ELECTROMAGNETIC WAVE SCATTERING BY PERIODICALLY STRUCTURED CHAINS OF CLOSELY SPACED SUB-WAVELENGTH WIRES

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Abstract – **Using the field expansions in local coordinates and addition theorems for cylindrical functions, we consider the problem of the H-polarized waves scattered by finite chains of circular wires. The scattering and absorption cross-sections are found numerically and localized-plasmon and periodicity-caused resonances are studied for the visible light scattering by the grids of closely spaced nanowires made of silver.**

I. INTRODUCTION

Small dielectric objects can exhibit resonance behavior at certain frequencies for which the object permittivity is negative and the free-space wavelength is large in comparison to the object dimensions. The latter condition clearly suggests that these resonances are electrostatic in nature. They appear at specific negative values of dielectric permittivity for which source-free electrostatic fields may exist. This is the physical mechanism of these resonances. For nanoscale metallic objects, these resonances occur in the optical frequency range and they result in powerful localized sources of light that are useful in the scanning near-field optical microscopy, nanolithography, and biosensor applications. General physical properties of plasmon resonances are actively studied now, together with techniques for direct calculation of negative values of dielectric permittivity, and the corresponding frequencies of electromagnetic radiation, for which resonances occur [1].

The plasmon resonances of particles with dimensions down to 2 nm can be investigated using Maxwell's theory [2]. Herein, the particles are characterized by their dispersion, i.e. by the complex permittivity as a function of the wavelength λ. For silver and gold this dispersion is quite complex in the optical range, as the plasma frequency ω_p of the conduction electron gas lies in this range. When the illumination frequency passes nearby ω_p, the real part $\varepsilon'(\lambda)$ of the dielectric function changes its sign, and for specific negative $\varepsilon'(\lambda)$ values, plasmon resonances can be excited in the structure. These specific values are strongly dependent on the particle size and shape [3]. The width of the plasmon resonances, which is related to the scattered or absorbed power, depends on the imaginary part $\varepsilon''(\lambda)$ of the permittivity. This imaginary part, which accounts for the bulk damping, becomes a function of the particle size when its dimensions are similar to the bulk electron mean free path. In that case, electron scattering at the particle boundary becomes a dominant effect, and the decrease of the electron mean free path leads to an increase of $\varepsilon''(\lambda)$.

Another kind of resonances is observed only in periodical structures: this is the grating resonance. Wood was the first who discovered drastic transformations of the scattering pattern and reflectance intensity if changing the light frequency or the angle of incidence [4]. One of such "anomalies," first theoretically explained by Lord Rayleigh is observed if one of the Floquet harmonics (terms of certain series in the theoretical description of an infinite grating) is "passing over horizon" [5]. In the simplest case of the normal incidence, such Rayleigh-Wood anomalies are observed if the period of the infinite grating is multiple to the wavelength. When solving a problem of diffraction by a finite grating, for example, by a grid of M wires, one cannot reduce it to one period and must treat the grid as an M-body scatterer [6]. It is quite interesting to find out what is the effect of periodicity and the number M on the scattering by finite grids. It is known that periodicity leads to the appearance of sharp lobes in the far-field scattering pattern in the directions of the Floquet-modes of similar infinite grid. Besides of that, among other Wood anomalies in the infinite-grating scattering one can observe the resonances on the "grating modes," whose frequencies are just below the Rayleigh frequencies if the wire diameter is a fraction of the period [7]. In this paper we study the formation of such a resonance in the scattering of the H-polarized plane waves by the large grids of sub-wavelength silver wires whose diameter is almost equal to the grid period.

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II. Formulation and Basic Equations

We consider the two dimensional scattering of the plane wave ($\sim e^{-i\omega t}$) by finite chains or grids of wires. A finite grid of equidistantly located parallel wires is shown in Fig. 1. The wires are assumed to be infinite circular cylinders with the same radius a and relative dielectric permittivity ε. The distance between adjacent cylinders is ρ and their number is M.

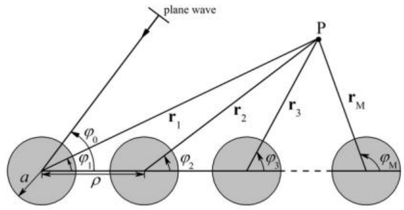


Figure 1. Scattering problem and notations.

The global coordinates have the origin at the centre of the first cylinder. For a 2-D problem, a scalar function U, which represents either E_z or H_z scattered-field component, must satisfy the Helmholtz equation with wavenumber $k_- = \sqrt{\varepsilon}k_0$ or $k_+ = k_0$ inside and outside of each cylinder, the total tangential field continuity conditions, the radiation condition at infinity, and the condition of the local power finiteness. The solution can be obtained by expanding the field function in terms of the azimuth exponents in the local coordinates (Fig. 1), using addition theorems for cylindrical functions, and applying the boundary conditions on all M cylinders.

The unknown coefficients of the field scattered by the q-th cylinder include the effect of all interactions between the cylinders [6,8]. They satisfy an infinite matrix equation, $(\mathbf{I}+\mathbf{U})\mathbf{X} = \mathbf{U}^0$, where $\mathbf{U} = \{\delta_{ij} U^{(i,j)}\}_{i,j=1,\ldots M}$, $\mathbf{X} = (x_m^{(1)} \ldots x_m^{(M)})$, $\mathbf{U}^0 = (U_m^{0(1)} \ldots U_m^{0(M)})$, $i, j = 1,\ldots M$, and $m = 0, \pm 1, \ldots$. It is necessary to cast the matrix equation to the $M \times M$ block-type Fredholm second kind form, where each block is infinite. Only provided that this is done, the solution of corresponding counterpart equation with each block truncated to finite order N converges to exact solution if the number N gets greater. To characterize far-field scattering properties of considered grids we have computed the total scattering cross section (TCS), absorption cross section (ACS) and backward cross section (BCS) frequency dependences. Here, we have tested our code and got a good agreement with the results published earlier in [3,9,10].

III. Numerical Results and Discussion

We have computed the scattering by densely packed finite grids of silver wires with radius of hundreds of nanometers, in the visible band. To characterize the dielectric permittivity of silver, we have used the bulk experimental data of Johnson and Christie [11] and a home-made interpolation code producing $\varepsilon'(\lambda)$ and $\varepsilon''(\lambda)$ at any given wavelength between 300 and 500 nm. Some of the obtained results are presented in Figs. 2 and 3.

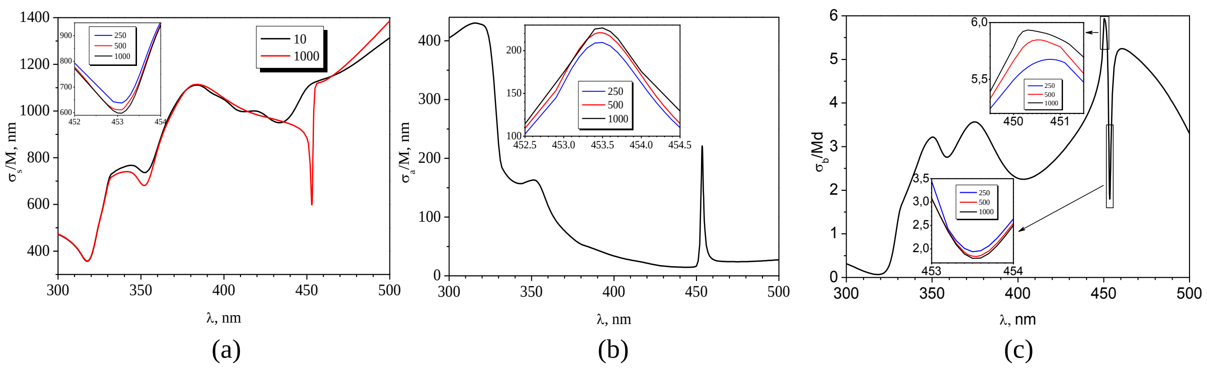
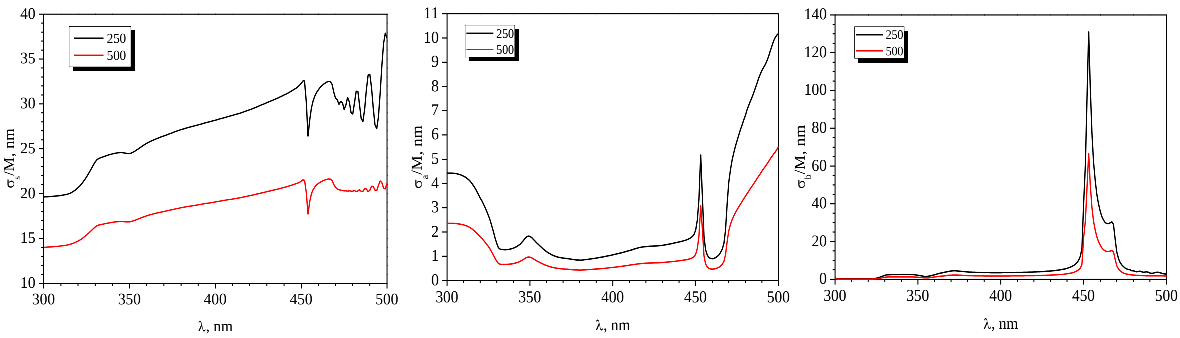


Figure 2. Normalized TCS (a), ACS (b) and BCS(c) as a function of wavelength for the H-wave broadside incidence ($\varphi_0 = 90^0$) on the different grids of silver nanowires with radius a = 200 nm and period ρ = 450 nm.



(a) (b) (c)

Figure 3. The same as in Fig. 2 however for the H-wave grazing incidence ($\varphi_0 = 0^0$) on the different grids of silver nanowires with radius a = 200 nm and period ρ = 450 nm.

First of all, one can see that the resonances display themselves as minima on the curves of TCS and simultaneous (i.e. at the same wavelengths) maxima on the curves of ACS.

Fig. 2 presents the data computed for the broadside incidence. In Fig. 2a, one can see three resonances: the first one at wavelength 342 nm, the second one at 384 nm and the third one at 451 nm. The first, smaller, resonance corresponds to plasmon of single cylinder with the same radius. The second resonance has larger amplitude that is conditioned by strong coupling effects between closely spaced wires. The third resonance can be seen only for the grids with large number of cylinders. At the wavelength around 453 nm, there is no observable TCS leap for the grid consisting of 10 cylinders, but for the grids of 250, 500 and 1000 cylinders one can see drastic TSC drops by some 40% of the off-resonance value. The TCS minimum becomes deeper if the number of wires in the grid increases.

Fig. 2b shows that ACS also has plasmon resonance maxima at the 316 nm and 352 nm wavelengths, blue-shifted in comparison to TCS, and a sharper grid-type resonance at the 453.5 nm wavelength. The latter resonance becomes larger if the number of grid wires increases.

Fig. 2c shows wavelength dependences of the BCS and displays two Plasmon-type (at the 350 nm and 375 nm wavelengths) and one grid-type (at the 451 nm wavelength) resonances. The first plasmon resonance is slightly red-shifted in comparison to the similar TCS resonance, while the second one is blue-shifted. The grid-type resonance is high-Q and sharp however has “double-extremum” character: after the maximum, BCS sharply decreases to the local minimum. This BCS minimum at the 453.2 nm wavelength coincides with the total scattering minimum on the TCS plot and absorption maximum on the ACS plot.

Fig. 3 shows the same dependences of the scattering and absorption cross-sections that in Fig. 2, respectively, but for the grazing incidence. In Fig. 3a, one can see wide and low-amplitude plasmon resonance associated with a single cylinder and does not see plasmon resonance caused by the presence of other cylinders. As for the grid-type resonance, one can see a sharp drop of TCS with the minimum at 453.4 nm wavelength. After this minimum, at the larger wavelengths, TCS shows oscillations, which get smaller for the larger grids. In Fig. 3b, one can see two absorption resonances. The first is the plasmon one, associated with a single silver cylinder and slightly red-shifted respectively to the similar one on the TCS dependence. The second resonance is the grid-type resonance. After it, at the larger wavelengths, absorption becomes large and ACS increases. In Fig. 3c, one can see the BCS wavelength dependence where only one resonance is seen, red-shifted with respect to the 450 nm wavelength. It is interesting that BCS curve has a hump at 467 nm wavelength, and this hump coincides with another minimum at the same wavelength for TCS (see Fig. 3a). Note that for such parameters of the scatterers and the angle of incidence the normalized by the number of grid elements M value of BCS becomes smaller if M increases.

So, we can see that if cylinders are placed periodically and closely to each other, their strong mutual influence gives some interesting effects. The difference between the broadside and grazing incidence is seen in the fact that the observed plasmon resonances are multiple (at least two) in the former case and only single in the latter case (relating to the stand-alone silver wire). Besides, these two incidence angles result in drastically different behavior of the BCS curves. Under the grazing incidence the BCS is uniformly low in the whole studied range except the vicinity of the grid-type resonance and does not depend much on the number M; this, in fact, is a sort of the Bragg phenomenon. In contrast, under the broadside incidence the BCS strongly depends on *M* and displays deep minimum at the grid-type resonance wavelength.

VI. CONCLUSION

We have presented accurate results for the scattering and absorption cross-sections for the H-polarized plane wave diffraction by finite linear grids of closely packed silver nanowires with the numbers of elements up to 1000. A 2-D diffraction problem with rigorous boundary conditions has been treated by the partial separation of variables using local polar coordinates of each scatterer. This has enabled us to reduce it to a Fredholm second kind block-matrix equation with favorable features. We have demonstrated two different kinds of resonances: plasmon and grid-type, and studied how they depend on the number of nanosize silver wires in the dense grids. If the number of grid elements gets larger, a narrow peak associated with the grid-type resonance on the wavelength very close to the grid period also gets sharper. An interesting observation, which seems to escape the attention of researchers earlier, is that, unlike the sparse grids (reported by us in [12]), in the case of dense grids the resonance values of TCS and ACS are opposite in the sense that they are seen as minima of the former quantity and maxima of the latter one.

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