

# Plasmon and Periodicity Assisted Scattering and Absorption of Light by Finite Nanowire Gratings

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## 1. Introduction

Two fundamental effects are known to influence the scattering of light by periodically structured metal scatterers. From the one hand, surface-plasmon resonances are observed for sub-wavelength noble-metal particles and wires in the mid-infrared and optical bands [1,2]. Nanosize objects can exhibit resonance behavior at certain frequencies for which the object permittivity is negative. This results in powerful enhancement of scattered and absorbed light that is used in the design of optical antennas and biochemical sensors for advanced applications. In the leading terms, the plasmon resonance wavelength depends on the object shape but not on its dimensions.

From the other hand, periodically structured scatterers, or finite and infinite gratings, arrays or chains of particles and holes in metallic screens (in 3-D) or wires and slots (in 2-D), are attracting large attention of researchers in today's nano-optics [3-6]. This is caused by the effects of extraordinarily large reflection, transmission, emission, and near-field enhancement that have been found in the scattering of light by periodic scatterers. Recently it has been discovered that these phenomena are explained by the existence of so-called grating resonances or poles of the field function [4-6] (a.k.a. geometrical, lattice and Bragg resonances). Their wavelengths lay near the Rayleigh wavelengths [7], i.e. near to period being a multiple of the wavelength if all elementary scatterers of a grating are excited in the same phase and their size is a fraction of the period. In the wave scattering by infinite gratings, they lead to almost total reflection of the incident field by a sparse thin-dielectric-wire grating in narrow wavelength bands [3,5]. The goal of our paper is extension of this study to more complicated periodically structured silver-wire configurations where both plasmon and grating resonances are present.

## 2. Scattering Problem

Consider finite collections of  $M$  parallel wires illuminated by an H-polarized plane wave shown in Fig. 1. The wires are assumed to be infinite circular cylinders, each having radius  $a$  and complex relative dielectric permittivity  $\varepsilon$ . For a 2-D problem, one has to find a scalar function  $H_z(\vec{r})$  that is the scattered magnetic-field  $z$ -component. It must satisfy the Helmholtz equation with corresponding wavenumbers inside and outside the cylinders, the tangential field components continuity conditions, the radiation condition, and the condition of the local power finiteness.

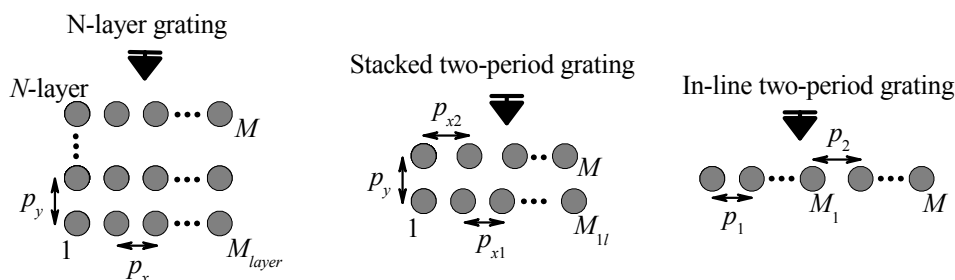


Figure 1: Scattering problem geometries and notations

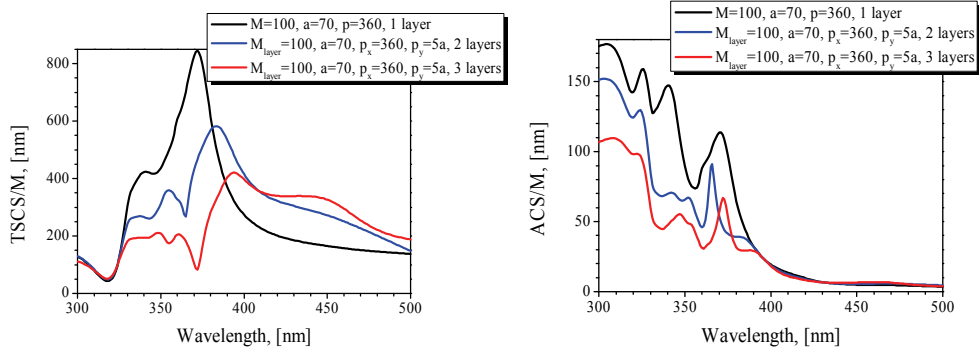


Figure 2: Normalized per number of wires TSCS and ACS as functions of the wavelength for the H-wave normal incidence on the 1, 2 and 3-layer gratings of silver wires

The full-wave numerical solution can be obtained similarly to [7,8], by expanding the field function in terms of the azimuth exponents in the local polar coordinates, using addition theorems for cylindrical functions, and applying the boundary conditions on the surface of each of  $M$  wires. This leads to an infinite  $M \times M$  block-type matrix equation where each block is infinite. Still a close inspection shows that the matrix equations used in the previous papers did not provide guaranteed convergence of solutions. Here, the convergence is understood in mathematical sense, as a possibility of minimizing the error of computations by solving progressively larger matrices. We fix this defect by re-scaling the unknown coefficients as explained in [6].

The obtained in such a way matrix equation is a block-type Fredholm second kind operator equation. In this case the convergence of solution, after truncation of each block to finite order  $N$ , to exact solution if  $N \rightarrow \infty$  is guaranteed by the Fredholm theorems. The results presented below were computed with  $N = 4-5$ ; this provided 3 correct digits in the far-field characteristics of the sparse gratings of silver wires with radii  $a \leq 75$  nm and periods  $p \geq 200$  nm. Note that denser gratings may need larger values of  $N$  to achieve the same accuracy.

### 3. Results and Discussion

We have considered three sparse ( $p - 2a > 2a$ ) configurations of finite number of sub-wavelength silver nanowires: two and three-layer gratings of the same periods, stacked two-period gratings, and in-line two-period gratings (Fig. 1). Dense configurations are also interesting objects however they deserve a separate study. To characterize the optical properties of considered discrete scatterers, we have used the wavelength dependences of the total scattering (TSCS) and the absorption (ACS) cross-sections and calculated the field patterns in the near zone. In Figs. 2 - 6, the

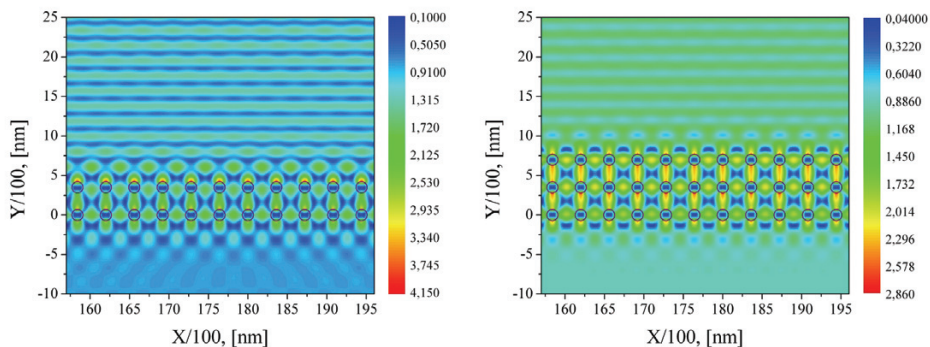


Figure 3. Near-field amplitude patterns of the central parts of the 2 and 3-layer gratings from Fig. 2 in the TSCS maxima at the 383 nm (left) and 394 nm wavelengths (right)

wavelength varies from 300 nm to 500 nm and the complex-valued dielectric function of silver has been borrowed from the classical paper of Johnson and Christy.

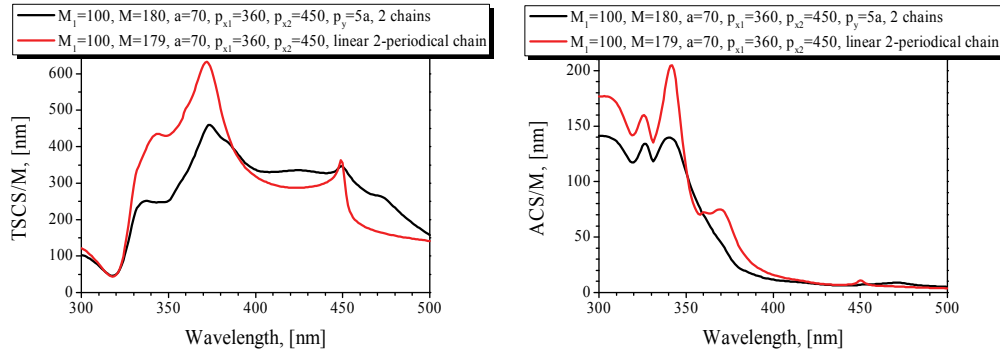


Figure 4: Normalized TSCS and ACS as functions of the wavelength for the H-wave normally incident on the stacked and in-line two-period gratings of identical silver nanowires

### 3.1 Two and Three-Layer Gratings

N-layer gratings are interesting structures for research because they are periodic along two axes and can be considered as finite-size photonic or plasmonic crystals. How this structuring influences the grating and plasmon resonances is a question that needs clarification. In Fig. 2 presented are per-wire TSCS and ACS as functions of the wavelength for the H-wave normal incidence on the gratings of two and three chains of 100 nanowires with radii  $a = 70$  nm, with periods along the  $x$ -axis  $p_x = 360$  nm and along the  $y$ -axis  $p_y = 5a$ . As known, the plasmon-resonance peak for a single silver nanowire in free space is near 350 nm, so both periods are in the vicinity of this value. As one can see, the resonant peaks of the averaged per-wire cross-sections are higher for 2-layer grating. This can be understood as a result of shadowing of the lower layers by the upper (better illuminated) ones. The TSCS wavelength dependences have several local maxima in the vicinity of 350 nm, and in the case of the 2-layer grating there is one resonance at 350 – 360 nm wavelengths, but in the case of the 3-layer grating there are two low-quality resonances in the same range. At the 383 nm and 394 nm wavelengths, one can see the highest resonances for the 2-layer and 3-layer gratings, respectively (see Fig. 3). Note that the ACS dependences show the peak values at the same wavelengths where TSCS dependences have their minima.

In Fig. 3 presented are near-field amplitude patterns for the central part of the 2 and 3-layer gratings in the main resonances. The visualized part of the grating includes 11 central wires from each layer. One can see that the most intensive H-field maxima are located at the illuminated side of the upper-layer wires for the 2-layer grating, and between the layers for the 3-layer grating. Besides, one can see a standing wave above the illuminated side for each grating.

### 3.2 Two-Period Stacked and In-Line Gratings

Two-period linear gratings which consist of two chains or arms with different periods are interesting for applications because of existence of two different grating resonances, in addition to

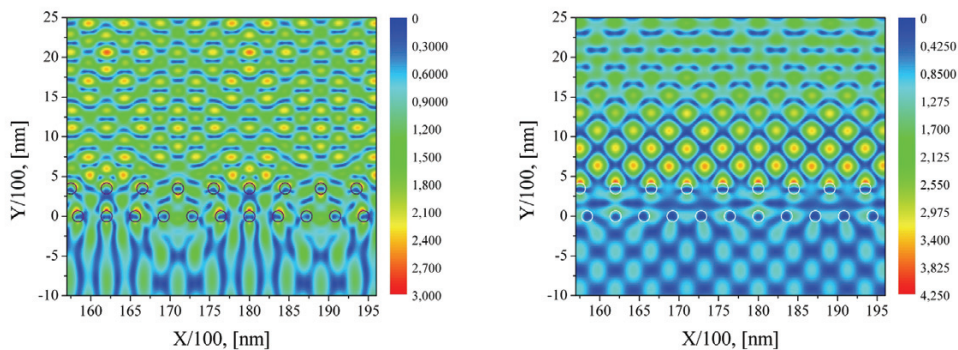


Figure 5: Near-field amplitude patterns of the central parts of the stacked two-period grating from Fig. 4 in the grating resonances at the 374 nm (left) and 449 nm wavelengths (right)

the plasmon resonance of each individual wire. This feature can be useful for electromagnetic engineering of the novel wideband absorbers for solar cells. Our analysis of such gratings have

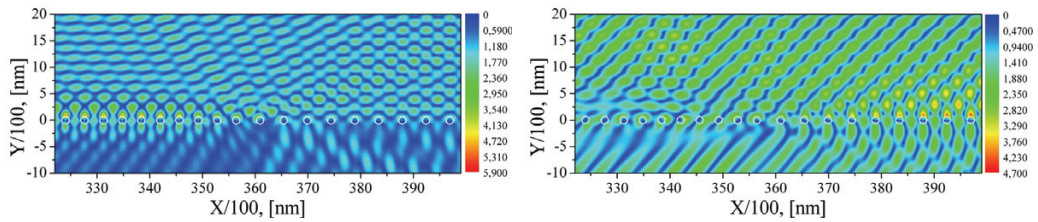


Figure 6: Near-field amplitude patterns of the central parts of the in-line two-period grating from Fig. 4 in the grating resonances at the 372 nm (left) and 449 nm wavelengths (right)

shown that it is indeed possible to combine the resonances and enhance the per-wire TSCS and ACS if the wire radius is 50 nm or larger and their number is at least 100.

In Fig. 4 presented are per-wire TSCS and ACS as functions of wavelength for the normal incidence of the H-wave on the stacked and in-line double-periodic gratings of silver nanowires with radii  $a = 70$  nm and periods  $p_1 = 360$  nm (180 or 179 wires) and  $p_2 = 450$  nm (100 wires); the distance between the layers in the stacked grating is 210 nm. One can see that the cross-sections for the in-line configuration have more intensive and sharper resonances of both types, apparently because such a configuration has no part shaded by other wires. The TSCS reaches its maximum value at 374 nm and 372 nm for the stacked and in-line two-period gratings, respectively. This resonance is associated with the smaller period and, in part, with the plasmon resonance of each wire. The other grating resonance in the vicinity of the 450 nm wavelength (the larger period value) has low intensity, especially for ACS because the bulk losses in silver are smaller there.

In Figs. 5 and 6, presented are the near-field amplitude patterns for the central parts of the stacked and in-line two-period gratings, respectively, in the grating resonances. In Fig. 5, we show the near-field patterns at 374 nm and 449 nm in the grating resonances associated with the smaller and the larger periods, respectively. Note that in the second case the field pattern demonstrates that the top grating (tuned into resonance) efficiently screens the bottom grating, which remains in the deep shadow. In Fig. 6, we show near-field patterns at 372 nm and 449 nm in the grating resonances for the smaller and the larger periods of the in-line grating. In each case, the standing waves are formed along the  $x$  and  $y$  axes however only near the arm of the grating that is in the resonance.

## Acknowledgments

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## References

- [1] J.P. Kottman, O.J.F. Martin, “Plasmon resonant coupling in metallic nanowires,” *Optics Express*, vol. 8, pp. 655–663, 2001.
- [2] D.R. Fredkin, I. Mayergoyz, “Resonant behavior of dielectric objects (electrostatic resonances),” *Phys. Rev. Lett.*, vol. 91, pp. 3902-3905, 2003.
- [3] M. Laroche, S. Albaladejo, R. Gomez-Medina, J.J. Saenz, “Tuning the optical response of nanocylinder arrays: an analytical study,” *Phys. Rev. B*, vol. 74, No. 9, pp. 245422-10, 2006.
- [4] F.J.G. Garcia de Abajo, “Colloquium: Light scattering by particle and hole arrays,” *Rev. Mod. Phys.*, vol. 79, no. 4, pp. 1267-1289, 2007.
- [5] V.O. Byelobrov, J. Ctyroky, T.M. Benson, *et al.*, “Low-threshold lasing eigenmodes of an infinite periodic chain of quantum wires,” *Optics Letters*, vol. 35, no 21, pp. 3634-3636, 2010.
- [6] D.M. Natarov, V.O. Byelobrov, R. Sauleau, T.M. Benson, A.I. Nosich, “Periodicity-induced effects in the scattering and absorption of light by infinite and finite gratings of circular silver nanowires,” *Optics Express*, vol. 19, no 22, pp. 22176-22190, 2011.
- [7] H. A. Ragheb, M. Hamid, “Scattering by  $N$  parallel conducting circular cylinders,” *Int. J. Electronics*, vol. 59, pp. 407-421, 1985.
- [8] D. Felbacq, G. Tayeb, D. Maystre, “Scattering by a random set of parallel cylinders,” *J. Opt. Soc. Am. A*, vol. 11, pp. 2526–2538, 1994.