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RADIATION OF AN ELECTRON FLUX MOVING OVER A GRATING CONSISTING OF CYLINDERS WITH LONGITUDINAL SLITS

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The characteristics of the radiation of an electron flux over a grating consisting of cylinders with longitudinal slits are investigated. It is shown that the radiation energy increases considerably with the excitation of resonance conditions in the structure.

The radiation of a two-dimensional electron flux moving above a periodic structure formed by parallel cylinders with longitudinal slits is considered (Fig. 1). The cylinder walls are assumed to be infinitely thin and to possess ideal conductivity. The cylinders have the radius a, the angular dimension of the slits is equal to 2θ, the orientation angle of the slits relative to the 0y axis is equal to φ_0, and the grating period is equal to l.

It is known [1, 2] that a grating consisting of metal bars with a rectangular cross section possesses resonance properties. This is connected with the fact that the flux excites operating conditions close to quasiintrinsic conditions, which affects materially the radiation energy. The structure considered here is also a resonance structure, while its geometry offers considerable scope for optimizing the characteristics of diffraction radiation. The solution obtained here makes it possible to explain the resonance properties of the structure by describing them analytically and performing a numerical analysis by means of a computer.

The method used for solving this problem is based on the results obtained in [2, 4]. It consists in separating in the integral equation of the first kind, to which the solution of the above problem is reduced, the integral operator pertaining to a separate grating element and then transforming it by using the method of the Riemann–Hilbert problem [3].

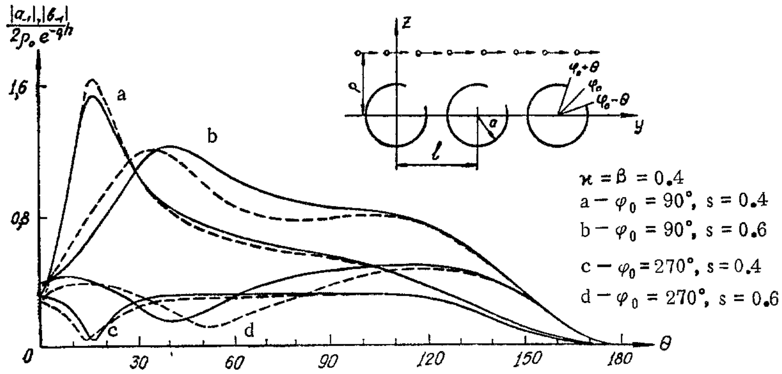

Fig. 1

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1. Reduction of the Problem to a System of Linear Algebraic Equations of the Second Kind

As is known, in the absence of the grating, the intrinsic electromagnetic field of a monochromatic two-dimensional electron flux with the velocity $v = \beta c$ and the charge density

$$\rho = \rho_0 \delta(z - p) e^{i(ky - \omega t)}, \tag{1}$$

has the form of an H-polarized nonuniform plane wave [5],

$$H_x^0 = 2\pi\beta\rho_0 \operatorname{sign}(z - p) e^{-q|z-p| + iky}, \tag{2}$$

where $q = (k_0/\beta)\sqrt{1-\beta^2}$ and $k_0 = \omega/c$. (Here and below, the time dependence $e^{-i\omega t}$ is omitted.) The other field components, E_y^0 and E_z^0, are determined from Maxwell's equations.

It will be subsequently necessary to represent the field (2) in the form of an expansion with respect to cylindrical functions in an r, φ coordinate system. In the range $z < p$, H_x^0 is given by

$$H_x^0 = -2\pi\beta\rho_0 e^{-qp} \sum_{n=-\infty}^{\infty} J_n(k_0 r) i^n \left(\frac{1-\sqrt{1-\beta^2}}{\beta}\right)^n e^{in\varphi}, \tag{3}$$

where $J_n(x)$ are Bessel functions.

The total field excited by the flux is determined by the unique, nonvanishing x-component of the magnetic field, which must satisfy the well-known conditions [2]. We shall represent it in the following form:

$$H(y, z) = H_x^0(y, z) - V(y, z). \tag{4}$$

We shall seek the function $V(y, z)$, which describes the scattered field, in the form of a superposition of double-layer potentials, distributed among the grating elements with the unknown current density $\mu(y, z)$:

$$V(y, z) = \pi \int_S \mu(y_s, z_s) \frac{\partial}{\partial \nu} K(y - y_s, z - z_s)\, dl_s, \tag{5}$$

where

$$K(y, z) = -\frac{i}{2} \sum_{n=-\infty}^{\infty} H_0^{(1)}\left(k_0\sqrt{(y-nl)^2 + z^2}\right) \exp\left(i\frac{k_0 l}{\beta} n\right) \cdot$$

$$= -\frac{i}{2\pi\varkappa} \sum_{n=-\infty}^{\infty} \frac{\exp\left\{ik_0\left[\left(\frac{1}{\beta}+\frac{n}{\varkappa}\right)y + \sqrt{1-\left(\frac{1}{\beta}+\frac{n}{\varkappa}\right)^2}\,|z|\right]\right\}}{\sqrt{1-\left(\frac{1}{\beta}+\frac{n}{\varkappa}\right)^2}} \tag{6}$$

[$\varkappa = l/\lambda$, $H_0^{(1)}(x)$ is the Hankel function, S is the contour of the zero element of the grating, and ν is the normal to its surface].

The second representation of the function $K(y, z)$ is obtained by using Poisson's summation formula and the $\sqrt{A}$ branch for which $\operatorname{Im}\sqrt{A} > 0$; if $\operatorname{Im}\sqrt{A} = 0$, then $\operatorname{Re}\sqrt{A} > 0$.

It follows from (5) that the scattered field constitutes a superposition of uniform and nonuniform plane waves, the amplitudes of which are determined by the geometry of the grating elements and the parameters of the electron beam.

The density of the surface current on the grating cylinders must be such that the Neumann boundary condition is satisfied, as well as the Meixner condition at the ribs. The first condition leads to an integrodifferential equation with respect to the function $\mu(y, z)$, while the second condition determines the class of allowable solutions of this equation. It can be shown that it is sufficient to use L_2 for such a class.

By representing the function $\mu(y, z)$ in the form of a Fourier series with respect to the azimuthal coordinate φ and using the summation theorem for Bessel functions, we pass from the integrodifferential equation to a system of functional equations with respect to Fourier coefficients for the current μ_m which are of the same type as in the case of plane wave diffraction on a single cylinder with a slit [6] or on a grating consisting of such cylinders [4]. By separating in these equations the part connected with the zero elements of the grating and transforming it by using the solution of the Riemann–Hilbert problem [3], we obtain a system of Fredholm linear algebraic equations of the second kind:

$$\mu_m = \sum_{n=-\infty}^{\infty} A_{mn}\mu_n + B_m \quad (m = 0, \pm 1, \ldots), \tag{7}$$

where

$$A_{m0} = i\pi (k_0 a)^2 J_0' \left[H_0^{(1)\prime} T_m^0 + \sum_{p=-\infty}^{\infty} (-1)^p \exp(ip\varphi_0) J_p' G_p(\varkappa, \beta) T_m^p\right],$$

$$A_{mn} = |n| (-1)^n \exp(in\varphi_0)\, \delta_n T_m^n + i\pi (k_0 a)^2 J_n' \sum_{p=-\infty}^{\infty} (-1)^p \exp(ip\varphi_0) J_p' G_{p-m}(\varkappa, \beta) T_m^p,$$

$$T_m^n = \begin{cases} -W_n(-u), & m = 0 \\ \dfrac{1}{m}(-1)^m \exp(-im\varphi_0) V_{m-1}^{n-1}(-u), & m \neq 0 \end{cases}$$

$$B_m = -i\pi (k_0 a)^2 \sum_{n=-\infty}^{\infty} \exp(in\varphi_0)(-i)^n J_n' T_m^n (1 - \sqrt{1-\beta^2})^n / \beta^n, \tag{8}$$

$$\delta_n = 1 + \frac{i\pi (k_0 a)^2}{|n|} J_n' H_n^{(1)\prime},$$

$$G_m(\varkappa, \beta) = \sum_{n=1}^{\infty} \left[\exp\left(i2\pi n\frac{\varkappa}{\beta}\right) + (-1)^m \exp\left(-i2\pi n\frac{\varkappa}{\beta}\right)\right] H_m^{(1)}\left(2\pi n\frac{\varkappa}{\beta}\right).$$

In these expressions, $(k_0 a)$ is the argument of the cylindrical functions, while the W_n and V_m^n functions are expressed in terms of Legendre polynomials [3], and $u = \cos\theta$.

The amplitudes of the three-dimensional harmonics are expressed in terms of the unknowns μ_m in the form of series. For instance, over the grid,

$$a_n = -\frac{2\beta\rho_0 e^{-qp}}{\varkappa\sqrt{1 - \left(\dfrac{1}{\beta} + \dfrac{n}{\varkappa}\right)^2}} \sum_{m=-\infty}^{\infty} \mu_m i^m J_m'(k_0 a) \exp(-im\varphi_n), \tag{9}$$

where $\varphi_n = \arccos[(1/\beta) + (n/\varkappa)]$ is the radiation angle of the harmonic with the number n.

2. Approximate Solution of the Infinite System of Equations and Characteristics of Diffraction Radiation

If we compare the matrix elements A_{mn} of system (7) with the analogous quantities in the problem of flux motion over one cylinder with a longitudinal slit [7], we notice that they differ from each other by the presence of a series with the functions $G_p(\varkappa, \beta)$. These functions actually describe the mutual effect of cylinders in the grating. It follows from the asymptotic expressions for Hankel functions that $\lim_{l\to\infty} G_p(\varkappa, \beta) = 0$, and, therefore, system (7) allows transition in the limit to the case of an isolated cylinder.

Since the Schlömilch series for G_p (8) converges slowly, it is more convenient to investigate system (7) by representing $G_p(\varkappa, \beta)$ in the form of rapidly converging series with respect to elementary functions, as was done in [8].

Analysis of the thus obtained expressions shows that the matrix elements A_{mn} may become infinite for certain values of the parameters $\varkappa$ and β. This occurs at the "sliding points" if $\varkappa[(1/\beta) \pm 1] = n$ $(n = 1, 2, \ldots)$ and also in the uninteresting case of the static charge distribution for $\beta = 0$.

It should be noted that, as a result of the substitution

$$G_p(\varkappa, \beta) = \widetilde{G}_p(\varkappa, \beta) + \alpha_{\mp} \exp\left(-ip\frac{\pi}{2}\right) + \alpha_{-} \exp\left(ip\frac{\pi}{2}\right),$$

where

$$\alpha_{\pm} = \frac{\exp\left(-i\dfrac{\pi}{4}\right)}{\pi\sqrt{\varkappa}} \sum_{n=1}^{\infty} \frac{\exp\left[i2\pi\varkappa\left(1 \pm \dfrac{1}{\beta}\right)n\right]}{\sqrt{n}},$$

system (7) is transformed into a system suitable for investigations for any values of $\varkappa$ and β $(\beta \neq 0)$.

Nevertheless, with the exception of the above values of $\varkappa$ and β, system (7) is preferable in all cases because of the simplicity of its elements. Since it is of the Fredholm type, its solution can be obtained by using the cutoff method [9].

Let us estimate the norm of the matrix of system (7) $q = \max_{m\neq 0} \sum_{n\neq 0, m} \left| \frac{A_{mn}}{1 - A_{mm}} \right|$. It follows from estimates of the functions V_m^n [3], the functions G_p [2], and the asymptotic expressions for cylindrical functions that

$$q < \sqrt{1-u^2}\left[C_1(k_0 a)^2 + C_2\left(\frac{a}{l}\right)^2\right] + C_3\sqrt{1+u}(k_0 a)\left(\frac{a}{l}\right) + C_4(1+u)\left(\frac{a}{l}\right)^2, \tag{10}$$

where C_i are constants.

Thus, for a grating consisting of sufficiently narrow strips ($u \to -1$) and in the case of a widely spaced grating ($s = 2a/l \ll 1$) consisting of cylinders with arbitrary slits – $q < 1$ – we can, therefore, use the method of successive approximations for the solution of system (7).

In the zero approximation, we obtain from (7)

$$\mu_0 = \frac{i\pi(k_0 a)^2 \sum_{p=-\infty}^{\infty} \exp(ip\varphi_0)(-i)^p J_p'\left(\frac{1-\sqrt{1-\beta^2}}{\beta}\right)^p W_p}{1 + i\pi(k_0 a)^2 J_0'\left[H_0^{(1)'} W_0 + \sum_{p=-\infty}^{\infty}(-1)^p \exp(ip\varphi_0) J_p' G_p W_p\right]}; \tag{11}$$

$$\mu_m = \frac{(-1)^m \exp(im\varphi_0)\, i\pi(k_0 a)^2}{m}\left\{\mu_0\left[J_0' H_0^{(1)'} V_{m-1}^{-1} + J_0' \sum_{p=-\infty}^{\infty}(-1)^p \right.\right.$$

$$\left.\left. \times \exp(ip\varphi_0) J_p' G_p V_{m-1}^{p-1}\right] + \sum_{p=-\infty}^{\infty} \exp(ip\varphi_0) J_p'\left(\frac{1-\sqrt{1-\beta^2}}{i\beta}\right)^p V_{m-1}^{p-1}\right\}$$

$$\times\left[1 - \frac{|m|}{m}\delta_m V_{m-1}^{m-1} - i\pi(k_0 a)^2(-1)^m \frac{\exp(-im\varphi_0)}{m} J_m' \sum_{p=\infty}^{\infty}(-1)^p \exp(ip\varphi_0) J_p' G_{p-m} V_{m-1}^{p-1}\right]^{-1}. \tag{12}$$

We shall now investigate in greater detail certain important particular cases.

a) Grating Consisting of Narrow Strips. The following approximate expressions are obtained from (11) and (12) for the amplitudes of the diffraction harmonics produced by interaction between the flux and a grating consisting of narrow cylindrical strips:

$$a_n = -\frac{2\pi\beta\rho_0(k_0 a)^2\cos^2(\theta/2)}{\varkappa\sqrt{1-\left(\frac{1}{\beta}+\frac{n}{\varkappa}\right)^2}}\left(\sqrt{1-\beta^2}\sin\varphi_0 - i\cos\varphi_0\right)$$

$$\times\cos(\varphi_n - \varphi_0)\exp[-q(p + a\sin\varphi_0)]\exp\left\{ik_0 a\left[\cos(\varphi_n-\varphi_0) - \frac{1}{\beta}\cos\varphi_0\right]\right\}. \tag{13}$$

The error of these expressions is of the order of $O[(k_0 a)^4\cos^4(\theta/2)]$. It follows from (13) that there is no radiation in the direction parallel to the grating strips, which agrees with the results obtained by other authors [10]. This is readily explained by the absence of a longitudinal current component at the strips. The factor $\exp[-q \times (p + a\sin\varphi_0)]$ reflects the fact that the efficiency of interaction between the flux and the grating depends exponentially on the impact parameter. Actually, the harmonic amplitudes are at their maximum at $\varphi_0 = 270°$, i.e., when the strips are closest to the flux. It should also be noted, that for $\varphi_0 = 90, 270°$, expressions (13) coincide with an accuracy to the phase factor with those that can be obtained from [5] for diffraction radiation over a plane narrow-strip grating.

b) Widely Spaced Grating Consisting of Cylinders with Narrow Slits. If the grid consists of sufficiently widely spaced cylinders with narrow slits, expressions (11) and (12) are also simplified considerably, and the amplitudes of the field harmonics of diffraction radiation assume the following form:

$$a_n = -\frac{2\beta\rho_0 e^{-qp}}{\varkappa\sqrt{1-\left(\frac{1}{\beta}+\frac{n}{\varkappa}\right)^2}}\left\{\mu_0 J_0' - (\mu_0 H_0^{(1)'} - 1)J_0' \sum_{m\neq 0}\frac{|m|}{m}\right.$$

$$\left.\times\frac{(-i)^m\exp[im(\varphi_0+\varphi_n)]V_{m-1}^{-1}}{H_m^{(1)'} + J_m' G_0} + \sum_{m\neq 0}\frac{|m|}{m}\frac{J_m'(-1)^m\exp(-im\varphi_n)V_{m-1}^{-1}}{H_m^{(1)'} + J_m' G_0}\left(\frac{\beta}{1+\sqrt{1-\beta^2}}\right)^m\right\}. \tag{14}$$

Expression (14) is determined with an accuracy of $O[(a/l)^2\sin^2(\theta/2)]$. Analysis of this expression shows that the harmonic amplitudes have extremums when the grating parameters satisfy the equation

$$| D_0(k_0 a, \theta) |' = 0, \tag{15}$$

where D_0 is the denominator of expression (11).

By solving the corresponding homogeneous problem of free oscillations, we can show that the resonance values of the parameters in (15) are close to the roots of the dispersion equation of the structure. These roots determine the quasiintrinsic operating conditions of the grating, and they are all complex. Physically, this means that, in the case of cylinders with narrow slits, the grating constitutes an open resonance structure, where quasinatural oscillations are damped in the course of time ("damped resonances" [2]). The damping rate is determined by the negative imaginary part of the resonance values of the frequency parameter $\varkappa$.

For $k_0 a \ll 1$, the dispersion equation assumes the following form:

$$1 + (k_0 a)^2 \left[1 - \pi \frac{(k_0 a)^2}{4} \operatorname{Im} G_0 \right] \ln \sin^2 \frac{\theta}{2} + O\left(\varkappa^3 s^4 \sin^2 \frac{\theta}{2} \right) = 0. \tag{16}$$

If, for instance, the radiation conditions are satisfied only for the −1st three-dimensional harmonic, then

$$\operatorname{Im} G_0(\varkappa, \beta) = -\frac{2}{\pi} \ln \frac{\gamma\varkappa}{2} - (\varkappa^2 - \nu^2) \frac{1.202}{\pi} + O(\nu^3),$$

where

$$| \nu | = \left| \frac{\varkappa}{\beta} - 1 \right| < \frac{1}{2}, \quad \gamma = 1.781\ldots .$$

The root of Eq. (16) is equal to

$$k_0 a = \left(-\ln \sin^2 \frac{\theta}{2} \right)^{-1/2} + \chi, \tag{17}$$

where the small complex addition χ has the order of $O[-\ln \sin^2(\theta/2)]^{-3/2}$. This root of the dispersion equation pertains to the so-called outflowing slit wave in a round waveguide with a slit. The Q-factor of the corresponding resonance, defined as $\operatorname{Re}\varkappa/\operatorname{Im}\varkappa$, has the order of $O[-\ln \sin^2(\theta/2)]$.

Analysis of the dispersion equation for arbitrary values of $k_0 a$ shows that the flux excites other quasi-intrinsic operating conditions if $k_0 a = \pi\varkappa s$ is close to the roots of derivatives of the Bessel functions, i.e., when the frequency is close to the resonance frequencies of a closed cylinder, but differs from them by a small addition. For instance, for perturbed symmetric H-oscillations, the resonance frequency is equal to

$$\begin{gathered} k_0 a = \nu_{0m} + \chi_1 + i\chi_2 + O(\delta^3), \\ J_0'(\nu_{0m}) = 0, \quad \chi_1 = -\delta + 2\delta^2 \eta_1, \quad \chi_2 = 2\delta^2 \eta_2, \quad \delta = \frac{1}{2} \ln^{-1} \sin \frac{\theta}{2}, \\ \eta_1 = \frac{4}{\nu_{0m}} + 2 \operatorname{Im} G_0 \frac{J_0'}{N_1'} + \frac{N_1''}{N_1'}, \quad \eta_2 = -2(1 + \operatorname{Re} G_0) \frac{J_0''}{N_1'} \end{gathered} \tag{18}$$

(ν_{0m} is the argument of the cylindrical functions).

The expression obtained indicates that the Q-factor of the corresponding resonance has the order of $O[\ln^2 \sin^2(\theta/2)]$, i.e., it is larger than in the case of resonance corresponding to a slit wave. The shift of the resonance frequency (with respect to ν_{0m}) is caused by nonuniformities in the form of slits in the cylinder walls and also by interaction between cylinders. It should be noted that, since the exciting field is not uniform, the radiation energy should be more critical with respect to the orientation angle φ_0 near resonances than at a distance from them.

3. Analysis of the Numerical Results

The approximate analytical expressions obtained by means of successive approximations can only be used if substantial constraints are imposed on the parameters θ, $\varkappa$, and s. For a more detailed investigation of the radiation characteristics, we performed a numerical analysis of the problem by solving system (7) by means of an M-222 computer, using the cutoff method. Figures 1-6 show some of the obtained amplitude dependences for the −1st three-dimensional harmonic, emitted into the upper (a_{-1}) and lower (b_{-1}) half-space.

The operating conditions where the entire energy of diffraction radiation is concentrated in a single undamped harmonic, emitted along the normal to the grid, are of the greatest interest in calculations. It follows from (6) that this occurs for the −1st harmonic if $\varkappa = \beta < 1$. It should be noted that, for those diagrams which do not pertain to rotation with respect to the orientation angle φ_0, the effective impact parameter does not change, i.e., the spacing h between the flux and the grid remains constant.

If the wave dimensions of the cylinders are fixed ($k_0 a = \pi \varkappa s$ = const), the dependences of $|a_{-1}|$ and $|b_{-1}|$ (dashed curves) on the angular dimension of the slit θ have a resonance character for different orientation angles of the slit φ_0 (Fig. 1). The increase or reduction in the radiation energy is connected with the excitation near the structure elements of a field close to the quasiintrinsic field of the grating, while the resonance values of θ and $k_0 a$ satisfy relationship (16). The character of the resonance depends essentially on the orientation angle of the slit φ_0. The calculation results indicate that the variation of φ_0 leads to changes in the phase and amplitude distributions of the current at the surface of a grating element, which affects the amplitudes of the emitted two-dimensional waves. The phenomenon indicates the nonuniformity of the excited field and is most strongly pronounced in the resonance case.

If the slit disappears ($\theta \to 0$), $|a_{-1}|$ and $|b_{-1}|$ tend to finite (not equal) values corresponding to radiation over a grating consisting of round bars. As was to be expected, the limiting values are independent of φ_0. In the other limiting case, when $\theta \to 180°$, i.e., the grating consists of narrow cylindrical strips, the amplitudes of the harmonics tend to zero, while almost equal amounts of energy are emitted into the upper and lower half-spaces. This agrees with the results obtained in investigating diffraction radiation over a grating consisting of flat strips [5].

Figures 2 and 3 show the amplitude of the −1st harmonic as a function of the space factor of the structure $s = 2a/l$. In this, the grating period remains constant, and only the wave dimensions of the cylinders change. The resonance character of the dependences is connected with the excitation of conditions close to quasiintrinsic conditions in the structure. For instance, if the resonances in Fig. 2 and the first of the resonances in Fig. 3a are slit-type resonances, the second (Fig. 3a) is connected with the excitation inside the cylinders of a field close to the oscillation field H_{11} of a closed cylinder. Actually, at resonance, $k_0 a = 1.92$; this is close to the first root of the function $J_1'(x)$, which is equal to 1.84 (the causes of the resonance frequency shift were mentioned earlier).

As was to be expected, the described resonance phenomena disappear when the angular dimensions of the slit are so large that the structure elements constitute cylindrical strips (Fig. 3b).

Let us consider in greater detail the effect of the orientation angle of the slit φ_0 on the radiation efficiency. As far as we know, this has not yet been discussed in problems of diffraction radiation. Figure 4 pertains to the case where the grating consists of narrow strips. It is also clear from elementary physical considerations that the energy of the harmonics is at a maximum when the flux is closest to the strips, and they form an almost plane grating ($\varphi_0 = 270°$). Since $\varkappa = \beta$, i.e., only the −1st harmonic is emitted along the normal to the grating, the radiation energy is evidently at a minimum when φ_0 is close to 0 or 180°, and the structure

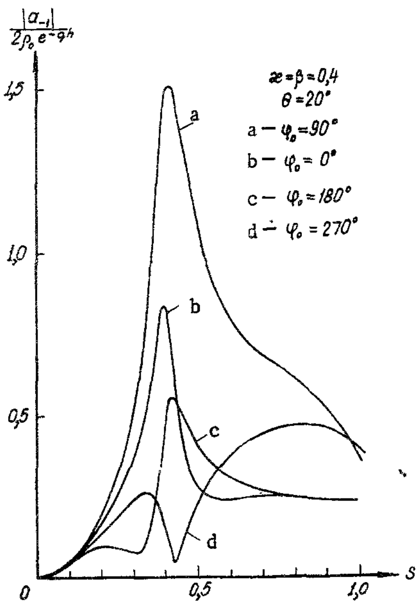

Fig. 2

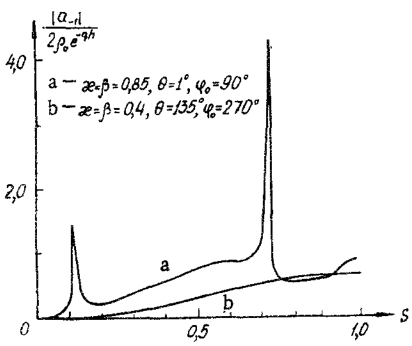

Fig. 3

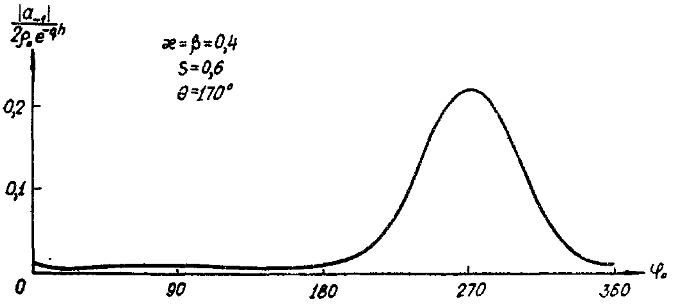

Fig. 4

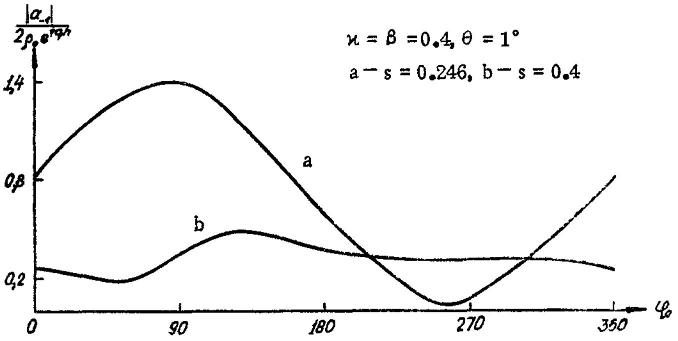

Fig. 5

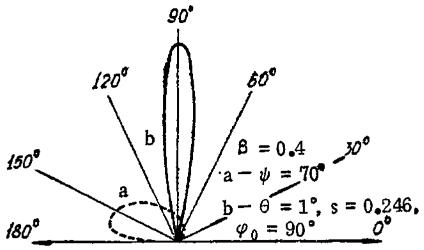

Fig. 6

is close to a knife grating. At the same time, this energy is not equal to zero, which is explained by the curvature of the strips forming the grating.

As θ diminishes, the character of the φ_0 dependence changes drastically. If the slit is narrow, for instance, if $\theta = 1°$ (Fig. 5), changes in the distance between the flux and the grating can be neglected in rotation with respect to φ_0. In the resonance case (s = 0.246), the angular dependence is strongly pronounced, while the maximum of the radiation energy is observed when the cylinder slits face the flux. With departure from the resonance with respect to any of the parameters, for instance, with respect to s, the dependence φ_0 becomes insignificant. Such behavior of the curves reflects the fact that resonance is connected with slit waves and is excited by the field of nonuniform plane waves.

In order to compare a grating consisting of open cylinders with gratings of other types, we also plotted normalized polar diagrams of radiation of the −1st harmonic (Fig. 6). (The dashed curve shows the diagram of radiation over a grating consisting of half-planes inclined at $\psi = 70°$ [10].) The direction of maximum radiation corresponds to the excitation in the structure of the quasiintrinsic conditions investigated above. By using suitable grating parameters, we can ensure, for instance, the maximum radiation in the vertical direction. This is of considerable importance in diffraction electronics and in designing continuous high-power diffraction radiation generators with electronic efficiency on the order of 20%. It should be noted that, for echelette-type reflection gratings and some others, radiation along the normal to the grating is weak according to fundamental considerations [10]. Moreover, the maximum radiation energy over a grating consisting of cylinders with longitudinal slits exceeds the radiation energy for the flux over a comb-type grating.

We shall mention in conclusion that analysis of the convergence of the calculation results shows that it is sufficient to solve a 17th order system in order to ensure an accuracy not worse than 0.5%. In order to verify the solution and the calculation algorithm, we checked the boundary conditions at the cylinder surface and the conditions at the rib and found that the results were entirely satisfactory for any parameters of the problem.

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COMPETITION OF MODES RESONANT WITH DIFFERENT HARMONICS OF CYCLOTRON FREQUENCY IN GYROMONOTRONS

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Oscillation of the working mode at the second cyclotron harmonic $2\omega_H$ has the effect of extending the region in which parasitic modes resonant with the first cyclotron harmonic ω_H can be excited. A study is made of the dependence of the conditions for self-excitation of parasitic modes (when the working mode is oscillating) on the basic parameters of the gyromonotron: beam current, intensity of magnetostatic field, frequency detuning of modes, and relative efficiency of interaction of electrons with the fields of the working and parasitic modes.

§1. In gyromonotrons with a spatially developed resonant system, it is possible for the excitation conditions to be satisfied simultaneously for several modes. The competition of modes resonant with the cyclotron frequency ω_H means that exciting one mode into oscillation reduces the region in which self-excitation of other modes can occur [1].

In gyromonotrons operating on a mode resonant with $2\omega_H$, "dangerous" competition comes, as a rule, from modes resonant with ω_H, which interact more effectively with the beam [2]. The interaction of electrons with a high-frequency field of frequency ω is different in nature for $\omega \approx \omega_H$ and $\omega \approx 2\omega_H$ [3]: For $\omega \approx \omega_H$ it is dipolar and for $\omega \approx 2\omega_H$ it is quadrupolar. One may expect, therefore, that the competition of modes resonant with the various harmonics of ω_H will differ from the case considered in [1] of the competition of modes resonant with the fundamental. In the present paper we report a study of the effect of the oscillation of the working mode, resonant at $2\omega_H$, on the conditions for self-excitation of parasitic modes resonant with ω_H.

§2. We consider the gyromonotron model adopted in [1]: The working space of the device is axially symmetric; the flux of weakly relativistic electrons has spread-free velocities and guiding center radii R_0; the resonator, of radius R, has a high Q, as a result of which the time to set the mode oscillating is considerably

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