Role of Periodicity in the Scattering by a Cloud of Randomly Located Plasmonic Nanowires

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Abstract: We consider the problem of the H-polarized plane wave scattering by a linear chain of silver nanowires in a cloud of similar arbitrarily located wires, in the visible range. Numerical solution uses the field expansions in local coordinates and addition theorems for cylindrical functions and has a guaranteed convergence. The total scattering cross-sections and near-zone and far-zone field patterns are presented. The observed resonance effects are studied and compared with their counterparts in the scattering by the same linear chain of wires in free space.

The periodically structured scatterers, for instance chains or arrays of finite number of separate elements display much greater frequency dependence of their characteristics than homogeneous scatterers of the same overall dimensions. The reason of this is specific resonant phenomena. Among several kinds of resonances observed on periodic structures, important role is played by the so-called grating resonances (Gresonances) [1-12]. If the size of the grating element (say, diameter of a wire) is a fraction of the period, then the Gresonance frequencies are just below the Rayleigh frequencies of the associated infinite gratings. The G-resonances lead to almost total reflection of the incident plane wave by a subwavelength wire grating in a narrow frequency band.

In this paper, we consider the two-dimensional scattering of the H-polarized plane wave by a linear chain of silver nanowires placed into a cloud of similar but randomly located nanowires (linear-chain-in-cloud, LCIC). As known, in the optical range the real part of the silver permittivity has negative values. This leads to the existence of the surface-plasmon resonances (P-resonances) on each of nanosize scatterers (i.e. for the wavelength being large in comparison to the wire diameter). P-resonance frequencies depend primarily on the object shape and much less on its dimensions.

Thus the goal of this work can be formulated in any of two reciprocal ways: this is a study of either the effect of periodic structuring on the scattering by random ensemble of identical scatterers or the influence of arbitrarily located additional scatterers on the scattering by a periodic structure.

In Fig. 1, one can see the scattering problem configuration. A linear chain of 100 silver wires with the radius of 60 nm each has the period of 450 nm; around this chain, in the area not exceeding the circle with the same diameter as the chain length, other randomly located 100 silver wires of the same radius are placed. This configuration is illuminated by the H-

polarized plane wave incident along the direction φ_0 counted from the *x*-axis.

To solve a 2-D scattering problem, one has to find a scalar function U, which represents the scattered-field H_z component. It must satisfy the Helmholtz equation with wavenumber vk_0 (v being the silver refractive index and k_0 being the free-space wavenumber) or k_0 inside or outside of each wire, the total tangential field continuity conditions across each wire contour, the radiation condition at infinity, and the



Fig. 1. Scattering problem configuration: linear chain of 100 silver nanowires with radius 60 nm and period 450 nm in the cloud of 100 randomly located similar wires.

condition of the local power finiteness. The solution can be obtained by expanding the field function in terms of the azimuth exponents in the local coordinates, using addition theorems for cylindrical functions, and applying the boundary conditions on all M cylinders [5]. The unknown coefficients related to the q-th cylinder include the effect of all interactions between cylinders. They can be cast to the $M \times M$ block-type Fredholm second-kind matrix equation, where each block is infinite [5]. Therefore the solution of corresponding finite-size equation with each block truncated to finite order N converges to exact one if $N \rightarrow \infty$.

To characterize the far-field scattering properties of considered wire scatterers, we compute the total scattering cross section (TSCS) in dependence of the frequency and the angle of incidence. Definition of TSCS can be found in [5] together with the far-field scattering patterns; we also present the total-field patterns in the near zone. For the refractive index of silver, we use experimental data of [13]. In computations, the block truncation number has been adapted to provide the 4-digit accuracy in the field expansion coefficients.

In Fig. 2a, presented is the relief of the normalized (i.e. perwire) TSCS as a function of the wavelength and the angle of incidence for the full LCIC configuration. Note that this is a composite scatterer whose total diameter is around 450 wavelengths. One can clearly see the areas of high values of TSCS which stretch along (but not coincide with) the solid lines corresponding to the ± 1 and ± 2 Rayleigh anomalies. This is the expected effect of the presence of periodicity of 100 wires, and, what is remarkable it is not much spoiled by the presence of the other 100 wires located randomly.

As one can see, the influence of the randomly located wires



Fig. 2. Relief of normalized TSCS as a function of wavelength and incident angle of the plane H-wave scattering by the 100-wire chain-in-cloud configuration (a) and the wavelength scans of TSCS for different incidence angles φ_{0} as compared to the same chain in free space (b).

in each quadrant of the angle of incidence variation $(0-90^{\circ})$, $(90^{\circ}-180^{\circ})$, etc. is similar. Therefore we can consider in detail only the case of φ_0 varying in the first quadrant

In Fig. 2b, presented are the wavelength dependences of the normalized TSCS for the angles of incidence $\varphi_0 = 30^\circ$, 60° and 90° . For comparison, the results are presented for both LCIC configuration and for a "bare" linear chain of 100 nanowires in free space.



Fig. 3. Near H-field and far-field scattering patterns for 10 central periods of LCIC for cases marked in Fig. 2b, i.e. at the G-resonance wavelengths of 352 nm (a) and 450 nm (c), and the incidence angles are 60° and 90° , respectively.

In both cases, one can see the effect of the resonances of both types. P-resonances are visible near to 350 nm on all curves, i.e. with and without a random cloud. This is not a surprise as P-modes are excited on each wire. The effect of the cloud is seen only in the shadowing of the part of wires by those wires, which are better illuminated. This damps the Presonance in approximately two times. Unlike them, the location of the peaks corresponding to the G-resonances depends on the angle of incidence (for the fixed period). Of course, G-resonances for a chain without random cloud have higher peak values however they are seen at the same wavelengths. Still it is amazing that the addition of random cloud of 100 nanowires does not prevent the G-resonances to play the leading role and display more intensive peaks than those of the P-resonances.

Finally in Fig. 3 we present the near and far-field patterns for then LCIC configuration illuminated under the incidence angles of 60° and 90° . In the near zone one can see a complicated chaotic field pattern because of the influence of non-periodically placed wires near to the chain. If $\varphi_0 = 60^{\circ}$, the G-resonance wavelength is close to the P-resonance wavelength, and one can see the field maxima near the wire contours. In the far-field scattering patterns, one can observe the shadow lobe, the specular-reflection lobe and the sidelobes in the directions parallel to the grating, corresponding to the grazing propagation of the Floquet diffractive orders.

Thus, in this paper we have presented accurate results for the H-polarized wave diffraction by LCIC. It has been shown that the presence of 50% periodicity in the ensemble of 200 arbitrarily placed scatterers is able to produce well-visible grating resonances in the total scattering cross-section under any angle of incidence except the grazing one.

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