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Lasing modes of a microdisk with a ring gain area and of an active microring

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Abstract Microcavity lasers shaped as thin circular disks are famous for the ultra-low thresholds of their whispering-gallery modes. We considered a two-dimensional model of such a laser in free space with a ring-like active region and compared the characteristics of its modes with the modes of an active microring, i.e. a similar disk with a concentric hole. The comparison showed that a microring has considerable rarefaction effect in terms of emission thresholds, accompanied by the blue-shift of emission spectra. If the ring becomes narrower than a half-wavelength in material, then all lasing modes obtain catastrophically high thresholds.

Keywords Lasers · Laser resonators · Lasing modes · Microdisk · Microring · Threshold of lasing

1 Introduction

Although flat thin circular "macro"-disk lasers were attempted first in the 1970s (Horvath 2012), it was the microdisk lasers standing on a pedestal or laying on a substrate that, since the 1990s (McCall et al. 1992), became an object of intensive research in photonics, as

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certified by the reviews (Nosich et al. 2007; Harayama and Shinohara 2011; He et al. 2013; Zhang et al. 2015). Their most remarkable feature is very low threshold of lasing, associated with the whispering-gallery (WG) modes famous for very high Q-factors in the pump-off situation. The working WG modes of a microdisk have almost periodically spaced frequencies and emit light mostly in the disk plane. The materials used for fabricating the microdisk lasers are semiconductors such as GaAs or GaN containing active layers of quantum dots, crystalline ones doped with erbium, and increasingly frequently met low-cost dye-doped polymers. The pumping is usually done by a wide optical beam illumination from the free space that makes all disk active however sometimes a shaped pump spot produced by a focused beam is used; spatially selective pumping is also common if one uses injection of carriers from electrodes.

Microring lasers are met less frequently however a ring is an attractive alternative to a disk because it supports smaller number of WG modes with low thresholds. If pumped with a hollow beam or from a ring-like electrode, microring laser needs smaller pump power or current to start lasing (Krauss et al. 1990; Schlehahn et al. 2013).

In terms of the modelling, thinner-than-wavelength dielectric cavities offer a chance to avoid a full 3-D modeling (which is still more adequate), and use the 2-D approximation. This implies replacement of the bulk refractive index by its effective value and analysis of the field in the cavity plane only. It predicts that in microdisks and microrings the WG mode Q-factors are exponentially large relatively to the optical size of the perfectly circular cavity although in reality they are restricted by the disk thickness and roughness. One should also keep in mind that each mode with azimuth index $m = 1, 2, 3,\ldots$ is double degenerate in a circular cavity. The in-plane emission on these modes has low directivity because their field varies in azimuth as $\cos m\varphi$ and hence displays many identical beams (their number being $2m$; Horvath 2012; McCall et al. 1992; Nosich et al. 2007; Harayama and Shinohara 2011; He et al. 2013; Zhang et al. 2015; Krauss et al. 1990; Schlehahn et al. 2013). This circumstance and practical need for more directive sources caused a quest for the microcavity lasers whose shape is distorted from circle, to provide better directionality of emission (Schwefel et al. 2004; Huang et al. 2006; Dubertrand et al. 2008; Wang et al. 2010; Smotrova et al. 2005, 2013). Still it is necessary to emphasize that real-life microdisk lasers have record-low thresholds, which obtain higher values if any distortion from the circular contour is made.

All simulations of microlaser modes made before the mid-2000s were performed using the model of passive cavity and hence, instead of the actual threshold, were looking for the mode quality factors. Moreover, passive modeling was used in all the mentioned above papers except of Nosich et al. (2007) and Smotrova et al. (2013). However this model is obviously not adequate to the lasing phenomenon and hence, in the 2000s, several theories appeared able to extract the thresholds of lasing from the linear electromagnetic-field formulations (Smotrova et al. 2005; Nojima 2005; Mock 2010; Smotrova et al. 2011; Chang 2012; Gagnon et al. 2014). We believe that fully adequate linear electromagnetic modeling of lasers is provided by the Lasing Eigenvalue Problem (LEP) specifically tailored to consider open resonators equipped with active regions (Nosich et al. 2007; Smotrova et al. 2005, 2011). The LEP formulation implies introduction of material gain in the whole microcavity or at least in the active region that enables extraction of the real-valued lasing frequencies and associated threshold values of material gain as eigenvalues.

In particular, LEP enabled to demonstrate that the WG-mode thresholds are exponentially low relatively to the optical size of the perfectly circular cavity (Smotrova et al. 2005). Still low threshold is not fully equivalent to high Q-factor although these two quantities are in inverse proportion to each other—the overlap between the active region and the modal electric field is equally important (Smotrova et al. 2011). It is worth noting that the LEP approach can be combined with any method of the analysis of electromagnetic fields, either analytical or numerical. A comparison of such methods, including the "billiard theory," the finite-difference time domain codes, and the integral-equation techniques can be found in Nosich et al. (2007).

In this paper, our task was to perform 2-D modeling, with the aid of LEP and the method of separation of variables, of two microlaser configurations: a circular disk with a ring-like gain area (i.e. active region) and a uniformly active circular ring (i.e. a disk with a circular hole in the center). We focused our analysis on the dynamics of the thresholds of lasing with respect to the variation of the ring-radii ratio.

2 Eigenvalue problems formulation

Geometries of a 2-D circular microresonator with a ring-like gain area (i.e. active region) and an active microring are shown in Fig. 1a, b, respectively. In either case we supposed that the cavity's outer radius is a. Considering the lasing eigenvalue problems (LEP), we assumed that, in either case, the gain γ is step-like, i.e. uniform inside the ring of the outer radius a and the inner radius b, and zero inside the circle of radius b. To compare the results, in computations we supposed that two microresonators had the same refractive index α_i and the environment had the same refractive index α_e.

In either case, we looked for the non-attenuating time-harmonic electromagnetic field $\sim\exp(-ikct)$, $k > 0$, inside and outside of the considered resonators (c stands for the free-space light velocity). Each point in the space is specified by the axial, radial, and azimuth coordinates, z, r, and φ. We assumed that the field does not vary along the z axis and could be characterized by a scalar function u, which represented either E_z or H_z component depending on the polarization. Off the boundary, this function had to satisfy the Helmholtz equation,

$$\Delta u(x) + k^2 \nu_j^2 u(x) = 0, \quad x \in \Omega_j, \quad j = 1, 2, e. \tag{1}$$

Here $x = (r, \varphi)$, $\Omega_1 = \{x \in R^2 : |x| < b\}$, $\Omega_2 = \{x \in R^2 : a > |x| > b\}$, $\Omega_e = \{x \in R^2 : |x| > a\}$, $\nu_1 = \alpha_i$ in the case of microdisk, $\nu_1 = \alpha_e$ for microring, $\nu_2 = \alpha_i - i\gamma$, and $\nu_e = \alpha_e$. The field was requested to satisfy the dielectric-boundary conditions at the circles $\Gamma_1 = \{x \in R^2 : |x| = b\}$, $\Gamma_2 = \{x \in R^2 : |x| = a\}$,

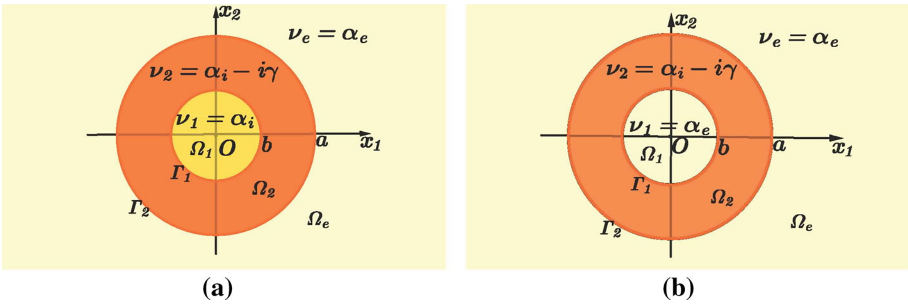

Fig. 1 Geometry of a microdisk with a ring gain area (**a**) and an active microring (**b**)

$$u^- = u^+, \quad x \in \Gamma_j, \quad j = 1,2, \quad \beta_1 \frac{\partial u^-}{\partial r} = \beta_2 \frac{\partial u^+}{\partial r}, \quad x \in \Gamma_1, \quad \beta_2 \frac{\partial u^-}{\partial r} = \beta_e \frac{\partial u^+}{\partial r}, \quad x \in \Gamma_2. \tag{2}$$

where $\beta_j = \nu_j^{-2}$ in the H-polarization case and $\beta_j = 1$ in the E-polarization case, $j = 1, 2, e$; $u^{\pm} = u(r \pm 0, \varphi)$, $r = a, b$. Following Smotrova et al. (2005, 2011), we considered the set of Eqs. (1), (2) with corresponding material parameters plus Sommerfeld radiation condition at infinity that selects outgoing field solutions (in view of the real value of the wavenumber k) as two eigenvalue problems (for microdisk and microring resonators) and looked for the eigenvalues as pairs of real-valued parameters (κ, γ). The first of them is the normalized frequency of lasing, $\kappa = ka$, while the second is the threshold value of material gain.

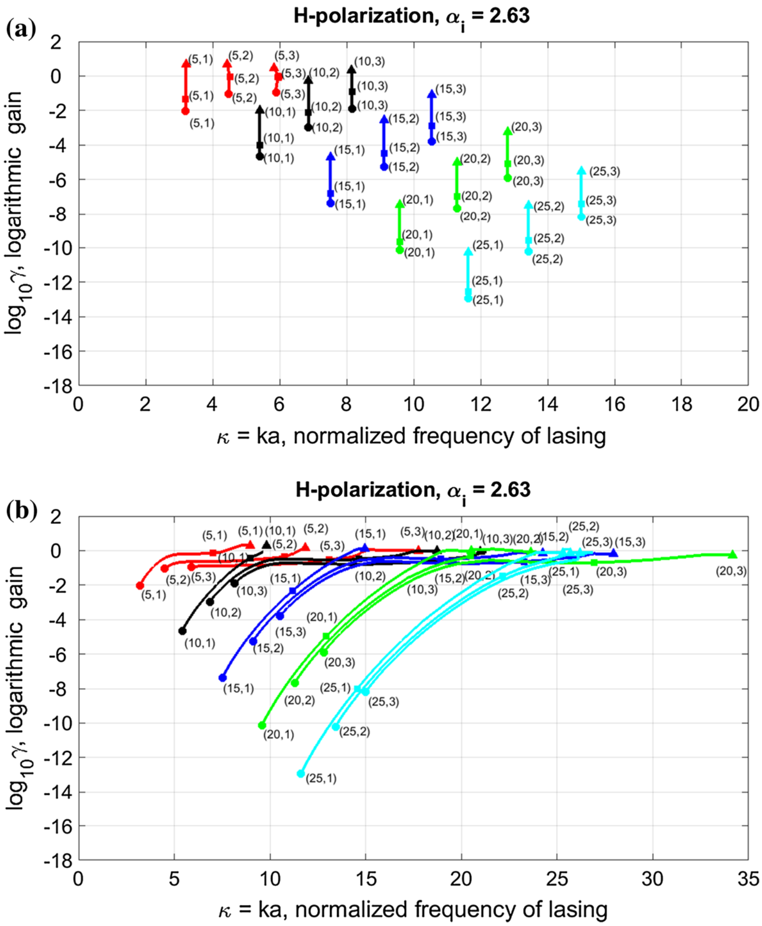

Fig. 2 Normalized frequencies of lasing and threshold gains for the H-polarized modes of a microdisk resonator with a ring gain area (**a**) and a microring resonator (**b**). Eigenvalues for the relative radius of the ring $\sigma = b/a = 0$ are marked by *circles*, for $\sigma = 0.9$ by *squares*, and for $\sigma = 0.999$ by *triangles*

3 Numerical results and discussion

The problem (1), (2) for a microdisk resonator was solved by the method of separation of variables in Smotrova et al. (2005). The solution for a microring resonator is analogous. In either case, all modes split into separate families according to the azimuth index m. Within each m-th family the eigenvalues (κ, γ) of both problems satisfy the following characteristic equation:

$$\frac{\beta_1 \nu_1 J_m'(\kappa\sigma\nu_1)}{J_m(\kappa\sigma\nu_1)} = \frac{\beta_2^2 \nu_2^2 H_m^{(1)}(\kappa\nu_2) P_m + \beta_2 \nu_2 \beta_e \nu_e H_m^{(1)'}(\kappa\nu_e) T_m}{\beta_2^2 \nu_2^2 H_m^{(1)}(\kappa\nu_e) P_m + \beta_e \nu_e H_m^{(1)'}(\kappa\nu_e) K_m}, \quad m = 0, 1, 2, \ldots, \qquad (3)$$

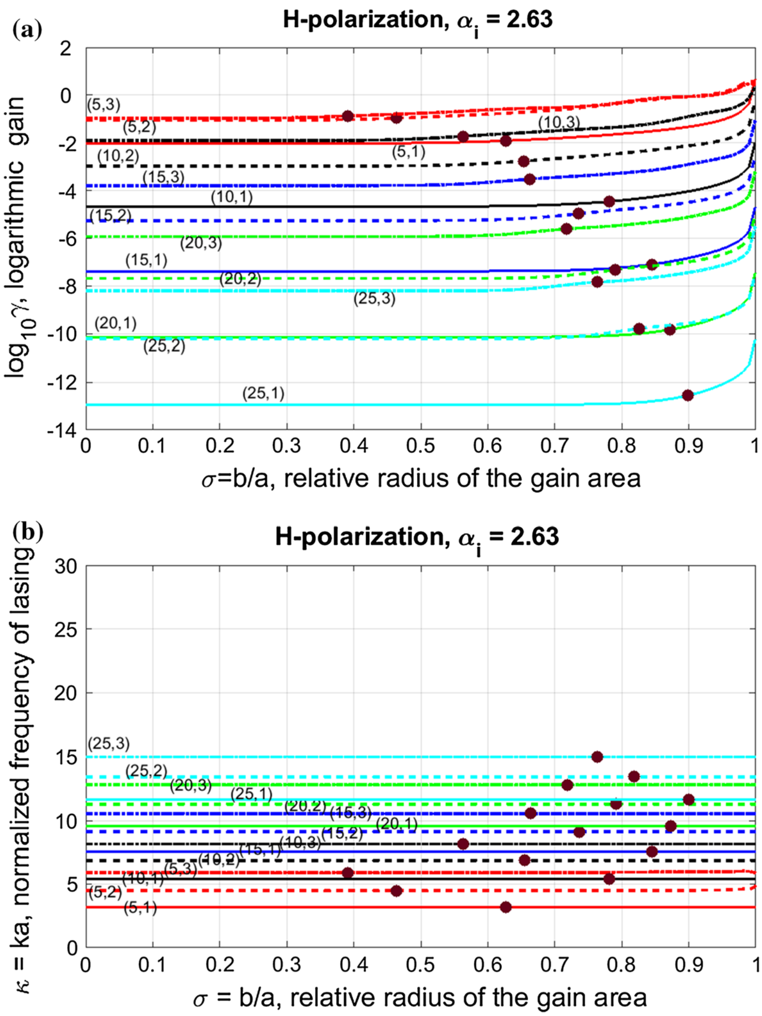

Fig. 3 Dependences of the threshold gains (**a**) and normalized frequencies of lasing (**b**) for the H-polarized WG modes of a microdisk resonator with a ring gain area on the radii ratio σ

where $\sigma = b/a$, and

$$P_m = J'_m(\kappa\nu_2)H_m^{(1)'}(\kappa\sigma\nu_2) - J'_m(\kappa\sigma\nu_2)H_m^{(1)'}(\kappa\nu_2), \tag{4}$$

$$T_m = J'_m(\kappa\sigma\nu_2)H_m^{(1)'}(\kappa\nu_2) - J_m(\kappa\nu_2)H_m^{(1)'}(\kappa\sigma\nu_2), \tag{5}$$

$$K_m = J_m(\kappa\sigma\nu_2)H_m^{(1)}(\kappa\nu_2) - J_m(\kappa\nu_2)H_m^{(1)}(\kappa\sigma\nu_2). \tag{6}$$

The theorems of complex calculus tell that the roots of (3) are discrete on the plane (κ, γ), infinite in number, and depend continuously on the parameters σ, α_i and α_e. To number the eigenvalues within one family, we used another index, $n = 1, 2, \ldots$, which characterized the number of the modal field variations along the radius of the resonator. In

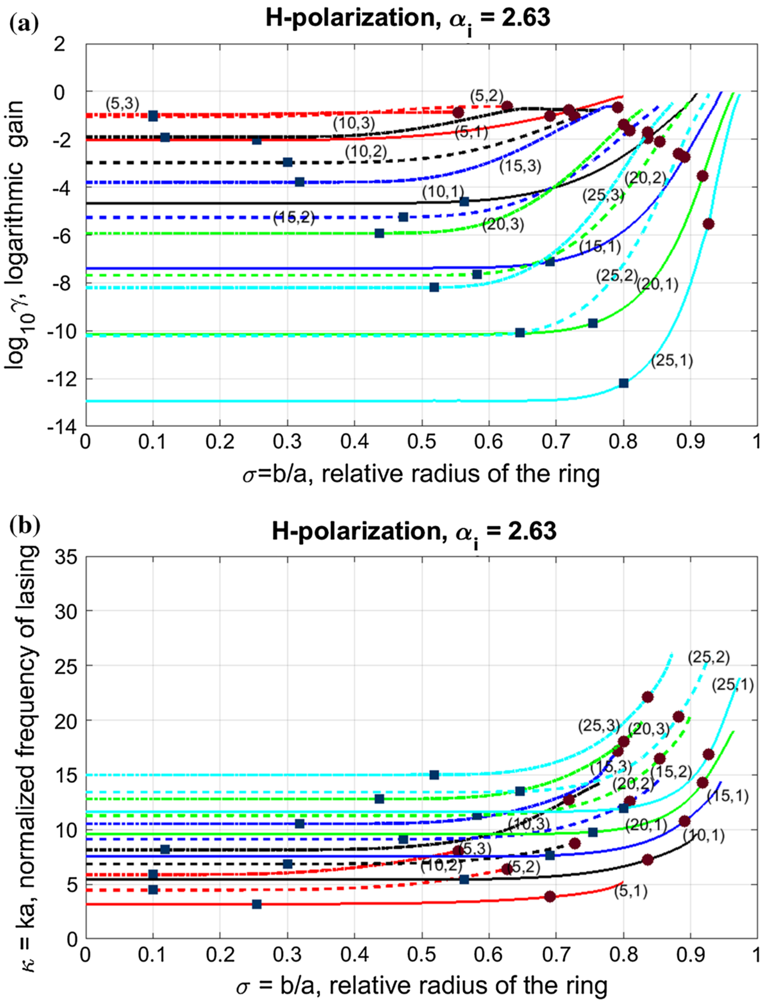

Fig. 4 Dependences of the threshold gains (**a**) and normalized frequencies of lasing (**b**) for the H-polarized WG modes of a microring resonator on the radii ratio σ

computations, we assumed that the cavity material had refractive index $\alpha_i = 2.63$ and the environment was air with $\alpha_e = 1$.

In Fig. 2, we present the trajectories of the LEP eigenvalues for the WG modes ["internal" modes in the sense of Dettmann et al. (2009)] with the azimuth indices $m = 5, 10, 15, 20$, and 25 and the radial indices $n = 1, 2$ and 3 for the ring-pumped solid microdisk [panel (a)] and the microring [panel (b)] on the plane (κ, γ). They correspond to the radii ratio $\sigma = b/a \in [0,\ 1]$ varying from 0 to 0.999 and enable one to compare the behavior of the H-polarized modes in two considered configurations of microlasers. If σ is smaller than 0.8, then the LEP eigenvalues of both ring-pumped microdisk and microring are close to the eigenvalues of a uniformly active microdisk (see also Figs. 3a, 4a). If $\sigma > 0.8$, then the material thresholds of microdisk WG modes start growing slowly, apparently because the overlap between the active region and the WG mode E-field remains good (Smotrova et al. 2011), while the material thresholds of microring modes grow much faster because this overlap gets spoiled quickly.

Figure 2b demonstrates the efficient rarefaction of the spectrum of modes with low thresholds: only the WG modes with radial index $n = 1$ keep low material thresholds at $\sigma = 0.9$, and even they obtain large thresholds if $\sigma > 0.99$. Note that shrinking of the active region of the solid microdisk has no effect on the lasing frequencies [see panel (a)] because of very low contrast (of the order of γ) between the active and passive parts of cavity. In contrast, deforming the disk into a ring shifts the frequencies to the blue side [see

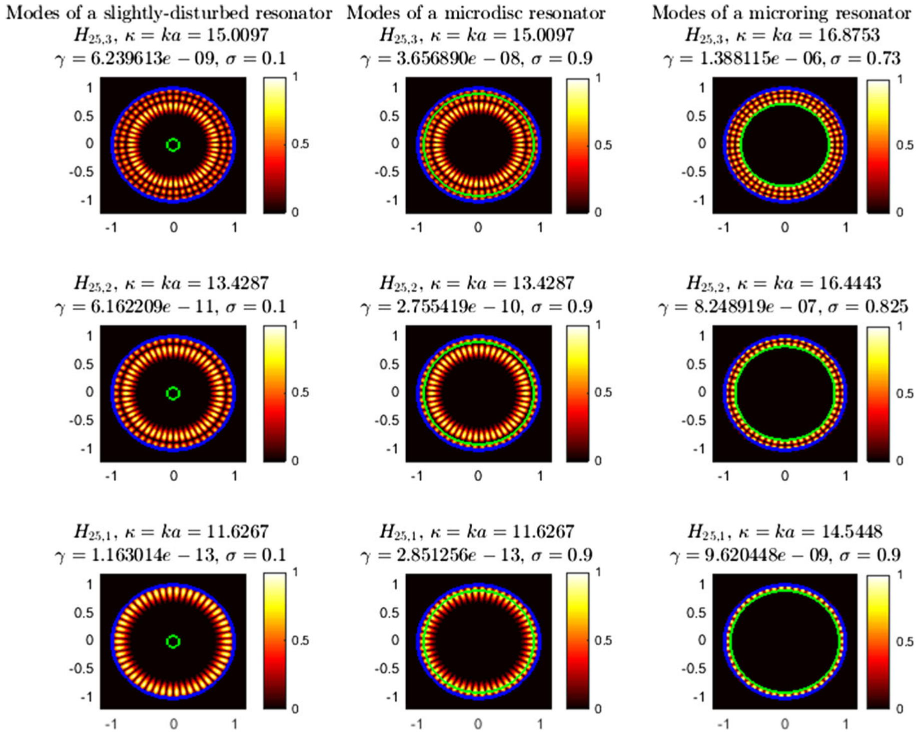

Fig. 5 Near fields of the $H_{25,n}$ modes for $n = 1, 2, 3$ of slightly-disturbed microresonators (*left column*), microdisk resonators with the ring active region having $\sigma = 0.9$ (*central column*), and microring resonators with some values of σ corresponding to stable calculations (*right column*)

panel (b)] because of the appearance of a high-contrast inner boundary between the microcavity and the environment (here, free space).

In Figs. 3 and 4, we show the dependences of the lasing thresholds and frequencies on the radii ratio σ for the same WG modes as in Fig. 2.

The round dots in Figs. 3 and 4 indicate the points satisfying the equation

$$w(\sigma) = n\lambda(\sigma)/(2\alpha_i), \tag{7}$$

where $w = a - b$ and λ/α_i is the wavelength in the microcavity material. As one can see from Fig. 3, the solutions of (7) approximately indicate the values of the relative radius of the ring gain area after which the thresholds begin growing. In Fig. 4, the solutions of (7) approximately indicate the values of the radii ratio σ of the microring resonator above which the calculations were unstable. The approximate points where the thresholds begin growing rapidly are marked by squares in Fig. 4 and satisfy the equation $w(\sigma) = n\lambda(\sigma)/\alpha_i$.

Figure 5 shows the transformation of the near fields of the WG modes $H_{25,n}$ in the partially-active microdisk and the active microring for $n = 1, 2, 3$ if the radii ratio σ varies.

As visible, the presence of small passive area or even a small hole in the center of the active microdisk ($\sigma = 0.1$) has no effect on the modal field. Making the ring-like active region of microdisk very narrow ($\sigma = 0.9$) does not change the field patterns (compare the panels in the left and the central columns in Fig. 5). The emission frequency is also stable and the threshold grows slowly, see Fig. 3. In contrast, for a microring, if the value of σ approaches the values given by (7), the material threshold value for the given mode starts growing rapidly while the emission frequency shifts to the blue (compare the left and the right columns in Fig. 5). If the value of σ becomes even larger, the threshold becomes very high ($\gamma \approx 0.1$) and the root-search algorithm is prone to jumping to the other eigenvalues inhabiting this part of the plane (κ, γ). It should be noted that, because of large optical contrast between the ring and the environment, the modes of the ring resonator are in fact "supermodes" built on the coupled modes of two partial domains, the ring and the hole (see Smotrova et al. 2008, 2011). Here we have studied the WG supermodes, which have predominantly ring-mode features; they can have low thresholds if the ring is not too narrow and hence the E-field overlap with the active ring is good. The other modes, which have predominantly hole-mode features, always have high thresholds because of intrinsically poor overlap of their E-field with active region. The same is true for the so-called "external" modes of the dielectric disk or ring that are not related to the strong internal reflections of the mode field and have large radiation losses in the passive cavity (see Dettmann et al. 2009).

4 Conclusions

We computed the lasing WG modes in a microdisk with a ring-like active region and in a uniformly active microring, placed into the same free-space environment. In this study we varied the inner-to-outer radii ratio and traced the variations in the mode frequencies and material thresholds. The comparison of two configurations showed that a microring had considerable rarefaction effect as only the WG modes with the radial index equal to 1 could keep relatively low material thresholds in a ring narrower than the wavelength in the material. If the ring becomes narrower than half-wavelength, then even these modes obtain catastrophically high thresholds. The growth of the threshold is accompanied with considerable blue shift of the lasing frequency. In contrast, shrinking the active region of a solid microdisk to a narrow ring at disk's rim has no effect on the frequencies and leads to a moderate growth of material thresholds. We believe that this accurate quantification of the advantages offered by the microring configuration can be useful in microcavity laser design.

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