characteristics of the scattered field in the scatterer far zone makes possible explicit physical interpretation, for example, explanation of the effect of significant increase in the radar section of a scatterer [5]. One can also explain the existence of an optimum (for fixed k_0b value) screen size, at which the reflected wave amplitude reaches its maximum.

LITERATURE CITED

1. S. S. Vinogradov, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 26, No. 1, 91 (1983).
2. S. S. Vinogradov, Dissertation [in Russian], Kharkov (1980).
3. S. S. Vinogradov and A. V. Sulima, Dokl. Akad. Nauk Ukr. SSSR, Ser. A, No. 5, 55 (1982).
4. V. P. Shestopalov, Summator Equations in Modern Diffraction Theory [in Russian], Naukova Dumka, Kiev (1983), p. 252.
5. A. V. Sulima, Izv. Vyssh. Uchebn. Zaved., Radiofiz., 26, No. 2, 258 (1983).
6. M. Born and E. Wolf, Principles of Optics, Pergamon (1960).
7. S. S. Vinogradov, A. V. Sulima, and V. P. Shestopalov, Preprint IRE Akad. Nauk Ukr. SSR, No. 218 [in Russian], Kharkov (1983).

ELECTRODYNAMIC MODELING OF OPEN RESONATORS WITH DIFFRACTION GRATINGS

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The problem of excitation of open resonators (OR), formed by a cylindrical mirror and different periodic gratings, by a fixed electromagnetic field is solved. The resonance properties of cylindrical or with a dense grating are studied in detail.

In this paper the problem of excitation of an open resonator (OR), consisting of a periodic grating and an open cylindrical screen, by a given electromagnetic field is solved. The problem is reduced to infinite systems of linear algebraic equations (SLAE) of the second Fredholm type.

1. Formulation of the Problem. The two-dimensional OR, formed by an ideally conducting cylindrical mirror consisting of part of a circular cylindrical with angular width $2(\pi - \theta_S)$ and a radius of curvature a and a periodic diffraction grating with period ℓ, placed at a distance b from the axis of the cylinder (Fig. 1), is excited by an H-polarized plane wave, arriving from the upper half-space. We denote by $H_z{}^i$ the component of the magnetic field of the wave parallel to the generatrix of the cylindrical:

$$H_z^i(x, y) = \exp[ik(\cos\alpha x - \sin\alpha y)],\ 0 \leqslant \alpha \leqslant \pi. \tag{1}$$

We shall represent the total field in the OR in the form

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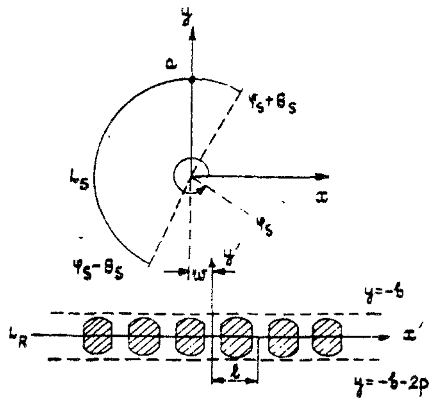


Fig. 1

$$H_z^{n\,\mathrm{Ri}}(x, y) = H_z^i(x, y) + H_z^R(x, y) + H_z^{Ri}(x, y). \tag{2}$$

Here H_z^{Ri} is the field of the plane wave reflected by the grating [1]; H_z^R is the component of the magnetic field, scattered by the structure and satisfying: 1) Helmholtz's equation outside the surface of the mirror L_S and the elements of the grating L_R; 2) the Neumann boundary condition on the surface of the mirror

$$\frac{\partial}{\partial n}\left(H_z^i(x, y) + H_z^R(x, y) + H_z^{Ri}(x, y)\right)\Big|_{L_S} = 0\,; \tag{3}$$

3) the system of boundary conditions in the region of the grating $[-b - 2p \le y \le -b]$, which includes the boundary conditions on the surface of the conductors; 4) the condition that the energy in an arbitrary bounded region of the (r, φ) plane is bounded; and, 5) the condition that there are no arriving waves as $r \to \infty$. These requirements ensure that the boundary-value problem is unique.

In the absence of the grating the field scattered by the cylindrical screen can be represented as the potential of a double layer [2]:

$$H_z^R(r, \varphi) = \int_{L_S} \mu(r_S)\frac{\partial}{\partial n_S} G^i(k, r, r_S)\,dl_S\,, \tag{4}$$

where

$$G^i(k, r, r_S) = \frac{i}{4} H_0^{(1)}(k|\mathbf{r} - \mathbf{r}_S|). \tag{5}$$

The field scattered by the cylinder in the presence of a diffraction grating is also best represented by an analogous integral transformation whose kernel is a derivative of the Green's function of the space containing the diffracton grating:

$$H_z^{\mathrm{total}}(r, \varphi) = \int_{L_S} \mu(\mathbf{r}_S)\frac{\partial}{\partial n_S} G(k, \mathbf{r}, \mathbf{r}_S)\,dl_S. \tag{6}$$

Here $G(k, \mathbf{r}, \mathbf{r}_S)$ is the sum

$$G(k, \mathbf{r}, \mathbf{r}_S) = G^R(k, \mathbf{r}, \mathbf{r}_S) + G^i(k, \mathbf{r}, \mathbf{r}_S), \tag{7}$$

$G^R(k, \mathbf{r}, \mathbf{r}_S)$ is a function that must be added to the function $G^i(k, \mathbf{r}, \mathbf{r}_S)$ in order to satisfy the boundary conditions in the region of the grating [3].

Using the representation of the function $H_0^{(1)}(k|\mathbf{r} - \mathbf{r}_S|)$ in the form of a Sommerfeld integral the function $G^i(k, \mathbf{r}, \mathbf{r}_S)$ can be written in the form of a Fourier integral:

$$G^i(k, \mathbf{r}, \mathbf{r}_S) = \frac{i}{4\pi}\int_{-\infty}^{\infty}\frac{1}{g}\exp\{ik[h(x - x_S) + g|y - y_S|]\}\,dh, \tag{8}$$

where $g = \sqrt{1 - h^2}$, $\operatorname{Im} g > 0$ as $h \to \pm\infty$.

We shall seek the function $G^R(k, \boldsymbol{r}, \boldsymbol{r}_S)$ in the form

$$G^R(\boldsymbol{k}, \boldsymbol{r}, \boldsymbol{r}_S) = \frac{i}{4\pi} \int_{-\infty}^{\infty} F(x, y; h) \exp(ikhx)\, dh. \tag{9}$$

Because the integral transforms (8) and (9) are linear the function F(x, y; h) can be regarded as the response of the grating to the incident plane wave:

$$H_z^0(x, y) = A^0(x_S, y_S; h) \exp(ikhx - ikgy),$$

$$A^0(x_S, y_S; h) = \frac{1}{g} \exp(-ikhx_S + ikgy_S).$$

The integrand in (9) is periodic in x whose period ℓ equals the period of the grating, and satisfies Helmholtz's equation and the boundary conditions in the region of the grating.

Expanding F(x, y; h) in a Floquet series and taking into account the condition that there are no arriving waves we obtain

$$F(x, y; h) = A^0(h) \sum_{n=-\infty}^{\infty} \begin{cases} a_n(h) \exp[ig_n(y+b)] \exp\left(i\frac{nk}{\varkappa}x\right), & y > -b \\ b_n(h) \exp[-ig_n(y+b+2p)] \exp\left(i\frac{nk}{\varkappa}x\right), & y < -b-2p \end{cases}, \tag{10}$$

where $g_n = \sqrt{1 - h_n^2}$, $h_n = h + n/\kappa$, $\kappa = \ell/\lambda$; and $\operatorname{Re} g_n > 0$ for $\operatorname{Re} g_n = 0$, $\operatorname{Im} g_n > 0$, a_n and b_n are the amplitudes of the spatial harmonics of the Floquet field scattered by the grating excited by a plane wave with unit amplitude.

Then

$$G^R(k, \boldsymbol{r}, \boldsymbol{r}_S) = \frac{i}{4\pi} \int_{-\infty}^{\infty} A^0(x_S, y_S; h) \times \sum_{n=-\infty}^{\infty} \begin{Bmatrix} a_n(h) \exp[ig_n(y+b)k] \\ b_n(h) \exp[-ig_n(y+b+2p)k] \end{Bmatrix} \exp(ih_nkx)\, dh. \tag{11}$$

The unknown coefficients a_n and b_n for fixed parameter h are determined by the currents induced on the elements of the grating.

Using the Jacobi–Anger formula

$$\exp(ix\cos\tau) = \sum_{n=-\infty}^{\infty} i^n J_n(x) \exp(in\tau),$$

we represent $H^R(r, \varphi)$ in the form

$$\begin{aligned} H_z^R(r, \varphi) = \frac{ki}{4\pi} \int_{L_S} \mu(r_S) \int_{-\infty}^{\infty} \frac{1}{g} \sum_{p=-\infty}^{\infty} (-i)^p J_p' (kr_S) \exp[ip(\varphi_S + \alpha)] \times \\ \times \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \begin{Bmatrix} a_n(h) \exp(ig_nkb) \exp[im(\varphi + \alpha_n)], & y > -b \\ b_n(h) \exp[-ig_nk(b+2p)] \exp[im(\varphi - \alpha_n)], & y < -b-2p \end{Bmatrix} \\ \times i^m J_m\, dh\, dl_S + \frac{ki}{4} \int_{L_S} \mu(r_S) \times \sum_{m=-\infty}^{\infty} \begin{Bmatrix} J_m'(kr_S) H_m^{(1)}(kr), & r > a \\ H_m^{(1)\prime}(kr_S) J_m(kr), & r < a \end{Bmatrix} \exp[im(\varphi - \varphi_S)]\, dl_S, \end{aligned} \tag{12}$$

where $r_S = x_S \cos\alpha$, $\varphi_S = \operatorname{arctg}(y_S/x_S)$, $\sin\alpha_n = -g_n$.

In our case for a circular cylinder the function $\mu(r_S) \equiv \mu(\varphi_S)$. Extended by zero values to the interval corresponding to the slit on L_S it is a periodic function of φ_S with period 2π. We expanded in a Fourier series

$$\mu(\varphi_S) = \frac{2}{i\pi ka} \sum_{n=-\infty}^{\infty} \mu_n \exp(in\varphi_S), \tag{13}$$

and replace in (12) the contour of integration L_S by a circle. Using the orthogonality of the exponentials we obtain the Fourier series for determining $H_z^R(r, \varphi)$ in the local coordinates (r, φ) tied to the cylindrical mirror:

$$H_z^R(r,\varphi)=\frac{1}{\pi}\sum_{m=-\infty}^{\infty} i^m J_m(kr)\exp(im\varphi)G_m^R+\sum_{m=-\infty}^{\infty}\mu_m\begin{Bmatrix} J'_m H_m^{(1)}, & r>a\\ H_m^{(1)\prime} J_m, & r<a\end{Bmatrix}\exp(im\varphi), \quad (14)$$

where

$$G_m^R=\sum_{p=-\infty}^{\infty}(-i)^p J'_p(kr_S)\,\mu_p G_{pm},$$

$$G_{pm}=\int_{-\infty}^{\infty} e^{-im\alpha}\sum_{n=-\infty}^{\infty} a_n(h)\exp(ig_n kb)\exp(iph_n k)\,dh.$$

Making (14) satisfy the system of boundary conditions on the screen we obtain a system of paired summation equations:

$$\sum_{n=-\infty}^{\infty}\mu_n e^{in\varphi}=0, \quad |\varphi-\varphi_S|<\theta_S,$$

$$\sum_{n=-\infty}^{\infty}\mu_n J'_n H_n^{(1)\prime} e^{in\varphi}=\sum_{n=-\infty}^{\infty} -i^n J'_n e^{in\varphi}\Big\{e^{in\alpha}+G_n^R+\sum_{p=-\infty}^{\infty} a_p(\cos\alpha)\exp[in\alpha_p(\cos\alpha)]\exp[ig_p(\cos\alpha)\,kb]\Big\}, \quad (15)$$

$$\theta_S<|\varphi-\varphi_S|\leqslant\pi.$$

Inverting the static part of the operator of Eqs. (15) by the method of the Riemann–Hilbert problem [2, 3] reduces them to an infinite SLAE of the second kind:

$$\mu_m=\sum_{n=-\infty}^{\infty}(A_{mn}^S+A_{mn}^{RS})\,\mu_n+B_m, \quad m=0,\pm 1,\dots, \quad (16)$$

where

$$A_{mn}^S=\Delta_n T_{mn},$$

$$A_{mn}^{RS}=i\pi(ka)^2 J'_n\sum_{p=-\infty}^{\infty} i^{p-n} J'_p G_{pn} T_{mp},$$

$$\Delta_n=|n|+i\pi(ka)^2 J'_n H_n^{(1)\prime},$$

$$B_m=i\pi(ka)^2\sum_{n=-\infty}^{\infty} i^n J'_n T_{mn}\times\{e^{in\alpha}-\sum_{S=-\infty}^{\infty} a_S(\cos\alpha)\times\exp[ikb(\sin\alpha_S+\sin\alpha)]\exp[in\alpha_S(\cos\alpha)]\}.$$

The values of the coefficients T_{mn} are presented in [2].

The system of linear algebraic equations of the second kind obtained above can be solved by the method of reduction, and thus the electrodynamic characteristics of the OR in the medium-wavelength and short-wavelength (so-called "strongly resonance") regions can be obtained.

The results of the numerical calculations performed can serve as an illustration of the effectiveness of the proposed model.

2. Approximate Solution of the Problem for a Dense Ribbon Grating. In finding the Green's function of a space with a periodic grating there arises the problem of diffraction of a plane wave with unit amplitude, incident at an arbitrary angle, by this grating. The result of the solution of this problem are the values of the amplitudes of the propagating and surface Floquet harmonics. Problems of this kind have been solved for the following periodic gratings: ribbon, comb, and echelon [4]. As an example we shall choose the simplest form of a periodic grating – an infinite ribbon grating; the electrodynamic properties of this grating were investigated in [4]. The calculations were performed for the problem of excitation of an OR with a grating consisting of flat, infinitely thin ideally conducting ribbons of width D placed in the plane $y = -b$ with a spacing of ℓ. Such a periodic grating permits modeling a half-transmitting (in the limiting case an ideally reflecting) plane placed in the near field of a cylindrical screen.

The calculations were performed in the region of values of the wave parameter $\kappa/2\pi \leq 0.3$ based on the Lamb approximation for the complex amplitudes of the harmonics of the field scattered by the grating with accuracy up to terms of order $O(\kappa^2)$ for an arbitrary space factor of the grating $0 \leq D/\ell \leq 1$ [3]:

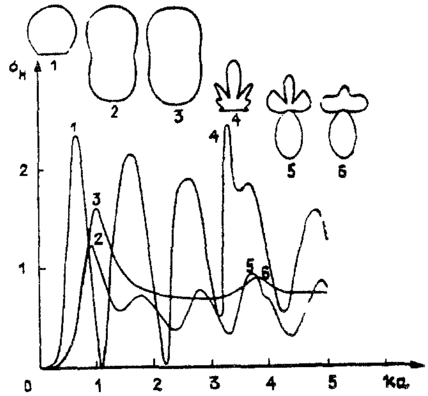


Fig. 2

$$a_0(h) = -b_0(h) \approx \frac{i\varkappa Qg}{1 - i\varkappa Qg}, \quad Q = -2\ln\cos\frac{\pi D}{2l}, \; a_n(h) = b_n(h) = 0 \quad n \neq 0. \tag{17}$$

Using (17) and calculating the integrals over h in (7)-(9) by the saddle-point method in the limit $r \to \infty$ we obtain an approximate expression for the directional pattern (DP) of the field scattered in the form of a cylindrical wave in the far zone

$$H_z^c(r, \varphi) = \sqrt{\frac{2}{\pi kr}} \exp\left[i\left(kr - \frac{\pi}{4}\right)\right] \Psi(\varphi),$$
$$\Psi(\varphi) = \sum_{n=-\infty}^{\infty} \mu_n J_n'(ka)(-i)^n \left[e^{in\varphi} + \begin{Bmatrix} a_0(\cos\varphi)\exp(2ikb\sin\varphi - in\varphi, & y > -b \\ -a_0(\cos\varphi)e^{in\varphi}, & y < -b \end{Bmatrix}\right]. \tag{18}$$

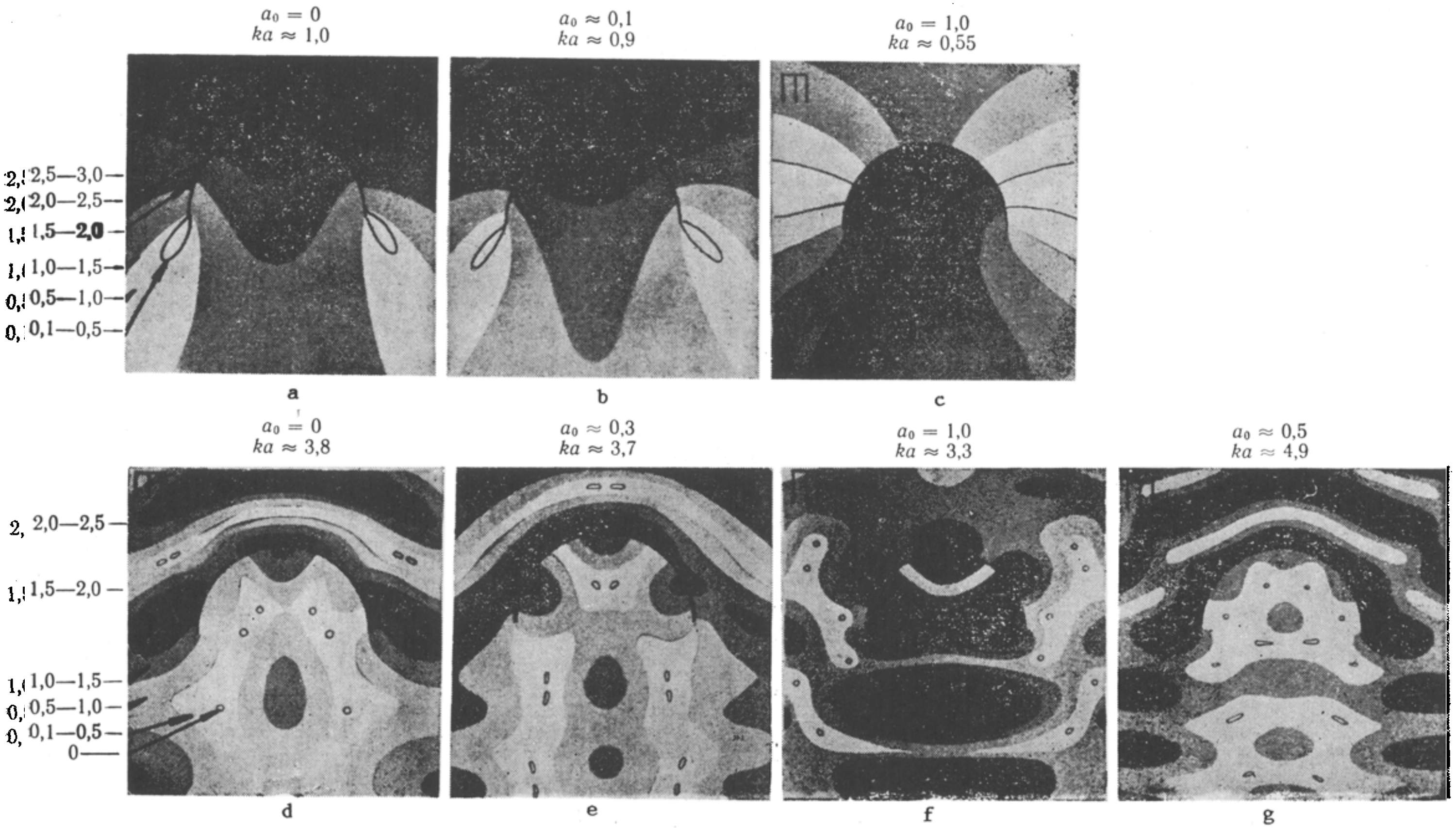
To investigate the integral characteristics of the electromagnetic field in the OR we introduce a quantity that is analogous to the total scattering cross section σ_H for a screen placed in free space:

$$\frac{\sigma_H}{4a} = \frac{1}{2\pi ka} \int_0^{2\pi} |\Psi(\varphi)|^2 \, d\varphi. \tag{19}$$

Calculation of $\sigma_H(ka)$ showed that when a half-transmitting or ideally reflecting plane is reduced into the region of the near field of the cylindrical screen two series of resonances are observed in the cross section (Fig. 2). Resonances of the first type (for a half-transmitting plane: $ka = 0.9, 3.7; \ldots$; for an ideally reflecting plane: $ka = 0.5, 3.3, \ldots$) are explained by the excitation of characteristic oscillations of the cylindrical mirror. The resonator formed by the cylindrical mirror and the half-transmitting plane is equivalent to a system of two strongly interacting cylindrical screens excited in phase [5]. As the reflection coefficient of the plane and the aperture angle of the slit of the cylinder are varied the maxima of σ_H shift relative to the maxima of σ_H for the cylinder itself, since the coupling between the screen and its mirror image changes. The calculations showed that $\sigma_{H,max}$ shift into the long-wavelength region of the parameter ka as the reflection coefficient of the plane increases and into the short-wavelength region as the aperture angle of the slit of the mirror decreases. Tilting the screen relative to the normal to the grating by angles of ~10° does not change the position of the resonances in σ_H in the structure and has no effect on the absolute values of $\sigma_{H,max}$; this indicates that this system is insensitive to tilting of the mirror.

Resonances of the second type (for a half-transmitting plane: $ka = 1.8, 2.8, \ldots$; for an ideally reflecting plane: $ka = 1.6, 2.6, \ldots$) arise owing to rereflection of the electromagnetic energy in the "screen-plane" space. For this reason their position and absolute magnitude depend on the method employed to excite the OR.

Figure 3 shows the dynamics of the change in the structure of the electromagnetic field in the near zone of the cylindrical mirror when ideally reflecting or a half-transmitting plane is inserted into this zone. Concentration of regions with maximum amplitudes of the field near the surface of the screen and in the interior cavity of the OR is characteristic



$a_0 = 0$
$ka \approx 1,0$
$a_0 \approx 0,1$
$ka \approx 0,9$
$a_0 = 1,0$
$ka \approx 0,55$
2,5—3,0
2,0—2,5
1,5—2,0
1,0—1,5
0,5—1,0
0,1—0,5
a
b
c
$a_0 = 0$
$ka \approx 3,8$
$a_0 \approx 0,3$
$ka \approx 3,7$
$a_0 = 1,0$
$ka \approx 3,3$
$a_0 \approx 0,5$
$ka \approx 4,9$
2,0—2,5
1,5—2,0
1,0—1,5
0,5—1,0
0,1—0,5
0
d
e
f
g

Fig. 3

for the structure of fields in resonances of κ_H of the first type the field virtually does not enter into the region of the OR (Fig. 3g).

The problem of excitation of a two-dimensional open resonator consisting of a cylindrical mirror and an infinite diffraction grating by an H-polarized field was solved under the condition that the solution of the problem of scattering of an arbitrary plane wave with unit amplitude by the given grating is known.

The resonances in the properties of the OR, one mirror of which consists of an open cylinder while the other is a dense ribbon grating modeling a half-transmitting (in the limiting case – ideally reflecting) plane placed in the region of the near field of the cylindrical mirror, were analyzed with the help of a computer. The calculations permitted determining the values of the parameters of such an electrodynamic structure for which it can be effectively employed as an OR or a screening device.

LITERATURE CITED

1. V. M. Skurlov, Radiotekhnika, No. 1, 69 (1965).
2. V. P. Shestopalov, Summation Equations in the Modern Theory of Diffraction [in Russian], Naukova Dumka, Kiev (1983).
3. V. P. Shestopalov, Method of the Riemann–Hilbert Problem in the Theory of Diffraction and Propagation of Electromagnetic Waves [in Russian], Vishcha Shkola, Kharkov (1971).
4. V. P. Shestoplaov, L. N. Litvinenko, S. A. Masalov, and V. G. Sologub, Diffraction of Waves by Arrays [in Russian], Vishcha Shkola, Kharkov (1973).
5. V. N. Koshparenok and V. P. Shestopalov, Zh. Vychisl. Mat., 18, No. 5, 1196 (1978).

SHORT-WAVELENGTH ASYMPTOTIC BEHAVIOR OF THE HIGHER-ORDER MODES OF A SMOOTHLY IRREGULAR WAVEGUIDE

A. D. Avdeev

UDC 621.372.8

A modification of the method of smooth perturbations, extending Brillouin's concept, is proposed.

One of the effective methods for constructing asymptotic expressions for the higher order modes of flat, smoothly irregular, multimode waveguides with reflecting walls is the method of smooth perturbations developed by Popov [1, 2]. In this method, which extends Brillouin's concept, the field of the normal wave is represented by a superposition of two quasiplane waves, and the problem reduces to integrating the eikonal and transfer equations, whose solutions are constructed in the form of expansions in the small smoothness parameter characterizing the slowness of the variation of the properties of the waveguide.

In [1, 2] expressions were found by the method of smooth perturbations for the fields of higher-order modes with an error in the phase on the irregular section proportional to the cube of the smoothness parameter. But to describe the exponentially small effects of the transformation of modes on analytical junctions [2] in this method one must employ artificial devices [3].

In this paper, whose basic ideas were formulated in [4], a modification of the method of smooth perturbations, which differs from Popov's method by the system of curvilinear coordinates employed and by the method employed to construct the solutions of the eikonal equations, making it possible to describe easily exponentially small effects of mode transformation, is studied. An expression is derived for the field of the propagating higher order modes with an error in the phase proportional to the fifth power of the smoothness

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