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Highly efficient design of spectrally engineered whispering-gallery-mode microlaser resonators

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Abstract. We present an accurate, reliable and versatile method for studying the effect of deformations on the characteristics of the high-Q whispering gallery (WG) modes supported by two-dimensional dielectric resonators (DR). An eigenvalue problem is formulated in terms of contour integral equations and further discretised with the method of analytical regularisation. Such a procedure significantly reduces the number of unknowns whilst providing high and controllable accuracy. Natural frequencies, Q-factors and field patterns of elliptical, flower- and egg-shaped DRs are calculated. Methods to enhance the Q-factors, provide directional light output, and parasitic mode suppression are discussed. The identification of WG modes is simplified by the symmetrical/asymmetrical mode separation included into the formulation of the problem and solution algorithm.

Key words: analytical regularisation, dielectric resonator, integral equations, microdisk laser, whispering gallery mode

1. Introduction

The ability to support high-Q whispering gallery (WG) modes has promoted a widespread use of circular disk and cylindrical dielectric resonators (DRs) in optical and microwave applications. The mechanisms that can affect the high Q-factors of circular WG-mode DRs are intrinsic material absorption, radiation loss caused by the DR surface roughness and shape deformations, and coupling from the resonator to adjacent waveguides. Two classes of shape deformations of DRs can be considered, distinguished by their origin. The first class is deformations appearing during the fabrication, i.e., surface roughness or an imperfect circular shape. The second class is deliberately formed deformations in order to split, otherwise double-degenerate, WG modes of circular DRs, to suppress parasitic (e.g., nonlasing) modes or to enhance coupling with transmission lines. Furthermore, substantial deviations from spherical or circular cylindrical symmetry can be introduced to obtain directional emission from WG-type lasers.

All WG modes of a circular DR are double degenerate due to the symmetry of the resonator. Such modes have the same radial and azimuthal mode orders and the same resonant frequencies, but have different phases in the azimuthal direction. However, in practice, any imperfection in the shape of the DR or the presence of mechanical supports may remove this degeneracy and cause undesired coupling between such modes. Therefore, special deformations of the symmetry have to be introduced to distinctly separate the resonant frequencies of the degenerate modes. The most common technique used to split degenerate modes is to break the symmetry of a DR, i.e., to elongate one of the axes of the circle to obtain either an elliptic or racetrack cross-section (Nöckel *et al.* 1994; Kogami *et al.* 1996; Boriskina *et al.* 2000). Elliptic and racetrack resonators are widely applied as bandstop and bandpass filters (Van *et al.* 2001), providing better coupling to optical waveguides due to the enlarged interaction region.

However, improving lasing characteristics or rarefication of the dense spectra of circular DRs not only requires separation of the degenerate modes but also the suppression of other, parasitic, modes. This goal has been achieved by making narrow sectorial cuts or inserts (Filipov *et al.* 1995), cutting the edge of a DR (Heide *et al.* 1994), or by corrugating the rim of the DR (Boriskina *et al.* 2000; Fujita and Baba 2001). Ring DRs, where all the volume oscillations and WG modes with more then one radial field variation are suppressed, find applications as filters and oscillators (Hagness *et al.* 1997; Boriskina and Nosich 1999). To couple light out of a WG-mode laser in a preferred direction, gratings and small indentations on the circumference of the DR are used (Levi *et al.* 1993). Another way to emit light in a certain direction and pattern is to increase the radius of curvature of the DR at certain points. This leads to the idea of egg-shaped (Levi *et al.* 1993) or fan-shaped (Sakai and Baba 1999) DRs.

The deformations of DR briefly described above, range from small contour imperfections, the effects of which can be studied by perturbation theory (Leung *et al.* 1994), through to substantial shape modifications that require more accurate treatment. Ray dynamics in highly asymmetric resonant cavities have been studied by Kolmogorov–Arnold–Moser theory in (Nöckel *et al.* 1994). However, ray-optics models seem to be more applicable to high-order modes of electrically large DRs, whereas in practice it is the smaller DRs that are of more interest as these offer lower threshold currents, stable single-mode operation, and compact optical designs. Versatile numerical techniques such as FDTD, have been applied to single-mode ring (Hagness *et al.* 1997), and rim-corrugated (Fujita and Baba 2001) DRs. Unfortunately, these require large computational and memory resources, and staircasing errors, introduced by mapping the geometry onto a discrete grid, can be significant for structures with many smooth corrugations. Another idea exploited in some previous work is to expand the field of a deformed DR in terms of the modes of unperturbed symmetric resonator (Filipov *et al.* 1995; Kogami *et al.* 1996). However, if large deformations are considered, the modes of a deformed DR can substantially differ from those of the circular one and such approaches can become inefficient.

Hence, to study the modal spectra of a wide class of optical cavities with arbitrarily smooth deformations from circular symmetry we apply an accurate and efficient technique developed in (Nosich 1999; Nosich and Boriskina 2002). The technique is based on the formulation of an eigenvalue problem in terms of singular contour integral equations (IEs), which are further cast into the Fredholm second kind matrix form by means of the method of analytical regularisation (MAR). At the core of the method lies the decomposition of the original integral operator into the main part, which has an explicit Fourier representation, and a remaining integral operator with a smooth kernel. Then, by applying the method of moments procedure with angular exponents as basis and trial functions, we obtain a discrete regularised homogeneous matrix equation. Such a discretisation scheme can have arbitrarily high convergence rate on the analytical contours (Saranen and Vainikko 1996). Although in the case under consideration here, the original integral operators do not have such main parts, the decomposition can actually be achieved by adding and subtracting an operator with the desired properties. The natural choice for a convolution part of the integral operator is the same operator defined on the circular contour as the scattering problem for a circular DR which has a well-known analytical solution in terms of series of cylindrical functions.

In (Nosich and Boriskina 2002) the MAR has been applied to a problem of a plane wave scattering from elliptical and super-elliptical dielectric cylinders, and the merits of the algorithm have been demonstrated. In this paper, we apply such a technique to searching for the natural frequencies of deformed optical cavities. Unlike a scattering problem, here the final matrix equation is a homogeneous one with solutions only existing for certain allowed values of the frequency parameter. These values are the natural frequencies of WG modes being sought. However, the problem must now be formulated in the complex domain, since all the modes of open DR are characterised by complex-valued natural frequencies. This reflects the fact that such modes lose energy due to evanescent leakage out of the DR. A further novel feature of the present approach is in accounting for degenerate mode splitting in deformed DRs by solving the determinantal equations for symmetrical and antisymmetrical modes separately. The complex natural frequencies of several interesting DR designs for practical applications are considered and the physical mechanisms of mode splitting, nonlasing modes damping, and directional light coupling out of DRs are discussed.

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2. Outline of the computational technique

2.1. EIGENVALUE PROBLEM FORMULATION

The simplest and the most common WG-mode DR is a circular cylinder or disk made of low-loss dielectric where the light is confined due to the total internal reflection mechanism. Such resonators are characterised by high Q-factors and find numerous applications in optoelectronic and microwave technology, spectroscopy, and metrology. Due to the spatial symmetry of the circular resonator, the wave equation describing its modes can be solved analytically with the separation of variables. Circular WG mode DRs support a series of $WGH^{\pm}_{mn}$ or $WGE^{\pm}_{mn}$ resonances where the principal field component lies in the plane of the DR cross-section. The subscripts n and m denote the number of azimuthal and radial variations of the mode field, respectively. All the WG modes are double degenerate due to the DR symmetry, which corresponds to a $\cos(n\varphi)$ or $\sin(n\varphi)$ angular field dependence. These two different relative phases of the same mode are denoted by the superscript $\pm$. However, the modes of deformed DRs have to be found by means of numerical techniques.

A dielectric cylindrical resonator with an arbitrarily smooth deformation from the circular cross-section is depicted in Fig. 1. The contour of the resonator is described by a smooth 2-D closed curve L, which can be presented in the parametrical form as follows:

$$L : x = a\mu r(t) \cos t, \qquad y = ar(t) \sin t, \quad 0 \leq t \leq 2\pi. \tag{1}$$

Here, a is the characteristic size of the resonator and μ is an elongation parameter along the x-axis. To make the following analysis complete and

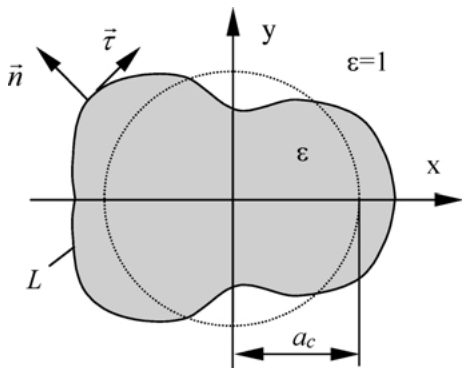

Fig. 1. Geometry of the deformed DR and its analytically solvable circular counterpart as well as a definition of the coordinate system used.

general, material absorption loss is taken into account ($\varepsilon = \varepsilon' + \varepsilon''$) as well as leakage of the field into the outer space.

The total field can be represented as a single scalar function U describing the z-component of the electric or magnetic field, depending on the polarisation and which should satisfy the Helmholtz equation in every region, together with continuity conditions across the resonator contour and the conditions at infinity. Expressing the fields inside and outside the DR in terms of single-layer surface potentials over the DR contour (Colton and Kress 1983) and following the technique of (Nosich and Boriskina 2002), the eigenvalue problem can be formulated in terms of two coupled IE for the unknown surface potential densities

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$$\int_0^{2\pi} \varphi(t_s) G_\varepsilon(t, t_s) L(t_s)\, dt_s - \int_0^{2\pi} \psi(t_s) G(t, t_s) L(t_s)\, dt_s = 0, \tag{2}$$

$$\frac{\varphi(t)}{2\alpha^{E(H)}} + \frac{\psi(t)}{2} + \frac{1}{\alpha^{E(H)}} \int_0^{2\pi} \varphi(t_s) \frac{\partial}{\partial n} G_\varepsilon(t, t_s) L(t_s)\, dt_s - \int_0^{2\pi} \psi(t_s) \frac{\partial}{\partial n} G(t, t_s) L(t_s)\, dt_s = 0. \tag{3}$$

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Here, $\varphi(t)$ and $\psi(t)$ are the unknown potential density functions; G_ε is Green's function of the homogeneous medium with permittivity ε, G the free-space Green's function; $\alpha^E = 1, \alpha^H = \varepsilon$; $L(t)$ is a Jacobian of the parametric curve describing the DR contour. The 2-D Green's functions in kernels of the IEs have logarithmic singularities at $t \to t_s$, whereas their normal derivatives have finite limit values at $t \to t_s$ on contours with a continuous curvature. The presence of these singularities makes a direct discretisation of IEs (2), (3) ineffective. To avoid the singularities, we exploit the fact that Green's functions and their derivatives for the case of a circular DR of radius b have the same set of orthogonal eigenfunctions: $\{e^{imt}\}_{m=-\infty}^{\infty}$. Therefore, adding and subtracting these circular-shape kernels to the actual kernels of the arbitrary-shape IEs, and using the exponents as an infinite global basis in Galerkin's scheme results in a regularised Fredholm second kind block-matrix equation for the Fourier coefficients

$$\alpha^{E(H)} \varphi_m H_m^\varepsilon J_m^\varepsilon - \psi_m H_m J_m + \sum_{n=-\infty}^{\infty} (A_{mn}\varphi_n - B_{mn}\psi_n) = 0, \tag{4}$$

$$kb\sqrt{\varepsilon}\varphi_m H_m^\varepsilon J_m^{\varepsilon'} - kb\psi_m H_m' J_m + \sum_{n=-\infty}^{\infty} (C_{mn}\varphi_n - D_{mn}\psi_n) = 0, \quad m = 0, \pm 1, \pm 2, \ldots, \tag{5}$$

where $J_n = J_n(kb)$, $J_n^\varepsilon = J_n(kb\sqrt{\varepsilon})$, $H_n = H_n^{(1)}(kb)$, $H_n^\varepsilon = H_n^{(1)}(kb\sqrt{\varepsilon})$ are the Bessel and Hankel functions, respectively; φ_m and ψ_m are the Fourier coefficients of the potential density functions; and A_{mn}–D_{mn} are the Fourier coefficients of the integral operators A–C defined as differences between the values of the operators on the original contour L and on the circle of radius b, respectively.

It can be proven (Saranen and Vainikko 1996; Nosich and Boriskina 2002) that all the discretised operators are compact on smooth contours and that the algorithm converges exponentially with respect to truncation number N of the matrix. If, however, the DR contour L has sharp corners, then matrix operators C and D loose compactness. Therefore, the application of the method is limited to smooth contours. This restriction does not mean that smooth bends or points of high however finite curvature are prohibited. Another limitation of the method is that a DR contour L should be described by a single-valued parametrical function, i.e., it is applicable to star-like shapes only. Thus, the method can still be successfully used to study natural frequencies of polygonal DRs with rounded convex or concave corners with high and controlled accuracy.

2.2. CALCULATION OF RESONANCE FREQUENCIES

Almost every practical resonator has symmetry of some kind. All DRs considered in this paper are symmetrical about the x-axis. The modes of such resonators are either symmetrical or asymmetrical about this axis. Thus, modal fields can be expanded in terms of either sines or cosines. Such a consideration allows us to split the problem into two uncoupled matrix equations, thus reducing the computational complexity of the algorithm and simplifying the mode identification. After the discretisation, the sine and cosine expansions of the density functions yield two systems of linear homogeneous algebraic equations

$$\sum_{n=0}^{\infty} (\delta_{mn} + C_{mn}) x_n^c = 0, \qquad \sum_{n=1}^{\infty} (\delta_{mn} + S_{mn}) x_n^s = 0, \tag{6}$$

where C_{mn} and S_{mn} are block moment matrices and $x_n^{c(s)}$ are column vectors containing the nth Fourier coefficient of the potential densities. The matrix equations (6) have nontrivial solutions only when the determinants of the matrices are zero

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$$\det(\delta_{mn} + C_{mn}) = 0, \qquad \det(\delta_{mn} + S_{mn}) = 0. \tag{7}$$

Equations (7) are solvable only at discrete complex values of the dimensionless parameter $ka = f + \mathrm{i}w$, where k is the free-space wavenumber. In the further discussion, we shall call the real part of the complex natural frequency, f, a resonant frequency, and the imaginary part, w, a resonance width. We assume the time dependence as $\exp(-i\omega t)$ and thus w can have only negative values. The search of the roots of Equations (7) has been performed in the complex plane of the parameter ka by means of the Powell hybrid method (Press *et al.* 1986), thus providing information for resonant frequencies and quality factors of the DR modes:

$$Q = -f/2w. \tag{8}$$

After the complex natural frequency is found, the near-field pattern can be calculated within a multiplicative constant. Although in practice the matrices used in Equation (7) are truncated, the analytical regularisation procedure reduces the impact of this and the method shows guaranteed and fast convergence with increasing truncation number.

2.3. CONVERGENCE AND ACCURACY OF THE ALGORITHM

Table 1 gives information on the numerical algorithm stability, accuracy, and efficiency of the method just described. It can be seen that the size of the DR and especially the smoothness of its contour affect the accuracy of the computations and, hence, the CPU time and memory requirements. The method works better for smooth contours like ellipses, and requires greater computational effort for the curves with higher curvature variations. Furthermore, the larger the size of the resonator, the greater the number of unknowns required to achieve the same level of accuracy. High accuracy is required if one is interested in both the resonant frequency and the quality factor. If, however, only resonant frequencies are of interest, the CPU time can be significantly smaller.

Table 1. Computational information

Shape	μ	δ	N	$\mathrm{Err_I}$	$\mathrm{Err_R}$	Mem	CPU
Ellipse	1.1		11	4.27e-5	2.92e-9	2.0	5
$\mathrm{WGH}_{6,1}$	1.5		13	9.99e-5	5.07e-8	2.05	5
Ellipse	1.1		17	9.85e-5	5.99e-9	2.1	6
$\mathrm{WGH}_{11,1}$	1.5		21	1.61e-5	4.64e-9	2.15	6
Egg	1.1		48	9.26e-5	4.49e-8	2.55	15
$\mathrm{WGH}_{11,1}$	1.5		55	3.74e-5	1.88e-9	2.6	15
Flower	1.0	0.05	53	2.25e-4	2.29e-6	4.4	54
$\mathrm{WGH}_{6,1}$	1.0	0.2	78	9.51e-4	4.57e-6	4.45	56

μ – DR elongation along x-axis; δ – corrugation depth; N – total number of unknowns; Err – relative error; $\mathrm{Err_R} = |f(N) - f(N-1)|/f(N)$, $\mathrm{Err_I} = -|w(N) - w(N-1)|/w(N)$; CPU – CPU time per iteration, s; Mem – memory requirements, Mb.

All computations have been done with double precision on a 1.0 GHz PC with 128 Mb of RAM. For a more detailed analysis on the uniqueness and convergence properties of the algorithm the reader is referred to (Nosich and Boriskina 2002). However, due to the separation of modes with different symmetry, here we have to solve matrices that are half the size of those in (Nosich and Boriskina 2002). Moreover, it should be noted that due to super-convergence of the trigonometric projection methods (Chatelin 1981), the rate of convergence of eigenvalues is twice that for the scattering or eigenvector problem solution.

3. Deformed optical cavities design

3.1. SPLITTING OF DEGENERATE MODES

In the following sections, we demonstrate the applicability of the algorithm developed to study the characteristics of several practical resonator designs. First, we consider an elliptical resonator ($r(t) = 1$) as the simplest example of a deformed DR. Fig. 2 shows the dependence of the resonant frequencies and Q-factors of two split $\mathrm{WGH}^{\pm}_{6,1}$ (a) and $\mathrm{WGH}^{\pm}_{11,1}$ (b) modes on the DR elongation. One can see that the increase of the ellipse aspect ratio shifts the resonance frequencies and spoils the Q-factors of the WG modes.

Electric field distributions for WG modes of two different orders and symmetries are plotted in Fig. 3. For both resonances the main part of the leakage occurs in the regions around the points $\varphi = 0$ and $\varphi = \pi$, which has been previously observed by (Nöckel *et al.* 1994; Kogami *et al.* 1996). However, unlike for very-high-azimuthal order WG modes in large optical DRs studied in (Nöckel *et al.* 1994), a distinct difference in the directional emission patterns can be observed for modes of different symmetry. Comparing the intensity patterns in Fig. 3(a) and (b) and Fig. 3(c) and (d), it can be noted that deformations of the same magnitude affect the lower-azimuthal-order modes more than the higher-order ones. Thus, low-azimuthal-order WG modes can only survive in those DRs that are only slightly deformed from a circle. If the deformation is increased further, the Q-factors of the WG modes decrease dramatically (Fig. 2a) due to increased leakage out of DR. The higher-azimuthal-order modes demonstrate better confinement and therefore can still survive even in DRs with severe deformations from circular symmetry (Fig. 2b).

Furthermore, it can be clearly seen that efficient splitting of high-order WG resonances does not occur for small or moderate DR deformations. However, for the more significant shape deformations, the Q-factors of both symmetrical and antisymmetrical modes are damped to a very low level. This reduces efficient practical applications of a resonator operating on such

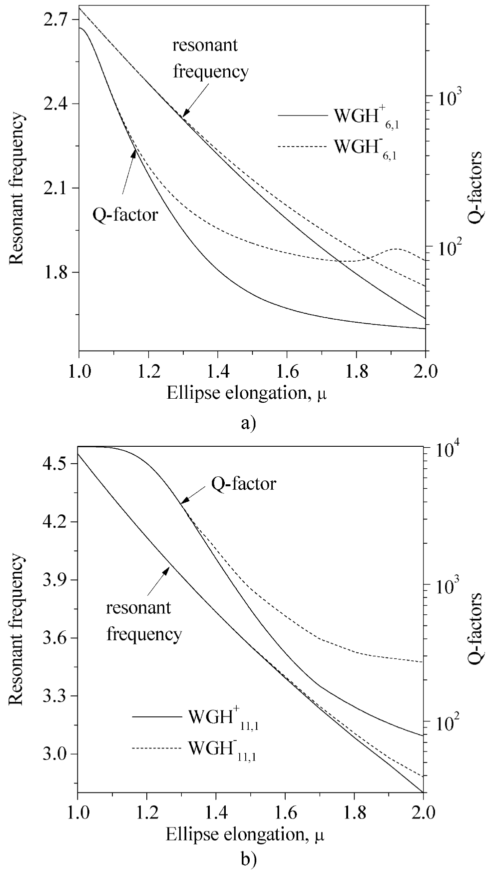

Fig. 2. Resonant frequencies and Q-factors of $\mathrm{WGH}^{\pm}_{6,1}$ (a) and $\mathrm{WGH}^{\pm}_{11,1}$ (b) modes of the elliptical DR versus the elongation parameter μ ($\varepsilon = 10 + \mathrm{i}10^{-3}$).

modes. The next section offers an analysis of another promising design of a deformed DR, which provides a more efficient way to separate the WG modes.

3.2. SUPPRESSION OF PARASITIC MODES

The spectrum of WG modes in circular DRs is very dense. The existence of a large number of high-order modes affects the efficiency of filtering and lasing operations. As was shown in the previous example, symmetrical deformations (elliptical, quadrupolar, etc.), affect all the modes of DR. Therefore, they cannot be effectively used to widen a parasitic-mode-free range of WG-mode DRs. To suppress parasitic modes without disturbing the preferred operational ones, it is desirable to introduce a deformation that is tailored to

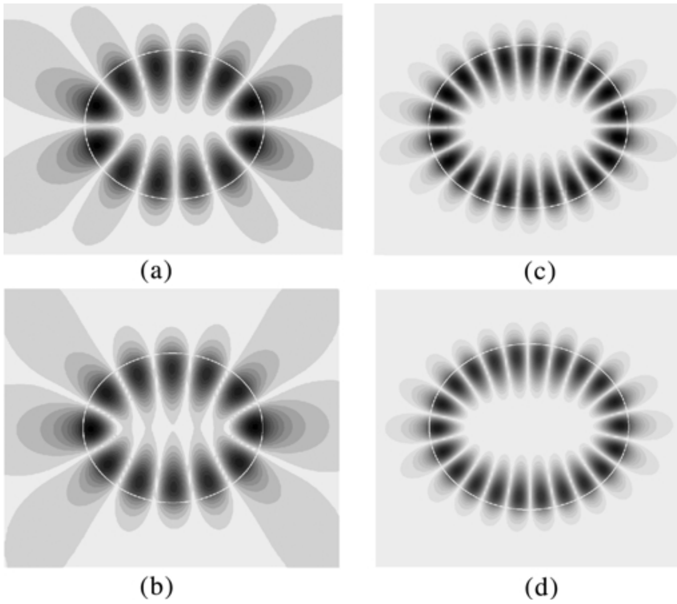

Fig. 3. Near-field intensity patterns of $WGH^{\pm}_{6,1}$ (a,b) and $WGH^{\pm}_{11,1}$ (c,d) modes in the elliptical DR with parameters: $\mu = 1.2$, $\varepsilon = 10 + i10^{-3}$.

account for the different field distributions of the modes. One of the popular approaches to achieve such a goal is to make a narrow sectorial cut in the DR (Filipov *et al.* 1999). Another design that has recently been suggested is a corrugated disk resonator with a grating of the same period as that of the azimuthal field variation of the lasing mode (Boriskina *et al.* 2000; Fujita and Baba 2001). Such a design provides splitting of a degenerate resonance and suppression of only one of two modes.

In contrast to the rectangular grating proposed in (Fujita and Baba 2001), here we study a smooth cosinusoidal corrugation of the resonator rim (flower-shape DR). A smooth contour corrugation fits more closely the WG mode field pattern and seems to be more attractive for fabrication purposes. The contour of the flower-shape resonator can be described by the following parametric function: $r(t) = (1 + \delta \cos \nu t)$, where ν is the number of the corrugations along the contour of DR and δ is the corrugation depth.

Fig. 4 shows the dependence of the resonant frequencies and Q-factors of the $WGH^{\pm}_{6,1}$ modes on the corrugation depth, δ. The corrugation period is chosen so that the number of maxima in the intensity pattern of the lasing mode is equal to the number of flower petals. The antisymmetrical $WGH^{-}_{6,1}$ mode is considered a parasitic one, and therefore is to be suppressed. Fig. 5 demonstrates the field profiles of the lasing, $WGH^{+}_{6,1}$, and parasitic, $WGH^{-}_{6,1}$, modes. It can be seen that the lasing mode has maxima of the field in the convex regions of the DR contour and practically is not disturbed by such a

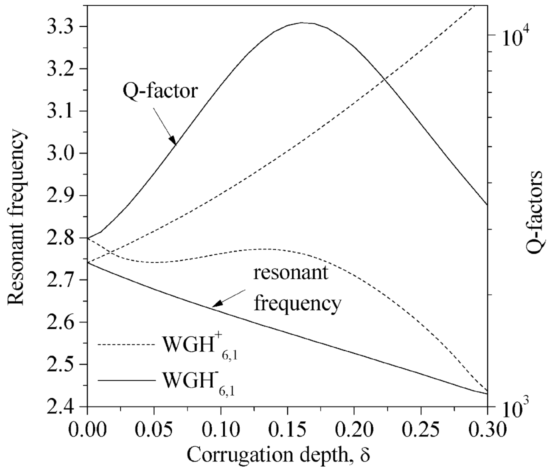

Fig. 4. Resonant frequencies and Q-factors of $WGH^{\pm}_{6,1}$ modes of the flower-shaped DR versus the corrugation depth δ. ($\varepsilon = 10 + i10^{-3}$, $\nu = 12$).

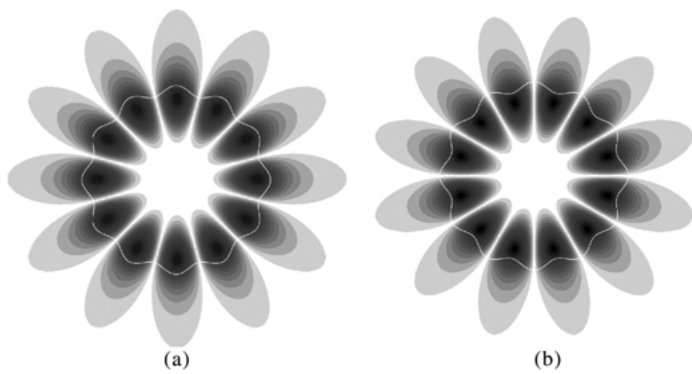

Fig. 5. Near-field intensity patterns of $WGH^{\pm}_{6,1}$ modes in the flower-shape DR with parameters: $\nu = 12$, $\delta = 0.05$, $\varepsilon = 10 + i10^{-3}$.

deformation. The effective DR radius for such a mode is increased and, therefore, its resonant frequency drops (Fig. 4). At the same time, the resonant frequency of the parasitic mode with different symmetry rises due to the decreased effective radius. Furthermore, the radiation loss is greater for the parasitic mode with maxima in the concave regions, which causes a damping of the mode quality factor. The most interesting fact is that the Q-factor of the lasing mode is not only unspoiled but even increased in the 0–30% range of the contour deformation. Thus, an efficient mode separation together with suppression of the parasitic mode and enhancement of the operational one can be achieved by corrugating the DR contour in a specific pattern according to the azimuthal order of the resonance.

However, the spectrum of DR also includes modes of higher radial orders. The natural frequencies of these modes are also affected by the DR contour corrugation and can be shifted into the close vicinity of the lasing mode. This can cause a coupling between the first and higher-radial-order modes (Levi *et al.* 1993). Moreover, the Q-factors of higher-radial-order modes are spoiled less by the contour deformations than Q-factors of the first-order ones because they are not closely confined to the perimeter of the resonator (Hagness *et al.* 1997). An efficient approach to suppress such modes and to rarefy the spectrum of the DR, is to make a hole in the central part of resonator. First-radial-order WG modes are characterised by a strong energy confinement within a small region between the outer rim and inner caustic and therefore are not affected (Hagness *et al.* 1997; Boriskina *et al.* 1999). Thus, we believe that a ring resonator with a corrugated outer rim will provide effective suppression of all the parasitic nonlasing modes.

3.3. DIRECTIONAL EMISSION

Another problem arising in the practical design of microdisk lasers is the extraction of the light with a strong spatial directionality. For large semiconductor WG-mode lasers directional light output can be achieved by means of Y-couplers or cleaved facets. Unfortunately, fabrication of such output couplers for microdisk lasers presents a fabrication challenge (Levi *et al.* 1993). An egg-shape DR can be used to provide a directional coupling of WG mode out of the microdisk laser. The egg-shape is obtained by elongating one of the axes of a circle in one direction only: $r(t) = 1$, $-\pi/2 \leq t \leq \pi/2$; $r(t) = 1/\mu$, $\pi/2 \leq t \leq 3\pi/2$. The orientation and intensity of radiation can be controlled by changing the DR elongation parameter.

Fig. 6 presents the resonant frequencies and Q-factors of the $WGH^{\pm}_{11,1}$ modes of the egg-shape DR as a function of the elongation parameter μ. It can be noted that the splitting of the WG mode occurs for smaller deformations than in the case of the elliptical DR. Moreover, the Q-factors of the modes decrease at a higher rate, which suggests more intensive leakage of the modal energy out of the egg-shape DR. Fig. 7 shows directional emission patterns of symmetrical and antisymmetrical $WGH^{\pm}_{11,1}$ modes of the egg-shaped resonator. From the ray-optics point of view, light confined in the resonator by a total internal reflection mechanism is more likely to escape at the regions of higher curvature of the DR contour. For the egg-shape, the region of the highest curvature is located near the point $\varphi = 0$. However, one can see that a directional beam in the positive direction of x-axis forms only for a symmetrical WG-mode. Apart from this beam, the emission patterns of both modes consist of several directional beams going in approximately the

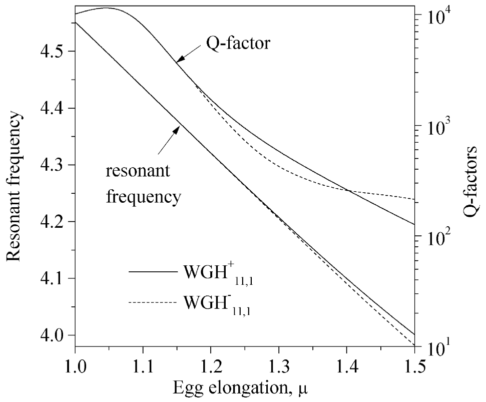

Fig. 6. Resonant frequencies and Q-factors of $WGH^{\pm}_{11,1}$ modes of the egg-shaped DR versus the elongation parameter μ. ($\varepsilon = 10 + i10^{-3}$).

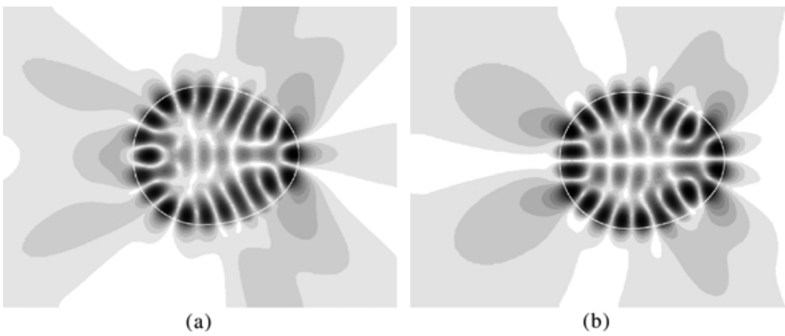

Fig. 7. Near-field intensity patterns of $WGH^{\pm}_{11,1}$ modes in the egg-shaped DR with parameters: $\mu = 1.4$, $\varepsilon = 10 + i10^{-3}$.

same directions. This important property of such a resonator can be useful for the designs of multimode lasers.

4. Conclusions

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An efficient method of calculating natural oscillations in DR deformed from circular symmetry has been presented. The approach is based on the contour IE formulation and the procedure of analytical regularisation applied to IEs. The final, fully discrete, scheme is proven to be stable and to have a very high convergence rate. Thus, the number of the unknowns required to reach a desired accuracy is significantly smaller than that of the conventional MoM, finite-difference or finite-element methods. The computational complexity of the algorithm was further reduced by solving the eigenvalue problems separately for the WG modes symmetrical and antisymmetrical with respect to the y-axis of the resonator. Such a procedure also simplifies the identification of WG modes. Our results show that the approach presented is very fast and economic in terms of computer resources. It enabled us to achieve very accurate solutions to a variety of eigenvalue problems on a desktop PC in reasonable amounts of time.

We have demonstrated on a number of practical examples that our approach can be successfully applied to study the dielectric microcavities and to the design of novel structures with improved characteristics. The shift of resonant frequencies and degradation of Q-factors of WG modes due to various deformations of DR contour was observed and we studied the characteristics of several spectrally engineered DR designs that enable one to improve the performance of microlaser resonator or to obtain a directional emission pattern.

The method is directly applicable to calculating threshold currents of WG modes, including field sources in the analysis (Nosich and Boriskina 2002), and can be generalised to consider a multilayered environment similar to (Boriskina and Nosich 1999). Furthermore, the method allows studying of Q-spoiling and shifting of WG-modes due to the DR sidewall roughness caused by the finite fabrication precision.

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(*) The field representation in terms of only the single-layer potentials that lead to IE (2), (3) is not the most general one. The same is true for the representation in terms of only the double-layer potential. Either of these representations leads to the appearance of the spurious (real-valued) eigenvalues of resulting IE that spoil the algorithm because the IE condition number has poles at the spurious-eigenvalue frequencies. The lowest of them, as can be easily found, lies near to the value where the largest "diameter" of the scatterer equals to one-half of the free-space wavelength. The severity of assocoated numerical error depends, however, on the details of the IE discretization scheme used.

In the wave-scattering problems, if it is a MAR-based scheme as in the current paper then the error is inacceptably large only in the domain whose width is of the same order as MAR's error and can be squeezed to machine-precision width by taking the matrix truncation order larger. However if the IE is discretized using a rougher scheme like a BEM or Galerkin MoM with local basis functions, then the domains of huge errors are much wider and overlap one another at the frequencies slightly larger than the first spurious-eigenvalue frequency. This makes any computations with such an algorithm completely senseless.

In the eigenvalue problems, the existence of real-valued spurious eigen-frequencies spoils the search for the complex-valued eigen-frequencies that may lay in the vicinity of a spurious frequency, i.e. those that have small imaginary parts or high Q-factors. For the frequencies that have considerable imaginary parts, the presented here algorithm works out quite well. Unfortunately, the modes with high Q-factors, like WGM modes, are the most interesting and important for applications in lasing and sensing.

The full remedy is the use of the Muller IE which is completely equivalent to the original boundary-value problem and thus free of spurious eigenvalues.