RAYLEIGH ANOMALIES IN THE E-POLARIZED WAVE SCATTERING BY FINITE FLAT GRATINGS OF SILVER NANOSTRIPS OR NANOWIRES

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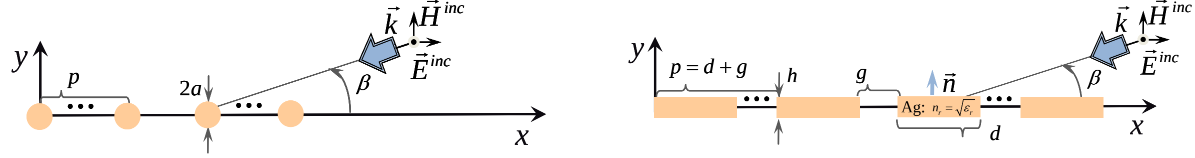
***Abstract* – We study numerically the optical properties of the finite periodic flat gratings made of multiple silver nanowires or silver nanostrips suspended in free space, in the context of periodicity-induced resonance effects. Our analysis is based on the use of two techniques: the angular field expansions in local coordinates and addition theorems for cylindrical functions for the nanowire gratings, and the generalized (effective) boundary conditions (GBCs) imposed on the strip median lines and the Nystrom-type discretization of the relevant singular and hyper-singular integral equations (IEs) for the strip gratings. In the E-polarization case, in contrast to the H-case, there are no localized surface plasmons and sharp periodicity-induced grating resonances are not visible in the scattering and absorption cross-sections. Instead, the suppression of the scattering near to the emerging Rayleigh anomalies is seen if the strip number gets larger.**

I. PROBLEMS FORMULATION AND BASIC EQUATION

The two-dimensional scattering and absorption of the E-polarized plane wave by two types of finite linear gratings of silver nanowires or nanostrips are considered. The corresponding freestanding geometries and the problem notations are shown in Fig. 1. The wires and strips are assumed to be identical infinite circular and rectangular cylinders, each having radius *a* and width *d* and thickness *h*, respectively, with complex relative dielectric function $\varepsilon_r(\lambda)$, λ being the wavelength. As known, for a 2-D problem one has to find a scalar function $E_z(x, y)$ that is the scattered field z-component.

In the scattering by finite nanowire gratings it must satisfy the Helmholtz equation with corresponding wavenumbers inside and outside the cylinders, the tangential field components continuity conditions, the radiation condition, and the condition of the local power finiteness. The full-wave solution can be obtained similarly to [1], [2], by expanding the field function in terms of the azimuth exponents in the local coordinates, using addition theorems for cylindrical functions, and applying the boundary conditions on the surface of all *N* wires. This leads to an infinite $N \times N$ block-type matrix equation where each block is infinite. However we emphasize that, unlike [1], [2], we cast it to the Fredholm second kind matrix equation. It is only in this case that the solution of equation with each block truncated to finite order converges to exact solution if $N \to \infty$. (for more details see [3,4]) The results presented below were computed with *N* = 4; this provides 3 correct digits in the far-field characteristics of the gratings of silver wires with radii $a \le 75\,\text{nm}$ and periods $p \ge 200$ nm .



(a) (b)

Fig. 1. Geometry of a freestanding finite gratings having period *p* and made of silver nanowires (a) with radius *a* and of silver nanostrips (b) with width *d* and thickness *h* .

In the scattering by finite nanostrip gratings, a reliable instrument for calculating the scattered and absorption characteristics is the developed by us earlier median-line integral equation method based on the two-side GBCs, combined with the Nystrom-type discretization of interpolation type [4-6]. In the core of our approach lays the fact that nanostrip thickness usually makes a small fraction of the optical wavelength while the width can be both comparable to and larger than the wavelength. The small thickness $h << \lambda$ suggests that the analysis can be simplified by neglecting the internal field of the strip and considering only the external field limiting values imposing the two-side GBCs at corresponding strip median lines (see [5,6] for details),

$$\partial[E_z^+(\vec{r}) + E_z^-(\vec{r})]/\partial\vec{n} = -i2kQ[E_z^+(\vec{r}) - E_z^-(\vec{r})], \tag{1}$$

$$[E_z^+(\vec{r}) + E_z^-(\vec{r})] = i2Rk^{-1}\partial[E_z^+(\vec{r}) - E_z^-(\vec{r})]/\partial\vec{n}. \tag{2}$$

Here, the coefficients R and Q are the so-called relative electrical and magnetic resistivities,

$$R = i\cot(kh\sqrt{\varepsilon_r}/2)/(2\sqrt{\varepsilon_r}), \qquad Q = i\sqrt{\varepsilon_r}\cot(kh\sqrt{\varepsilon_r}/2)/2, \tag{3}$$

which contain the strip characteristics such as electric thickness kh and relative dielectric function ε_r, $\vec{n}$ is the unit vector normal to the strip grating, and the superscripts $\pm$ denote the limit values of the field at the top and bottom faces of the strip, respectively. These GBCs are valid if $kh \ll 1$ and $|\varepsilon_r| \gg 1$ [1,2].

This problem reduces to two independent sets of N IEs of the second kind for the electric and magnetic surface currents induced on the strips. Note that one set of IEs contains equations with logarithmic singularities and the other – with hyper-type singularities. Here, the hyper-singular integrals are understood in the sense of finite part of Hadamard. For discretization of IEs we use Nystrom-type method with two different quadrature rules of interpolation type: one based on the Gauss-Legendre quadrature formulas of the n_v-th order with nodes in the nulls of Legendre polynomials $P_{n_v}(\tau_j) = 0$, $j = 1,...,n_v$, and the other – on the Gauss-Chebyshev quadrature formulas of the n_w-th order (with the weight $(1-t^2)^{1/2}$) which is more efficient, with nodes in the nulls of Chebyshev polynomials of the second kind, $t_j = \cos(\pi j/n_w)$, $j = 1,...,n_w$. Thereby, applying the above mentioned quadrature formulas, we arrive at two independent sets of matrix equations of the orders $N \cdot n_v$ and $N \cdot n_w$, respectively. These matrix equations represent discrete models of our IEs. On solving them we obtain the surface currents as interpolations polynomials. The chosen quadrature formulas ensure rapid convergence of numerical solutions to the accurate ones if $n_v, n_w \to \infty$. Thus, such an approach leads to very economic and rapidly convergent algorithms, whose results agree very well with full boundary and volume IE results.

III. Numerical Results and Discussion

In Fig. 2-3 we present the some numerical results related to the normally incident plane E-polarized wave.

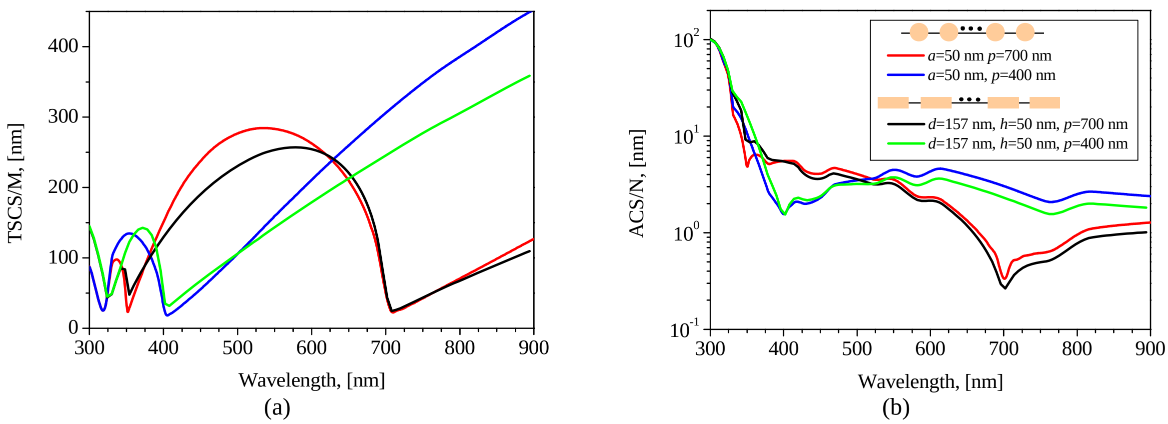
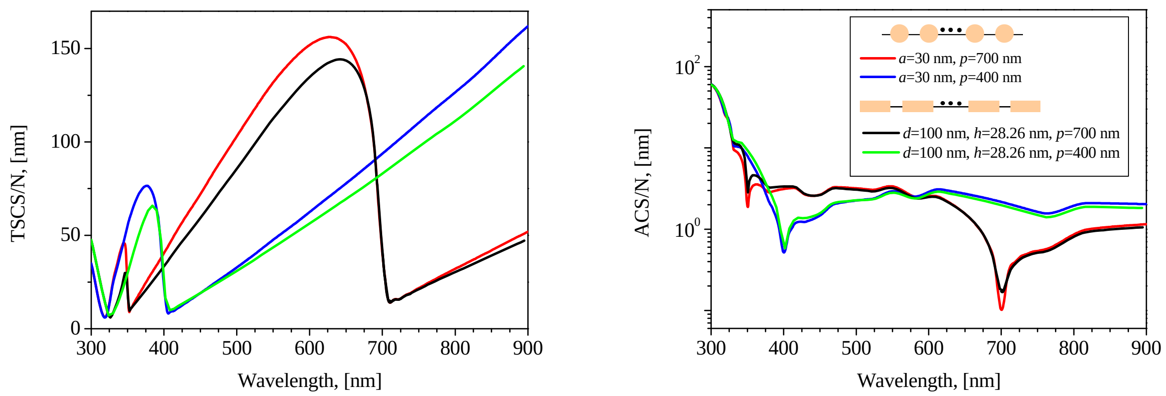


Fig. 2. Normalized TSCS (a) and ACS (b) versus the wavelength for the gratings of N = 50 silver nanostrips of width d = 157 nm and thickness h = 50 nm, and N = 50 silver nanowires of a = 50 nm for two values of the periods p = 400 nm and p = 700 nm under the normally incident ($\beta = \pi/2$) E-wave.

To study the features in the E-polarized light scattering and absorption by the finite periodic silver nanoparticle gratings, we have investigated the wavelength dependences of the total scattering (TSCS) and absorption (ACS) cross sections, obtained via integration of the far-field scattering patterns and the optical theorem, and visualized the near-field patterns. To characterize the complex dielectric permittivity of silver we took the experimental data of Johnson and Christy [9] with spline interpolation.

We show the wavelength scans of TSCS and ACS normalized by N for N = 50 nanostrips or nanowires arranged in the linear grating. As two comparison criteria, the equal cross-sectional areas of corresponding nanoscatterers $S = \pi a^2 = dh$ and the grating period have been chosen. In Fig. 2, we consider gratings with S = 7850 nm^2 and two different periods of p = 400 and 700 nm, and in Fig. 3 we focus on the gratings of less massive silver elements, i.e. with a smaller cross-sectional area of S = 2826 nm^2.

As one can see, under the E-wave illumination finite linear silver nanostrip and nanowire gratings have quite similar behavior and demonstrate the formation of the Rayleigh anomalies near the corresponding wavelengths of associated infinite grating, that is $\lambda_m^R = p/m$, $m = 1, 2, \ldots$, at the normal incidence.

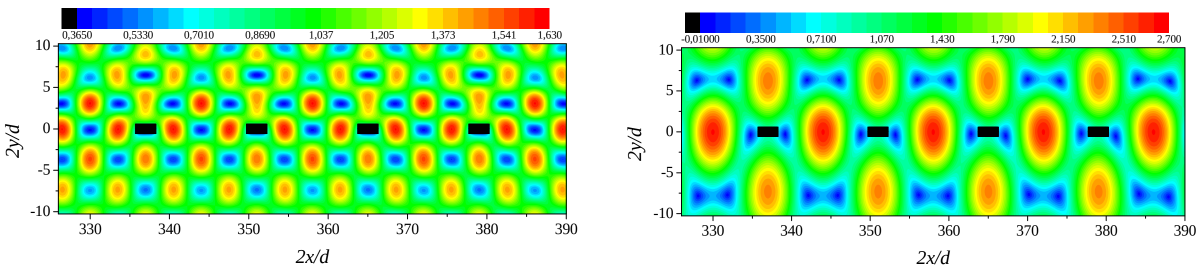


(a) (b)

Fig. 3. Normalized TSCS (a) and ACS (b) versus the wavelength for the gratings of N = 50 silver nanostrips of width d = 100 nm and thickness h = 28.26 nm and N = 50 silver nanowires of a = 50 nm for two values of period p = 400 nm and p = 700 nm under the normally incident ($\beta = \pi/2$) E-wave.

An unexpected result is that not only the E-wave scattering but also the absorption per element of the grating is suppressed at the wavelengths of the emerging Rayleigh anomalies. This seems to be a unique feature of the silver wire and strip gratings in the-polarization case.

Further we demonstrate the total near electric field patterns in the vicinities of the ±2-nd and ±1-st Rayleigh anomalies, namely near to λ_1^R = 350 nm and λ_2^R = 700 nm for the N = 50 nanostrip grating with parameters d = 100 nm, h = 28.26 nm and p = 700 nm (see Fig. 4). These patterns demonstrate characteristic maps of double and single maxima of the E-field in the gap between the strips.



(a) (b)

Fig. 4. Normalized total E-field patterns for the normally incident E-wave for the gratings of N = 50 silver strips (at three central periods) of d = 100 nm, h = 28.26 nm and p = 700 nm at the wavelengths λ_1^R = 350.6 nm (a) and λ_2^R = 700.7 nm (b).

The patterns in Fig. 5 show the same quantities for the scattering of the normally incident E-wave by the grating of N = 50 silver cylinders (wires) having the radii of a =30 nm.

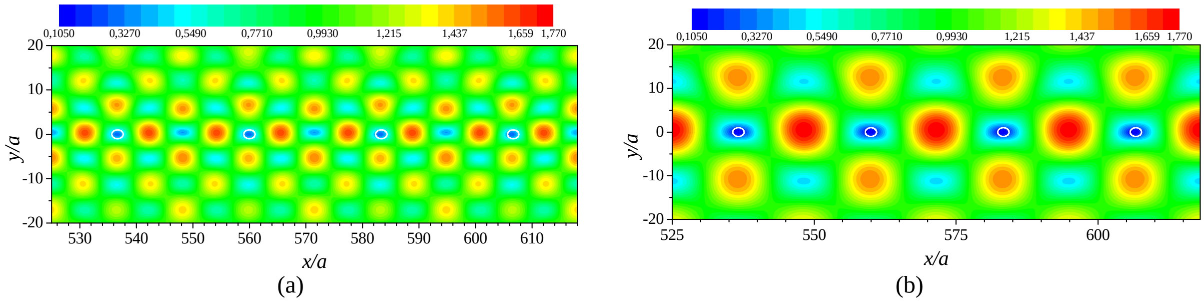


Fig. 5. The same as in Fig. 4 for the nanowire gratings with a =30 nm at the wavelengths of λ_1^R = 350.5 nm (a) and λ_2^R = 700.5 nm (b).

ACKNOWLEDGEMENT

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