Scattering of Light by a Discrete Cross Made of Silver Nanowires

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Abstract. We consider the two-dimensional (2-D) problem of the H-polarized plane wave scattering by a discrete cross made of periodically arranged circular cylindrical wires, using the field expansions in local coordinates and addition theorems for cylindrical functions. Resulting block-type matrix equation is cast to the Fredholm second-kind form that guarantees convergence of numerical solution. The scattering and absorption cross-sections and near-field and far-field patterns are found and the interplay of plasmon and grating-type resonances is studied for the crosses made of nano-size silver wires in the visible-light range of wavelengths.

Keywords: cross-shaped grating, silver nanowires, partial separation of variables, plasmon resonance, grating resonance
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INTRODUCTION

Periodically structured scatterers, or finite-periodic gratings, arrays or chains of particles and holes in metallic screens (in 3-D) or wires and slots (in 2-D), are attracting large attention of researchers in today's nanophotonics [1-9]. This is caused by the effects of extraordinarily large reflection, transmission, emission, and near-field enhancement that have been found in the scattering of light by periodic scatterers. Recently it has been discovered that these phenomena are explained by the existence of so-called grating resonances [4] (a.k.a. geometrical, lattice and Bragg resonances). Their wavelengths are just above the Rayleigh wavelengths [10] (period being a multiple of the wavelength) if all elementary scatterers of a grating are excited in the same phase and their size is a fraction of the period. In the wave scattering by infinite gratings, they lead to almost total reflection of the incident field by a thin-dielectric-wire grating in a narrow wavelength band [3].

Another type of resonances is observed for sub-wavelength noble-metal particles and wires in the mid-infrared and optical bands [11,12]. Excitation of plasmons results in powerful enhancement of scattered and absorbed light that is used in the design of optical antennas and biosensors for advanced applications. Plasmon resonances have unique physical property: in the leading terms, their wavelengths depend on the object shape but not on its dimensions. In [13], we have studied the effect of grating resonances on the plasmon-assisted scattering by finite linear gratings of sub-wavelength silver wires. The goal of this paper is extension of this study to a 2-D periodically structured silver wire configuration in the shape of cross.

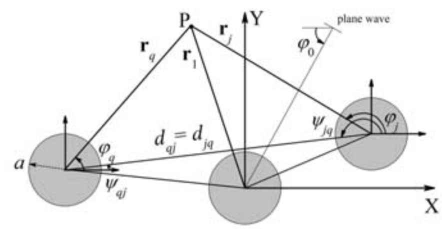

FIGURE 1. Generic problem geometry and corresponding notations: a plane wave is incident on a finite number of identical parallel circular cylinders.

THE SCATTERING PROBLEM

We consider the scattering of the plane wave by collections of wires as shown in Fig. 1 that consist of finite number of M identical circular wires. The wires have the same radius a and relative dielectric permittivity ε (although they can be different).

The global coordinates have the origin at the centre of the first cylinder. For a 2-D problem, a scalar function U, which represents either E_z or H_z scattered-field component, must satisfy the Helmholtz equation with wavenumber $k\sqrt{\varepsilon}$ or k inside and outside of each cylinder, the tangential field continuity conditions, the radiation condition at infinity, and the condition of the local power finiteness.

We proceed to the problem solution by expanding the field function in terms of the azimuth exponents in the local coordinates (Fig. 1), using addition theorems for cylindrical functions, and applying the boundary conditions on all M cylinders.

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The unknown field-expansion coefficients related to the q-th cylinder include the effect of all interactions between the cylinders. They satisfy a $M \times M$ block-type matrix equation, $(\mathbf{I}+\mathbf{U})\mathbf{X}=\mathbf{U}^0$, where each block is infinite, $\mathbf{U}=\{\delta_{ij}U^{(i,j)}\}_{i,j=1,\ldots M}$, $\mathbf{X}=(x_m^{(1)}\ldots x_m^{(M)})$, $\mathbf{U}^0=(U_m^{0(1)}\ldots U_m^{0(M)})$, $i,j=1,\ldots M$, and $m=0,\pm1,\ldots$.

This method of solution is not new and can be traced to the papers of V. Twersky is in the 1950s [14-19]. However we have found that the matrix equations used in the previous papers had not guaranteed convergence of numerical solutions. Here, we understand the convergence in mathematical sense, as a possibility of minimizing the error of computations by solving progressively larger matrices. (Frequently, convergence is mixed up with accuracy. Many divergent numerical schemes are able to provide a few first digits correctly however fail when tasked with better accuracy.) Fortunately, this defect of earlier papers can be fixed by re-scaling the unknown coefficients. The obtained in such a way matrix equation is a block-type Fredholm second kind equation, i.e. $\|\mathbf{U}^{(i,j)}\|_{l^2}<\infty$ for all $i,j=1,\ldots M$. Therefore the solution of corresponding counterpart equation with each block truncated to finite order N is destined to converge to exact solution when the number N gets greater.

NUMERICAL RESULTS

To characterize the optical properties of considered periodically structured scatterers, we have computed the total scattering (TSCS) and absorption (ACS) cross-sections as a function of the incident plane wave length, in the visible range. The dielectric function of silver was taken from [20].

As we have been interested in the configurations where the grating-type resonances are well developed (their Q-factors grow up with larger M), we have computed the cross-shape scatterers with M up to 101 in four arms of cross, i.e. with one central wire and 20-wire arms. We have studied the sparse-periodic configurations with $a < p-2a$, where p is the period. In this case, the order of truncation of each block of the derived matrix equation was taken as N = 4 to guarantee 4 correct digits in the numerical solution. In the case of dense-periodic configurations this order must be larger, for the same accuracy, and adapted to the separation between the wires. The wires cannot touch each other as this spoils the Fredholm property of the matrix equation. Dense configurations are also interesting objects however they deserve a separate study.

In Figs. 2 and 3, we present the spectra of TSCS and ACS for the same cross-shape scatterers, in the visible range, under two variants of the plane wave incidence, along a cross arm and along a diagonal, respectively.

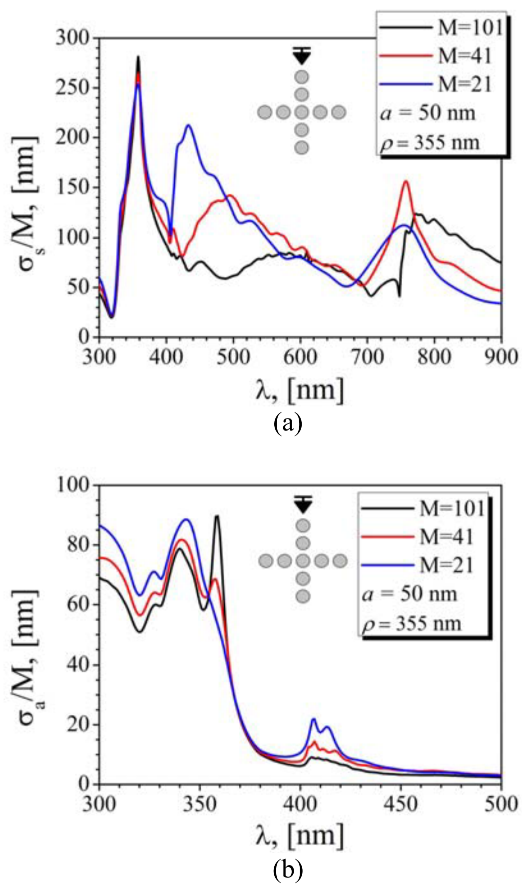
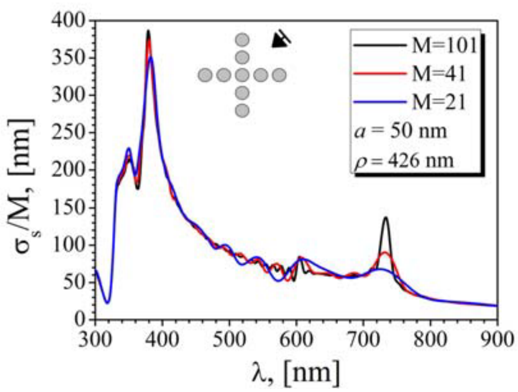


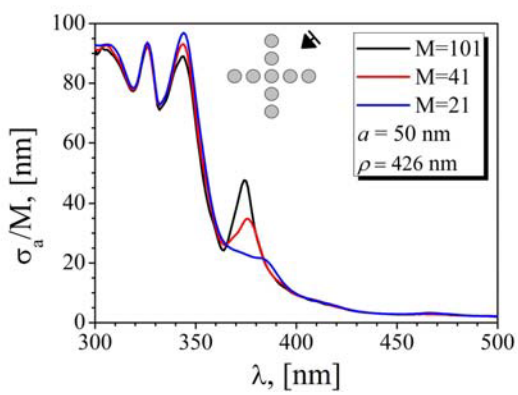
FIGURE 2. TSCS (a) and ACS (b) as a function of wavelength for different discrete crosses from wires of the radius 50 nm, period 355 nm. Normal incidence.

The plots in Fig. 2 correspond to the H-polarized (i.e. with electric field being orthogonal to the wires) plane-wave incidence along one of the cross arms. This means that the other two arms are illuminated in broadside manner. It is interesting to see that contrary to the common view, ACS is a better tool for visualizing the resonances than TSCS. The plasmon resonance of a single silver wire is known to sit at the wavelength of approximately 340 nm, slightly red-shifted from the value that provides $\mathrm{Re}\,\varepsilon(\lambda)=-1$. If the wires are assembled into a sparse linear grating, the main (dipole) plasmon resonance is still present at the same wavelength. From Fig. 2 (a), one can see that increasing the number of wires in the cross arms inhibits the plasmon resonance.

In contrast, under sin-phase excitation the principal grating-type resonances have the wavelengths slightly red-shifted from the period value, which has been taken, in computations, as $p = 355$ nm. Because of sizable losses in silver, these resonances manifest themselves as one combined absorption resonance just to the right from plasmon resonance. Unlike plasmon, this resonance is getting sharper and more intensive if the number of wires increases. The same is valid for the most intensive peak in TSCS (Fig. 2 (b)) that happens to be tuned to plasmon resonance in the scattering. Note another grating-type resonance developing around 750 nm on the TSCS plots. This is apparently the resonance of the grating having twice-larger period of 710 nm. It is not observed on the ACS plots because the silver becomes less lossy in the red part of the visible range.



(a)



(b)

FIGURE 3. The same in Figure 2 for discrete crosses with period 426 nm and angle of incidence $\varphi_0 = \pi/4$.

If the plane wave illuminates the same cross of silver wires along its diagonal, the mentioned above features of the scattering and absorption are kept. The difference is that the plasmon and the grating-type resonances are slightly detuned from each other both in the scattering and the absorption. Once again, they are clearly identified by their different dynamics with respect to the growth of the number of wires.

The data presented in this paper are preliminary. However they demonstrate that a cross of silver wires, if considered as a sensor assisted with combined plasmon-grating resonance, is less dependent on the direction of arrival of the incident illumination than a conventional linear chain of nanowires.

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