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Radiation conditions, limiting absorption principle, and general relations in open waveguide scattering

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Abstract–The purpose of this paper is to give solid ground to rigorous studies of open-waveguide-related scattering problems. In the core of the difficulties associated with them there lies the presence of infinite but penetrable boundaries. This calls for a revision of the condition of radiation, and to the necessity of justifying the limiting absorption principle. The analysis is based on the Fourier-transform approach and study of the analytic properties of transform functions with respect to a parameter. Except for simple 2-D geometries, regularized integral equations and operator theory theorems are used. The result differs from the free-space expression and involves the guided natural modes spectrum. It reveals also that the direction of power flow associated with each particular mode is to be taken into account. The obtained expressions validate the intuitively clear equations of power conservation and reciprocity. A numerical example is given and extension to more complicated open waveguides is discussed.

1. INTRODUCTION

The funds for military-oriented studies like RCS simulation via free-space scattering of waves are clearly shrinking now. This adds to the motivation for intensive study of electromagnetic problems associated with more complicated media, particularly open waveguides. More reason can be seen in the widening applications of such waveguides in millimeter waves and optoelectronics. Although the propagation on regular guides is of obvious interest, the principle of operation of any device as filter, mixer, or coupler is based on the modification of a surface mode field by means of irregularities. Therefore, the scattering and mode conversion in open waveguides is of great practical importance.

The design and optimization of such devices calls for rigorous treatment of corresponding boundary-value problems. But, it should be noted that open-waveguide scattering is a somewhat nonclassical branch of wave diffraction theory, due to the combination of two 'infinitives' in any relevant geometry. These are the infinite albeit regular boundaries of the waveguide's elements, and the infinite domain of the cross-section. Although not always clearly realized, this fact causes quite a number of more or less formal modifications with respect to free-space and closed-waveguide scattering of time-harmonic waves.

Indeed, assume that the wave field depends on time as $e^{-i\omega t}$ where $\omega = kc$ with $k = Re k > 0$, and c is the free-space propagation velocity. Such a field is known to satisfy a certain partial differential equation having a variety of solutions. In the open domain, a frequently used basis to single out the unique solution is

The Principle of Radiation. It eliminates any form of the scattered field which implies sources at infinity. Formally this principle is realized by introducing a certain restriction on the far field behaviour, i.e., a condition at infinity [1].

In the *free-space* scattering from *finite* obstacles, the well known Sommerfeld radiation condition [2, p. 188] is used. It may be expressed in several equivalent forms. One of them is an asymptotic request for the scattered field to represent an outgoing space wave sufficiently far from the obstacle. For example, in the scalar case it is

$$U^{sc}(\vec{R},k) \underset{R\to\infty}{\sim} \Phi(\Omega_\nu)R^{(1-\nu)/2}e^{ikR} \tag{1}$$

where Ω_ν is a point on a unit sphere, ν is the space dimension number.

However, (1) obviously fails in waveguide problems containing *infinite* boundaries. Instead of decaying uniformly far from the obstacle, the scattered field here can be guided along the boundary.

For the scattering in *closed waveguides*, another condition is valid (known as the Sveshnikov condition in Soviet literature after [3]). It requires the field to be expandable in terms of outgoing or decaying normal modes far from the obstacle,

$$U^{sc}(\vec{R},k) = \sum_{q=1}^{\infty} a_q V_q(\vec{r})e^{ih_q|z|}, \qquad |z| \geq l = const \tag{2}$$

where $h_q = (k^2 - g_q^2)^{1/2}$, and $g_q, V_q(\vec{r}), q = 1,2,...$ are the (real) eigenvalues and eigenfunctions of cross-sectional problem. So, at fixed frequency, the first Q terms in (2) correspond to outgoing guided modes, and the rest to cutoff, i.e. decaying, ones of the waveguide.

However, for *open waveguides* (2) is clearly not appropriate because it neglects the scattering to surrounding space through the infinite penetrable boundaries. So, a modified expression serving as adequate condition of radiation is to be used. In deriving it we follow [4,5]. The result differs from both (1) and (2), and leads to an important conclusion: the usual and seemingly obvious concept of the scattered field as a superposition of waves propagating to infinity can be erroneous for open waveguides, and must be replaced by the concept of waves carrying power to infinity. Besides giving solid ground to studying the relevant problems, this expression plays an important role in deducing general relations for the far-field characteristics.

It should be noted that there exists another way of extracting out the unique physical solution of lossless open-domain problems. It is *The Principle of Limiting Absorption*, according to which the solution is taken as

$$U^{sc}(\vec{R},k) = \lim_{\tilde{k}\to+0} \tilde{U}^{sc}(\vec{R},k+i\tilde{k}) \tag{3}$$

where $\tilde{U}^{sc}(\vec{R},k+i\tilde{k})$ is the vanishing at infinity solution of a lossy problem, i.e., the one with losses assumed in unbounded domain. The latter problem is a self-adjoint one, and hence always has an exponentially decaying solution. The limit in (3) is understood in $L^2(D)$ sense, for an arbitrary bounded region D. Unfortunately, in spite of obvious simplicity, there are no general theorems ensuring a universal validity of this principle. Existence of a limit like (3) must be proved independently for different classes of boundary-value problems. For free-space and closed-waveguide scattering this was done in [6] and [3], respectively. For open waveguides it needs a separate treatment.

The remainder of this paper is organized as follows. In Section 2, a modified radiation condition is derived for impedance plane as the simplest 2-D open waveguide. The analysis utilizes Fourier transform in space domain and complex plane integration. In Section 3, the treatment is extended to multimode 2-D open waveguides such as dielectric slabs. Based on the modified expression and Green's formula, two general relations known as power conservation (leading to the so-called 'Optical Theorem'), and reciprocity equations are obtained. Derivation for the 3-D case is discussed in Section 4. It is not so straightforward as could be expected. The reason is that the solution of the key problem of point-source radiation is not available explicitly in Fourier-transform domain. Nevertheless the Fredholm theory for operator equations enables us to arrive at the expression sought. Power conservation and reciprocity relations are further obtained using the Lorentz Lemma for Maxwell equations. Section 5 deals with justifying the Limiting Absorption Principle. In Section 6, a review of relevant literature is given, and an illustrative numerical example is presented. Section 7 gives an insight to extending the results to more complicated and realistic geometries, and to some modifications needed when analyzing the junctions of open waveguides.

In the following treatment, time dependence is suppressed throughout the analysis.

2. THE CASE OF IMPEDANCE PLANE AS A SINGLE-MODE 2-D OPEN WAVEGUIDE

Any element of the obstacle in a waveguide can be viewed as a secondary radiating source induced by the incident field. Thus, it is evident that when analyzing far-field behaviour, the key problem is that of point-source radiation in a regular guide. Let us start from a 2-D case and consider the simplest open-waveguide model available. Assume that a magnetic line-source is positioned at $\vec{r'} = (0, y')$ in lossless halfspace $D^+ = \{y > 0\}$ (see Fig. 1) bounded by the plane $y = 0$ at which an impedance type boundary condition is imposed:

$$\left(\alpha + \frac{\partial}{\partial y}\right) G(\vec{r}) = 0 \tag{4}$$

Thus the field function $G(\vec{r}) \equiv H_x$ must solve the Helmholtz equation with Dirac-delta in the right-hand part, and (4). The condition at infinity, however, remains unspecified.

Let us decompose the total field as a sum of a primary one G^0, singular at $\vec{r'} = \vec{r}$, and a secondary one G^{sc}. To ensure approaching the correct limit when the boundary plane vanishes, we have to take the first term as the free-space 2-D Green's function

$$G^0 = i/4 H_0^{(1)}(k|\vec{r} - \vec{r'}|) \tag{5}$$

Being independent of z, (4) offers a way to use integral Fourier transformation in the z-domain, thus arriving at the representations

$$G^0(\vec{r}) = \frac{i}{4\pi} \int\limits_{C(-\infty,\infty)} \frac{1}{g} e^{ig|y-y'|+ihz} dh \tag{6}$$

where $g = (k^2 - h^2)^{1/2}$, and similarly,

$$G^{sc}(\vec{r}) = \frac{i}{4\pi} \int\limits_{C(-\infty,\infty)} F(y, y', h, g) e^{ihz} dh \tag{7}$$

Integral (6) is known as the Sommerfeld one, with the integrand being a two-valued function of h. The branch of $g(h)$ is chosen so as to ensure the integration convergence, which implies $\operatorname{Im} g > 0$, and is known as "proper" sheet, C_h^1. So, the branch cut from $h = k$ goes up, while from $h = -k$ down in C_h^1. The integration path C is composed of real h-axis sections and infinitesimal semicircles around the branchpoints, i.e., below $h = k$ but above $h = -k$.

Substituting (6) and (7) into the Helmholtz equation leads to a 1-D problem for Fourier-transform being subject to (4) and satisfying

$$\left(\frac{\partial^2}{\partial y^2} + g^2\right) F(y, h) = 0 \tag{8}$$

The general solution of (8) is known to be a linear combination of e^{+igy} and e^{-igy}. As both h and k are real, g can be either purely real or imaginary on C. However note that for Fourier-transform as a function of y we have no infinite boundary (i.e., one of the initial 'infinitives' is eliminated). So, to single out $F(y, h)$, the conventional condition of radiation (1) for $|h| < k$, or the condition of decay for $|h| > k$, can be applied. That means we can request

$$F(y, h) = Ae^{igy}, \qquad y \geq 0 \tag{9}$$

Then, from condition (4) at $y = 0$ we obtain an explicit solution for the transform function

$$F(y, y', h) = g^{-1} e^{igy'} R(h) e^{igy} \tag{10}$$

where

$$R(h) = (ig - \alpha)/(ig + \alpha) \tag{11}$$

Note that unlike the Fourier-transform of G^0, that of G^{sc} is a meromorphic function of h having a pair of symmetrical poles at $h = \pm h_0$, where

$$h_0 = (k^2 + \alpha^2)^{1/2} \tag{12}$$

Provided that α is purely real (no losses), these poles lie on the path of integration C and must be bypassed. Actually this means that the Fourier transformation must be understood in a generalized way, and to the integrand must be added a Dirac-delta term at the pole. Bypassing is arranged along infinitesimal semicircles again, however its direction may be arbitrary. Let us introduce a number $\gamma = \pm 1$ such that $\gamma = 1$ corresponds to bypassing the pole at $h = h_0$ from below, and $\gamma = -1$ from above (see Fig. 2). From the symmetry considerations it is clear that the pole at $h = -h_0$ is to be bypassed in the opposite direction.

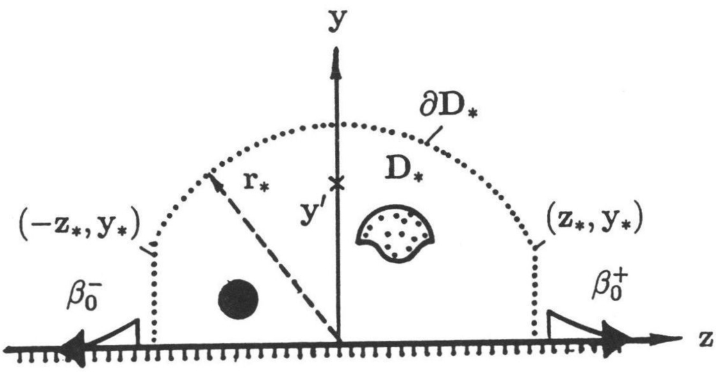

Figure 1. Two-dimensional geometry of localized scatterers near an impedance plane.

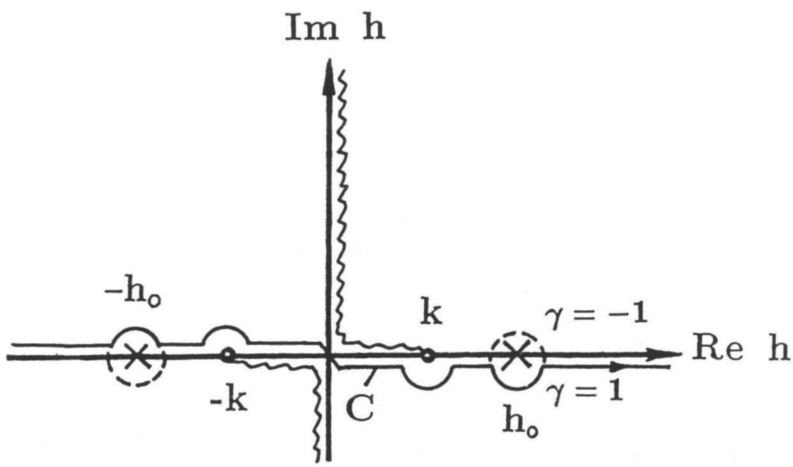

Figure 2. Complex h-plane and the contour of integration.

Another important observation reveals that the function (10) decays exponentially as $|h| \to \infty$ in the "proper" sheet C_h^1. This offers a way of evaluating the integral (7) asymptotically through a contour-deformation technique provided that $kr = k(z^2+y^2)^{1/2}$ is large enough. Introduce the observation angle θ such as $y/z = \tan\theta$. Deforming the path of integration to pass through the saddle-point $h = k\cos\theta[\equiv h_s]$ yields

$$G^{sc}(\vec{r}) = I_{SD} + 2\pi i U(\theta) \underset{h=sign(z)\gamma h_0}{Res} \{F(y,y',h)e^{ih|z|}\} \tag{13}$$

where I_{SD} stands for the integral along the steepest-descent path. Here $U(\theta) = 1$ if $\theta < \theta_{cr}$ or 0 otherwise, θ_{cr} defined by $\sin(\theta_{cr}) = (1 + \alpha^2/k^2)^{-1}$ being the angle of pole interception.

Note that the pole contribution depends on which pole is intercepted when deforming the path. This depends, obviously, on both the position of observation point with respect to the source, and on the direction of original bypassing the pole by C. Further derivation involves expanding the integrand in a power series at $h = h_s$, and term-by-term integration in I_{SD}. However the radius of convergence of this expansion is determined by the distance from h_s to the nearest pole. That is why, strictly speaking, this procedure brings a result which is less accurate and even discontinuous within a certain angular sector of θ variation. Rigorous analysis should be carried out by using the modified saddle-point technique [7]. However *at infinity* ($r \to \infty$) this sector tends to zero [7], so asymptotically the ordinary saddle-point method is correct. Besides, the factor $U(\theta)$ in (13) may be omitted, as at $\theta > \theta_{cr}$ the pole contribution is exponentially small for $r \to \infty$, and asymptotically it does not make any difference whether one takes account of it or not.

Bearing this in mind, we collect the leading terms of asymptotics from both field terms to arrive at

$$G(\vec{r}) \underset{r\to\infty}{\sim} \Phi_0(\theta)\left(\frac{2}{i\pi kr}\right)^{1/2} e^{ikr} + \beta_0 e^{-\alpha y + i\gamma h_0|z|} \tag{14}$$

where

$$\Phi_0(\theta) = 1 + R(k\cos\theta)e^{2iky'\sin\theta}, \qquad \beta_0 = i\alpha h_0^{-1}e^{-\alpha y'} \tag{15}$$

Note that the first term in (14) meets the Sommerfeld condition (1), while the second one violates it since it does not decay at $z \to \pm\infty$, i.e., at $r \to \infty$, $\theta = 0$ or π. Note also that $R(\theta) \to -1$ as $\theta \to 0$ or π, so the first term vanishes at grazing directions.

Strictly speaking, the above treatment is not valid at $\theta = 0$ or π where the saddle point $h_s = \pm k$. Here another way of contour deformation is to be applied [8]. However the main terms of far field are contributed by the pole and the integrand expansion at the branch point $h = \pm k$. That is why the result is exactly the same as the limit of (14) at $\theta \to 0$ or π.

One can easily see that the function

$$H_0(\vec{r}) = e^{-\alpha y \pm ih_0 z} \tag{16}$$

satisfies the homogeneous Helmholtz equation in D^+ and (4). So, we conclude that the residue of $F(y,y',h)e^{ihz}$ at $h = \pm h_0$ represents a natural guided mode (eigenmode) of the impedance plane open waveguide. There is only one such a mode. Its field has surface character, decaying away from the boundary. Thus, along the waveguide, it is the guided mode which solely carries the power.

Note that γ has not been specified so far. Physical intuition gives a hint that one should take $\gamma = 1$, to obtain surface-wave terms in (14) travelling from source to infinity. However, this conclusion may be arrived at quite formally. To show this let us apply Green's formula

$$\int_{D_*} (u\Delta v - v\Delta u) d\vec{r} = \int_{\partial D_*} \left(v\frac{\partial u}{\partial n} - u\frac{\partial v}{\partial n} \right) d\vec{r} \tag{17}$$

to the difference $u = G_1 - G_2 \, [\equiv \delta G]$ between any two solutions of the scattering problem, and its complex conjugate, i.e. $v = u^*$, in a bounded domain D_*. In the left-hand part we obtain identically zero. Now the trick is to take D_* of appropriate shape, constructed of circular and straight parts as shown in Fig. 1. We expand D_* to the whole D^+ by letting $z_*, y_* \to \infty$ but $y_*/z_* \to 0$ (for example, take $z_* = y_*^2$), and make use of asymptotic expression (14). When integrating along the circular arc, it is only the first term in (14) that gives nonvanishing contribution, while at the straight parts of ∂D_* only the second, and cross-product terms vanish at $\partial D_* \to \infty$. Thus, the result is the expression

$$\frac{2}{\pi} \int_0^\pi |\delta\Phi_0(\theta)|^2 d\theta + 2\gamma \frac{h_0}{\alpha} |\delta\beta_0|^2 = 0 \tag{18}$$

which obviously leads to the conclusion that only for $\gamma = sign(h_0/\alpha) \equiv 1$ do we have $\delta\Phi_0(\theta) \equiv 0$ and $\delta\beta_0 = 0$, i.e. a unique solution.

Now, assume the scattered wave field H^{sc} to be produced by a collection of active or/and induced sources contained in a finite domain D_0 in D^+ with a density $\rho(\vec{r})$ (Fig. 1). Apply (17) to the functions H and G in $D_* \to \infty$ to obtain

$$H^{sc}(\vec{r}) = \int_{D_0} \rho(\vec{r'}) G(\vec{r}, \vec{r'}) \partial \vec{r'} + \int_{\partial D_* \to \infty} \left(H^{sc} \frac{\partial G}{\partial n} - G \frac{\partial H^{sc}}{\partial n} \right) d\vec{r'} \tag{19}$$

If we subject now the field H^{sc} to the same condition at infinity as the Green's function G, the second term vanishes. Due to linear character of integration, the far field must behave asymptotically as

$$H^{sc}(\vec{r}) \underset{r \to \infty}{\sim} \Phi(\theta) \left(\frac{2}{i\pi kr} \right)^{1/2} e^{ikr} + \begin{Bmatrix} \beta^+, & z > 0 \\ \beta^-, & z < 0 \end{Bmatrix} e^{-\alpha y + ih_0|z|} \tag{20}$$

with $\Phi(\theta) \to 0$ for $\theta \to 0$ or π.

3. EXTENSION TO MULTIMODE 2–D OPEN WAVEGUIDES

Let us extend the treatment to multimode open waveguides of greater practical interest. Assume that there is a lossless layered dielectric slab D_ϵ of thickness $2d$ and permittivity $\epsilon(y)$ sandwiched between two lossless halfspaces $D^\pm$ with permittivities $\epsilon_\pm$ as in Fig. 3. Function $\epsilon(y)$ is assumed to be of a step-constant type.

Suppose there is a finite number of cylindrical inhomogeneities and sources placed in parallel to the x-axis. With respect to this axis, any electromagnetic field can be decomposed into E and H-polarized ones. They may be completely characterized by either E_x or H_x components, and are treatable separately.

We start from the problem of seeking the E and H-type Green's functions of the slab. Using Fourier-transform in z-domain, we can reduce it to 1-D problem for ordinary differential equation like (8) but with a step-constant coefficient. Subjecting its general solution to boundary conditions and condition like (9) off D_ϵ we can obtain the Fourier-transform explicitly, as a meromorphic function of h on the four-sheet Riemann surface of $g_+(h) + g_-(h)$, where $g_\pm^2 = k^2\epsilon_\pm - h^2$,

$$F(y, y', h) = f(y, y', g_-, g_+)D(g_-, g_+)^{-1} \tag{21}$$

Here, expression $D(h) = 0$ stands for eigenvalue (dispersion) equation, the residues of (21) solving a source-free problem. All the poles of $F(h)$ are known to be simple and have no accumulation points. The "proper" sheet C_h^{11} is defined by assuming $Img_+ > 0, Img_- > 0$, transform (21) being a decaying function of $|h| \to \infty$ in C_h^{11}: $|F(h)| < Ce^{-Img_\pm|y|}$.

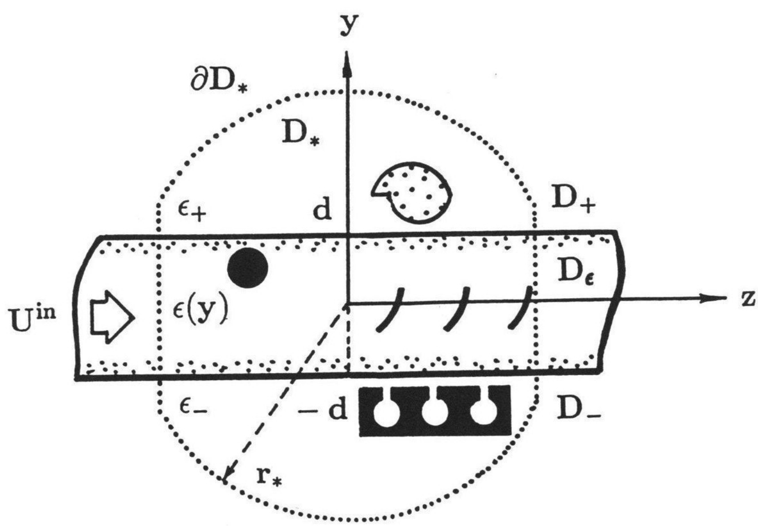

Figure 3. Two-dimensional geometry for a dielectric slab open-waveguide scattering.

At any fixed frequency the slab is known to support a finite number $Q \geq 1$ of natural modes of TE and TM type, nondecaying along the z-axis, namely

$$[E_x \, or H_x \equiv] U_{\pm}(\vec{r}) = V_p(y) e^{\pm i h_p z} \tag{22}$$

where $h_p : k \, inf \epsilon \leq h_p < k \, sup \epsilon$ and $V_p(y)$ are the proper real eigenvalues (simple real zeros of $D(h)$ in C_h^1) and eigenfunctions of the cross-sectional problem. These functions decay away from the slab according to (9), and are known to satisfy the orthogonality relation

$$\int_{-\infty}^{\infty} w(y) V_p(y) V_q(y) dy = \delta_{pq} N_p^2 \tag{23}$$

where N_p^2 is the norm of the mode, and the weight function $w(y)$ is 1 for TE modes and $\epsilon^{-1}(y)$ for TM modes. To prove this relation, it is enough to use the Green's formula (17) with weight $w(y)$ for functions V_p, V_q in a rectangular domain, and pass to a limit $y_* \to \infty$.

Assume that there is no pole at a branch point, i.e. $h_p \neq k_{\pm}$ for any p. Retracing all the derivations from the impedance-plane case and taking into account (23), we come to a generalized 2-D condition of radiation

$$U^{sc}(\vec{r}) \underset{r \to \infty}{\sim} \begin{Bmatrix} \Phi^{\pm}(\theta)(i\pi k_{\pm} r/2)^{-1/2} e^{ik_{\pm} r}, |y| > d \\ 0, \quad |y| < d \end{Bmatrix}$$

$$+ \sum_{q=1}^{Q} \begin{Bmatrix} \beta_q^+, z > 0 \\ \beta_q^-, z < 0 \end{Bmatrix} V_q(y) e^{i\gamma_q h_q |z|} \tag{24}$$

Here, $\gamma_q = sign(N_q^2)$. One can see from (23) that at least while $\epsilon(y) > 0$ (which covers true dielectrics and some models of plasma media), $N_q^2 > 0$, so $\gamma_q = 1$ for all modes of both types. In other words (24) has the same meaning as the Sommerfeld condition: '*In the scattered field, only outgoing waves exist*'. However the field may now contain nonuniform plane waves, i.e. proper natural (guided) modes besides a cylindrical wave. One should note that unlike the case of the impedance plane, the slab waveguide Green function transform has also complex poles h_m (zeros of $D(h)$). Actually, there exists an infinite number of poles in the "improper" sheets $C_h^{2,3,4}$ travelling towards $\pm k_{\pm}$ as the frequency is increasing. However, they give either exponentially small contribution at large r (as for a leaky-wave pole trapped within a certain angular sector of θ), or even (as for a so-called open-mode pole in the real axis of the "improper" sheet, never trapped by contour deformation) negligible at $r > 10/|k \cos\theta - h_m|$ independently of θ (see [1,7]).

It can be shown that $\Phi^{\pm}(\theta)$ is formed by a product of $\sin\theta$, and a function nonvanishing at $\theta \to 0$ or π. So, the far field pattern always has nulls at grazing directions, the power being carried by the guided field.

Now assume that the p-th guided natural mode is incident from $z = \pm\infty$ on a localized irregularity inside or near the slab. Let us obtain the relation for power conservation. To this end use (17) again, with weight $w(y)$, apply it to the functions $U = U_p + U^{sc}$ and U^* within appropriate domain $D_* \to \infty$ (see Fig. 3), and take account of far-field behaviour (24) and orthogonality (23). For convenience, we replace here β_q^+ by $T_{qp} - \delta_{qp}$, and β_q^- by R_{qp}, as they stand now for mode conversion coefficients. After integrating we obtain

$$N_p^2 = \sum_{q=1}^{Q} N_q^2 \left(|T_{qp}|^2 + |R_{qp}|^2\right) + \sigma_p^{sc} \tag{25}$$

where

$$\sigma_p^{sc} = \frac{2}{\pi}\int_0^{\pi}\left[\frac{1}{k_+}|\Phi_p^+(\theta)|^2 + \frac{1}{k_-}|\Phi_p^-(-\theta)|^2\right] d\theta \tag{26}$$

stands for the scattering cross-section of the obstacle. Index p attached to mode amplitudes and far-field patterns recalls that these quantities depend on the type of the incident mode.

The other relation of interest is reciprocity which couples mode conversion coefficients for two mirror-opposite positions of the obstacle. Let us use the Green's formula once more, for the functions $U^+ = U_p + U^{sc+}$ and $U^- = U_{-q} + U^{sc-}$. Following a procedure similar to the previous one, we arrive at the expression

$$N_q^2 T_{qp}^+ = N_p^2 T_{pq}^- \tag{27}$$

Note that setting $p = q$ we conclude that the transmission, i.e., the amplitude and phase of the incident mode in forward direction, does not depend at which end of the inhomogeneous section is the input.

4. SCATTERING IN 3-D OPEN WAVEGUIDES OF FINITE CROSS-SECTION

In 3-D electromagnetic scattering from *finite* objects, the vector analog of Sommerfeld's radiation condition (1) is known as Silver-Müller condition after [9]. Although being referred to in a number of papers on open waveguides, it obviously fails here because guided field (even the longitudinal components) does not decay along the guide axis.

Consider now a model of 3-D open waveguide formed by a finite number of infinite but regular dielectric rods $D \times z$ placed in lossless free space, as shown in Fig. 4. An observation point is characterized by the vector $\vec{R} = (\vec{r}, z)$. Permittivity $\epsilon(\vec{r})$ is again assumed to be a step-constant function of cross-section (but not z), equal to 1 outside the guiding region D. Cross-sectional contour ∂D is supposed to be a simple smooth curve.

As before, the key problem is associated with time-harmonic point-source excitation. Depending on the type of source (electric or magnetic) we introduce two 6-component Green's functions $\vec{G}^{e,m}(\vec{R},\vec{R}') = \{\vec{E}^{e,m}(\vec{R},\vec{R}'), \vec{H}^{e,m}(\vec{R},\vec{R}')\}$. On determining these vector-functions, one can obtain the dyadic Green's function of the open waveguide. Functions $\vec{E}^{e,m}, \vec{H}^{e,m}$ are singular at $\vec{R} = \vec{R}' \equiv (\vec{r}', 0)$, and satisfy Maxwell's equations with continuity conditions at the surface of a guiding region. Our aim is to obtain a 3-D counterpart of expression (24).

However, an attempt to follow the 2-D case faces the problem that Green's functions are not available explicitly. Indeed, we may still use Fourier-transform in z-domain. More accurately, it is enough to assume that $\vec{G}^{\alpha}(\vec{R}), (\alpha = e, m)$ exists and increases with $|z| \to \infty$ no more rapidly than some power of $|z|$, to be sure that the Fourier-transform

$$\vec{F}^{\alpha}(\vec{r}, \vec{r'}, h) \equiv \{\vec{U}^{\alpha}, \vec{V}^{\alpha}\} = \int_{-\infty}^{\infty} \vec{G}^{\alpha}(\vec{R}, \vec{R'}) e^{-ihz} dz \tag{28}$$

exists as a function of h, at least in the sense of distributions. The latter remark means that it may have pole singularities at the integration path of the inverse transform, i.e., on the real h-axis. Thus the integration in

$$\vec{G}^{\alpha}(\vec{R}, \vec{R'}) = \frac{1}{2\pi} \int_{C(-\infty,\infty)} \vec{F}^{\alpha}(\vec{r}, \vec{r'}, h, g) e^{ihz} dh \tag{29}$$

is actually to be performed in a generalized sense, along contour C bypassing the poles.

To analyze far-field behaviour, we need certain properties of the integrand of (29) to hold, namely, it must be at most a meromorphic function decaying at $|h| \to \infty$ in a "proper" sheet of the domain of analytic continuation. Before, in the 2–D case, these properties were evident, now we must prove them.

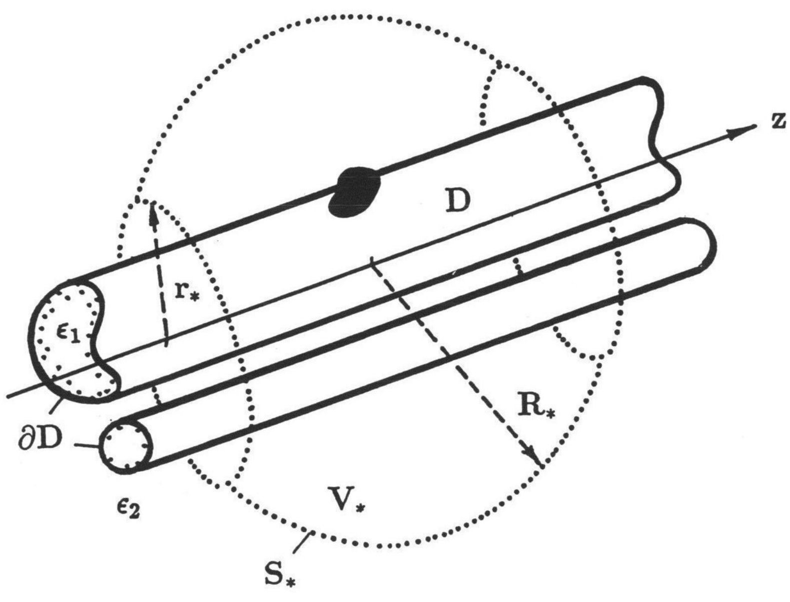

Figure 4. Three-dimensional geometry for a dielectric core open-waveguide scattering.

The functions $\vec{U}^\alpha, \vec{V}^\alpha$ satisfy a boundary-value problem for reduced harmonic Maxwell equations in the plane of cross-section. These equations can be combined in known ways to produce 2-D Helmholtz ones, with the operator $\nabla^2 + k^2\epsilon(\vec{r}) - h^2$. The most important point for us is that the cross-sectional problem is of conventional diffraction-theory type, as it does not involve any infinite boundary. Thus the unique solution may be extracted by the 2-D Sommerfeld condition of radiation if $|h| < k$, or the condition of decay if $|h| > k$. Combining these two cases, we can follow Reichardt [10] and request

$$\vec{F}^\alpha(\vec{r}, h) \equiv \{\vec{U}^\alpha, \vec{V}^\alpha\} = \sum_{n=-\infty}^{\infty} \{\vec{a}_n^\alpha, \vec{b}_n^\alpha\} H_n^{(1)}(gr)e^{in\phi}, \qquad r \geq a \tag{30}$$

to be valid uniformly for all ϕ, where cylindrical coordinates (r, ϕ) have the origin on the axis of circular cylinder of radius a containing all the elements of the guide, and $g^2 = k^2 - h^2$.

Expression (30) is based, of course, on the fact that the 2-D Green's function is given by G^0 (5), and the addition theorems for cylindrical functions. What is remarkable, is that it serves as analytic continuation of (1) at $\nu = 2$ to all complex g [10]. It also reveals that the domain of analytic continuation of the Fourier-transform is more complicated than in the case of 2-D open waveguides. Now it is a set in the Riemann surface of the function $Ln(k+h)(k-h)$. The latter has two branch points $h = \pm k$ of logarithmic type, and an infinite number of complex sheets. The "proper" one C_h^{01} is specified by conditions $-\pi/2 < \arg\gamma(h) < 3\pi/2, \operatorname{Im} g(h) > 0$, while the "conjugate improper" sheet C_h^{02} differs by a request that $\operatorname{Im} g(h) < 0$, and other "improper" ones $C_h^{n1,n2}$, $n \neq 0$ by adding $2\pi n$ to $\arg g(h)$.

Further treatment is based on two fundamental results.

First, Green's functions' Fourier-transforms $\vec{F}^\alpha(\vec{r}, \vec{r'}, h), (\alpha = e, m)$ do exist as unique functions of h at $|h| < k, arg g(h) = 0, \pi$. Moreover, they may be continued analytically onto the whole "proper" sheet C_h^{01} and onto all the other "improper" sheets of the $Ln(k-h)(k+h)$, except for a discrete set of poles. The latters are of finite multiplicity and have the only point of accumulation at infinity. Besides, $|\vec{F}^\alpha(h)| < Ce^{-Im gr}$ as $|h| \to \infty$ in C_h^{01}.

Second, the residues at these complex poles do solve a source-free problem, and hence represent generalized natural guided modes of the open waveguide, i.e.,

$$\operatorname*{Res}_{h=h_{\pm p}} [\vec{F}^\alpha(\vec{r}, \vec{r'}, h)e^{ihz}] \equiv \vec{W}_{\pm p}(\vec{R}) = C(\vec{r'})\vec{w}_{\pm p}(\vec{r})e^{\pm ih_p z}$$
$$\vec{W}_p \equiv \{\vec{E}_p, \vec{H}_p\}, \qquad \vec{w}_p \equiv \{\vec{U}_p, \vec{V}_p\} \tag{31}$$

for a simple pole. In the case of an m th-order pole, the residue extracting procedure, besides of a natural mode, results in a finite chain of ***associated*** guided modes (see a remark before (28))

$$\vec{W}_p^{(n)}(\vec{r}, h_p) = \frac{1}{n!}\frac{d^n\vec{W}_p(\vec{r}, h)}{dh^n} \underset{z\to\infty}{\sim} z^n e^{ih_p z}, \quad n = 1, 2, ..., m-1 \tag{32}$$

The proof involves reducing the 2-D problem for Fourier transforms to a set of the Fredholm 2-nd kind integral equations over the contour ∂D, with smooth enough kernels. The latters are of Müller's type [9], i.e., depend on combinations of two free-space Green's functions: $G^0(g)$ and $G^0(g_\epsilon), g_\epsilon^2 = k^2\epsilon - h^2$. We do not write this set here to save the space but note that in operator notation, it can be written as a regularized equation $[I + T(h)]\overline{X} = \overline{X}_0$, in the Hilbert space of $L^2(\partial D)$. Here $\overline{X}$ stands for a vector of four boundary values of transformed tangential field components, I is identity operator, and $T(h)$ is a compact operator.

Studying the analytic properties of operator $T(h)$ and taking account its invertibility for $|h| < k$ in the real h-axis of C_h^{01}, we arrive at the final result, guided by the operator Fredholm theorems of Steinberg [11].

Further investigation reveals that real poles on C_h^{01} can exist (actually, do exist) only if all the guide's elements are lossless, i.e. $Im\epsilon = 0$, and $Re\epsilon > 1$. Due to the discreteness, corresponding wavenumbers h_p form a finite set at the interval $k < |h| < k\sup\epsilon$. Such poles represent proper natural (guided) modes with fields decaying exponentially outside the guide, as it is clear from (30). Guided modes are known to be orthogonal in the cross-section, that is

$$\int_0^\infty \int_0^{2\pi} (\vec{U}_q \times \vec{V}_p^*)\vec{z}\, r\, dr\, d\phi = \delta_{qp} N_p^2 \tag{33}$$

where the number N_p^2 is the norm of the mode. For other (complex) modes with $h_m \in C_h^{01}$ this number is identically zero. The proof is based on Green's vector formula (known also as the Lorentz Lemma in the theory of Maxwell equations),

$$\int_V (\vec{A} rotrot \vec{B} - \vec{B} rotrot \vec{A}) d\vec{R} = \oint_S (\vec{B} \times rot\vec{A} - \vec{A} \times rot\vec{B})\vec{n}\, d\vec{R} \tag{34}$$

applied to functions $\vec{W}_q, \vec{W}_p^*$ in a circular cylinder of radius $r_* \to \infty$, terminated by two planes normal to the z-axis.

These properties of Green's functions' Fourier transforms, and of guided modes, enable one to arrive at a final expression very similar to (14) but in 3-D space. Further, it is due to this expression that for any two fields the far-zone integration gives a vanishing result:

$$\lim_{S_* \to \infty} \oint_{S_*} [(\vec{E}_1 \times \vec{H}_2) - (\vec{E}_2 \times \vec{H}_1)] d\vec{r} = 0 \tag{35}$$

So, provided that all guided-mode poles are simple and lie off the branch points, the 3-D condition of radiation for a scattered field $\vec{W}^{sc} \equiv \{\vec{E}^{sc}, \vec{H}^{sc}\}$ is obtained as

$$\vec{W}^{sc}(\vec{R}) \underset{R\to\infty}{\sim} \left\{ \begin{matrix} \vec{\Phi}(\phi,\theta) R^{-1} e^{ikR}, r > a \\ 0, \quad r < a \end{matrix} \right\} + \sum_{q=1}^{Q} \left\{ \begin{matrix} \beta_q^+ \vec{w}_{+q}(\vec{r}), z > 0 \\ \beta_q^- \vec{w}_{-q}(\vec{r}), z < 0 \end{matrix} \right\} e^{i\gamma_q h_q |z|} \tag{36}$$

where again $\gamma_q = sign(N_p^2)$.

To prove uniqueness, one should apply formula (34) to the difference between any two solutions, in a finite domain $V_* \to \infty$ bounded by a closed spherical-terminated surface S_* as shown in Fig.4. Again the passing to the limit is arranged so that $R_*, r_* \to \infty$ but $r_*/R_* \to 0$.

Note, however, that now the numbers N_q^2 are not always positive, even if $\epsilon > 0$, hence γ_q are not necessarily equal to 1. As a matter of fact, there exist examples of backward-power-carrying modes on 3-D dielectric waveguides. Any such a mode can be only a hybrid one (neither TE nor TM). So, the condition of radiation (36) has a more general meaning than (1), namely, *'In the scattered field, only waves carrying power to infinity exist'.*

Suppose now there is a localized irregularity in an open 3-D waveguide, and a guided natural mode of p-th number is incident from $z = +\infty$. In this case we replace in (36) coefficients of mode excitation $\beta_q^{\pm}$ by mode conversion coefficients $T_{qp} - \delta_{qp}, R_{qp}$. Now we use again the Lorentz Lemma in $V_* \to \infty$, for the function $\vec{W} = \vec{W}_p + \vec{W}^{sc}$ and its complex conjugate, or for the functions $\vec{W}^+ = \vec{W}_p + \vec{W}^{sc+}$, and $\vec{W}^- = \vec{W}_{-q} + \vec{W}^{sc-}$, to arrive at the following expressions, respectively

$$\gamma_p N_p^2 = \sum_{q=1}^{Q} \gamma_q N_q^2 (|T_{qp}|^2 + |R_{qp}|^2) + \sigma_p^{sc} \tag{37}$$

where the scattering cross-section is given by

$$\sigma_p^{sc} = \frac{1}{k^2} \int_0^{\pi} \int_0^{2\pi} (|\Phi_p^{Ez}(\phi,\theta)|^2 + |\Phi_p^{Hz}(\phi,\theta)|^2) \sin\theta d\theta d\phi \tag{38}$$

and

$$\gamma_q N_q^2 T_{qp}^+ = \gamma_p N_p^2 T_{pq}^- \tag{39}$$

These expressions serve as 3-D counterparts of equations (25)-(27). However, they need certain alterations if an associated guided mode (due to a multiple pole) exists.

5. ON THE PRINCIPLE OF LIMITING ABSORPTION

As it was pointed out before, the Limiting Absorption Principle (3) must be separately justified for open waveguide scattering. Its validity follows neither from free-space nor from closed-waveguide scattering treatments.

The clue lies in studying the analytic properties of poles $h_q(k,\epsilon)$ of the Green's function transform as functions of wavenumber k or permittivity ϵ. Again, in 2-D case the transforms are available explicitly, therefore below we consider the more difficult case of the 3-D arbitrary dielectric rod of Section 4.

Assume that $k' = k + i\tilde{k}, \tilde{k} > 0$. Integral equations identical to the lossless case can be obtained again, but now with a complex k'. However, using formula (34), it can be verified that now all the poles of the transform functions in C_h^{01} are complex. The theory developed above gets simpler as no guided-mode residues appear. Further, Steinberg's theorems [11] enable us to conclude that the poles $h_p(k')$ are analytic functions of k', except the values where the poles coalesce. It means that the limit of $\Delta h/\Delta k'$ at $\Delta k' = i\tilde{k} \to 0$ exists at any point of analyticity, and does not depend on $\Delta k'$. In other words there exists a real number $v_p \equiv (dh_p/dk)^{-1}$ known as *group velocity.* Formula (34) applied to a particular mode $\vec{W}_p$ having $h_p \in C_h^{01}$ reveals that, for poles approaching the real h-axis at $\tilde{k} \to 0$,

$$v_p = N_p^2 [\int_0^\infty \int_0^{2\pi} (\epsilon |\vec{U}_p|^2 + |\vec{V}_p|^2) d\vec{r}]^{-1} \tag{40}$$

otherwise $v_p \equiv 0$.

Expression (40) shows that if $\tilde{k} \to +0$ the poles forming the set of guided modes in the lossless limit, they deform the inverse Fourier-transform integration contour C in exactly the same manner as required by the condition of radiation (36). Further, Green's formula offers a way to show that no real value of k can be an eigenvalue of Maxwell's equations. This is enough to validate the Limiting Absorption Principle and its equivalence to the Principle of Radiation realized in terms of power flow.

A similar treatment can be developed for the parameter ϵ instead of k. It is worth to note that based on (40) and its counterpart for $dh_p/d\epsilon$, one comes to the conclusion that

$$\gamma_p = sign(v_p) = sign(dh_p/dk) = sign(dh_p/d\epsilon) \tag{41}$$

This observation removes the problem of interpretation of the radiation condition (36), for a linear problem, in terms of nonlinear quantities N_p^2.

6. DISCUSSION AND NUMERICAL EXAMPLE

From a practical viewpoint, the relations obtained serve as convenient tools for checking the correctness of the approach and the accuracy of computations. What is important is that they are valid for an arbitrary open waveguide containing arbitrary discontinuity of a compact nature. Indeed, our derivations involved only those equations which specify field behaviour far from the irregular part of the guide.

In addition, they can help to reduce the volume of computations in certain situations. For example, if the problem of finding the transmission coefficients for all Q modes propagating in $+z$ direction has been solved, then similar quantities for propagation in $-z$ direction are also known automatically from (39). Furthermore, (37) can be used to determine the scattering losses characterized by σ_p^{sc} without integrating the far-field pattern, provided that mode conversion coefficients have been calculated.

The power conservation equation may also be transformed into the so-called *Optical Theorem.* The latter was first discovered in free-space scattering of plane waves from localized obstacles (for a simple derivation based on the condition (1) see [12], p.98). For the scattering from an obstacle in a closed conducting-wall waveguide the power conservation relation differs from (37) by the absence of the last term, i.e., the radiation term. One can easily extract from it the value of "extinction" due to the presence of the scatterer

$$\sigma_p^{ext} \equiv \sum_{q=1}^{Q} N_q^2 (|T_{qp} - \delta_{qp}|^2 + |R_{qp}|^2) = -2N_p^2 Re(T_{pp} - 1) \tag{42}$$

As for open waveguides, although the modified condition like (36) was first discussed in [4], the power conservation equation had been in use before, based of physical intuition. Thus, for impedance plane and slab scattering it was treated in [13,14], and for a circular fiber in [15], respectively. For grounded-slab geometry and under TE mode incidence this relation was derived also in [16] in a more regular way based on the discrete+continuous -spectrum representation. However, the approach of [16] fails in the general 3-D case because it lacks a rigorous procedure of obtaining continuous spectrum eigenmodes. Morita [17] was the first to transform the power conservation relation into the *Optical Theorem*, that is the same as (42) but with σ_p^{ext} replaced by $\sigma_p^{ext} + \sigma_p^{sc}$. After (37) it is clear that this result is true under the assumption that all the guided modes are forward-power-carrying ones.

The reciprocity relation for 2-D open waveguides was used by Tanaka, *et al.,* in [18], based on physical intuition. Probably, the correct proof was first reported in [5] (see also [19]). Again, it has a well-known analogue in free-space diffraction: the forward scattering amplitude is the same for two mirror directions of plane wave incidence (see [12], p.102). In closed-waveguide scattering, a similar property is even more obvious, but a correct proof needs the condition (2) plus orthogonality of modes.

The reciprocity has a remarkable consequence for scatterers having a plane of symmetry. Assume α to be the angle between the plane of symmetry and the cross-sectional plane of the waveguide. Then obviously $T_{qp}^{\pm} = T_{qp}^{+}(\pm\alpha)$, and from (39) it follows that one must have $T_{pp}^{+}(\alpha) = T_{pp}^{+}(-\alpha)$, unlike the rest mode conversion coefficients at $+\alpha$ and $-\alpha$.

A note should be made on the comparison between open and closed waveguides. In the latter case one can easily combine power conservation and reciprocity relations and conclude that in a closed single-mode waveguide not only the transmission but also the absolute value of reflection is the same for two opposite directions of mode incidence. In an open waveguide this is not the case.

As an example, let us consider the scattering of a $TM_p (p = 0, 1, ...)$ guided mode of a dielectric slab waveguide caused by a circularly curved perfectly conducting screen (Fig. 5). The corresponding problem is obviously H-polarized. The accurate approach to solving such problems was reviewed in [19]. It is based on using the Green's function of the slab, reducing the problem to dual series equations, and applying the partial inversion procedure. The resulting matrix equation is of the Fredholm 2-nd kind, and can be solved numerically to any desired accuracy.

Fig. 6 shows the dependences of mode conversion coefficients on the relative radius a/d of the screen placed at the center of the two-mode slab. The resonant phenomena observed are due to excitation of the screen's damped natural oscillations. The resonance at $a/d \approx 0.16$ corresponds to $ka \approx 0.36$ and is associated with so-called low-frequency, or the Helmholtz, mode H_{00} (see [19]), for which $k^2a^2\epsilon \approx -1/2 \ln^{-1}\cos(\delta/2)$. As the screen is positioned symmetrically with respect to the slab's interfaces, the even and odd guided modes excite separate splitted families of the screen's resonances. They are $H_{00}, H_{11}^{+}, H_{21}^{+}$ ..., or $H_{11}^{-}, H_{21}^{-} \ldots$, depending on the even/odd nature of the incident mode.

Note that unlike the transmission, the reflection coefficient exhibits quite a different behaviour for two mirror-opposite positions of the screen. More numerical results on this problem and similar ones concerning finite periodic gratings of screens in a slab may be found in [20,21]. Actual agreement between $|T_{jj}^{+}|$ and $|T_{jj}^{-}|$ (i.e., the reciprocity) was within 7 and more digits in that analysis. Of course, this was achieved due to analytical inversion of a part of the dual series operator. Another example can be found in [18], where the domain integral equation with extracted guided-mode contribution was applied to study a slab waveguide branching. The power conservation relation was satisfied within 3 digits while reciprocity was satisfied within only 1 digit. This forced the authors to conclude about the necessity of improving the MoM-based numerical algorithm. So, the check of reciprocity seems to be more fine than that of power conservation. That is why 3-4 digits for the latter achieved in [17] may be not quite enough for validating the results. However, the reciprocity test cannot be used for symmetrical discontinuities.

Another important note should be made about the fact that both power conservation and reciprocity are necessary but not sufficient, as they have far-field nature. Sufficiency test can be performed only by checking the accuracy of satisfying the near-field boundary conditions.

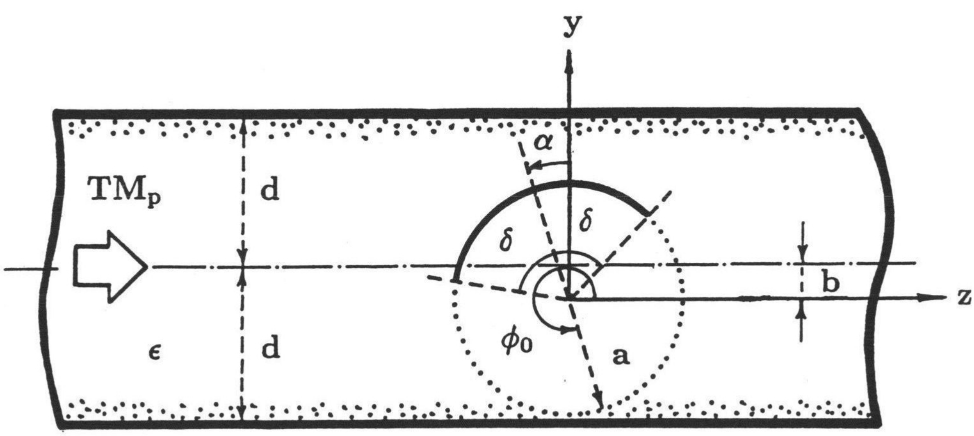

Figure 5. A particular 2-D geometry for a circular screen inside a dielectric slab. One of the natural guided modes is incident.

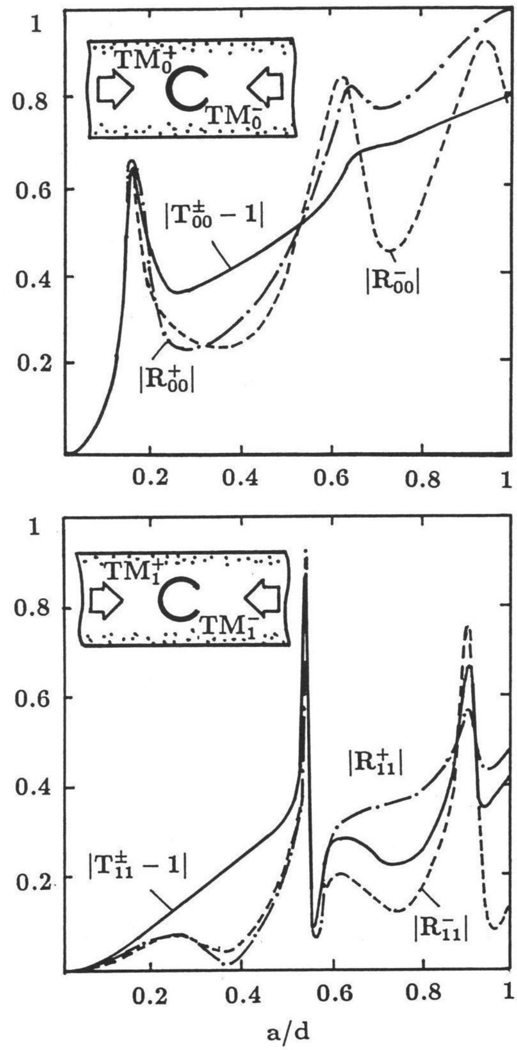

Figure 6. Amplitudes of the scattered field coefficients for a cavity shaped screen inside the two-mode slab. Incident mode is travelling either in the positive or negative z-direction. $kd = 2.3, \epsilon = 2.25, b = 0, \theta = 30^o, \phi_0 = 0$.

7. ON THE EXTENSION TO MORE COMPLICATED OPEN WAVEGUIDES

1) Cross-sectionally nonhomogeneous dielectric waveguides. As it has been demonstrated in Section 4 for 3-D dielectric guides of step-constant ϵ, a rigorous far-field analysis is ensured by reducing the key problem to a Müller-type *contour* integral equation, i.e., a regularized operator equation. A similar technique involving *domain* integral equations was developed in [22] for treating 2-D and 3-D waveguides with parameters being functions of the cross-section (Fig. 7a).

Provided that the function $\epsilon(\vec{r})$ is continuously differentiable, $\lim_{\vec{r}\to\partial D} \epsilon(\vec{r}) = 1$ (i.e., there is no sharp boundary), and strictly positive, a Müller-type integral equation within D, with a weakly-singular kernel is obtained. Thus, it forms a Fredholm 2-nd kind operator equation in the Hilbert space $L^2(D)$. As the kernel is an analytic function of wavenumber h in the Riemann surface of same function $Ln\, g(h)$ as before, the whole theory developed above is valid here as well, resulting in identical expressions for the far field behaviour.

The prerequisite of the absence of sharp boundary seems to not be critical, due to a recent analysis of Viola and Nyquist [23].

2) Combined metal-dielectric waveguides. The next step is to treat open waveguides composed of both dielectric and metal rods and/or zero-thickness surfaces (strips) of finite cross-section (see Fig. 7b). The corresponding analysis done in [22] reveals that regularized integral equations in $L^2(\partial D)$ or $L^2(D)$ may be obtained in a similar way. However, the kernels of these equations are now determined not by free-space Green's function $G_0(\vec{r}, \vec{r'}, g)$ but by certain Dirichlet and Neumann-type Green's functions $G_E(\vec{r}, \vec{r'}, g)$, $G_H(\vec{r}, \vec{r'}, g)$. The latter correspond to free-space 2-D problems of a line-source excitation of a *metal* structure obtained by setting $\epsilon(\vec{r}) \equiv 1$.

After Müller [9] and Reichardt [10], for smooth bodies, $G_E(g)$, $G_H(g)$ are known to exist in the Riemann surface of $Ln\, g$, like $G_0(g)$. What is different, is that they are no more holomorphic functions but *meromorphic* ones, i.e. they have a discrete set of poles, of finite multiplicity. However, this is quite enough to use operator Fredholm theorems of [11] and arrive at the previous principal results. Moreover, after Muravei [24] the same properties are justified for the Green's function of 2-D problem of the 3-rd kind. It means that impedance boundary condition does not change anything essential. Finally, Sukhinin [25] had proved that the functions $G_E(g)$, $G_H(g)$ have the same features on the sheets of $Ln\, g$ in the 2-D problem associated with a finite number of smooth open curves (zero-thickness scatterers).

So, expressions (36)-(39) obtained above are valid for all open waveguides of finite cross-section shown in Fig. 6b. In [22], this gave a ground for a study of modal propagation and excitation in microstrip/slot lines on circular and circularly-wrapped substrates.

3) 3-D waveguides of infinite cross-section. Such waveguides as open microstrip/slot lines on infinite substrates (Fig. 7c) can be treated by using the double Fourier transformation in space domain [23]. In this case the structure of guided field is more complicated as the substrate itself may support a number of guided modes, nondecaying in any lateral direction. Then, under a localized excitation, these latter contribute to plane-cylindrical waves, so

$$\vec{W}^{sc} \underset{R\to\infty}{\sim} \begin{Bmatrix} \vec{\Phi}^{\pm}(\phi,\theta)R^{-1}e^{ik_{\pm}R}, r>a, |y|>d \\ 0, \quad r<a, |y|<d \end{Bmatrix}$$

$$+\sum_{q=1}^{Q} \begin{Bmatrix} \beta_q^+ \vec{w}_{+q}(\vec{r}), z>0 \\ \beta_q^- \vec{w}_{-q}(\vec{r}), z<0 \end{Bmatrix} e^{i\gamma_q h_q |z|}$$

$$+\sum_{n=-\infty}^{\infty}\sum_{p=1}^{P} \tau_{pn}\vec{V}_{pn}(y)H_n^{(1)}[\gamma_{pn}h_{pn}(x^2+z^2)^{1/2}]e^{in\theta} \tag{43}$$

Corresponding modifications should be made in the power conservation and reciprocity relations, due to the presence of the last term in (43). The domain D_* involved in the derivations is of more complicated shape than in the previous case, being bounded by a spherical surface terminated by two planes normal to the z-axis and also by a circular cylindrical one normal to the substrate plane.

4) Junctions of open waveguides. Bends, or more generally, branchings of open waveguides constitute a practically important class of models of irregular waveguides. They may be considered as compositions of a finite number of semi-infinite guides, so the result is obviously the superposition of expressions like (36) for each particular guide. For example, in case of 2-D bend or M-branching (see Fig. 7d), excited by the p-th guided mode from the j-th branch, the far field behaviour is given by

$$U^{sc}(\vec{r}) \underset{r\to\infty}{\sim} \begin{Bmatrix} \Phi_{pj}^{\pm m}(\theta)(i\pi k_{\pm m}r/2)^{-1/2}e^{ik_{\pm m}r}, r\in D_{\pm m} \\ 0, \quad |y_m|<d_m \end{Bmatrix}$$
$$+\sum_{m=1}^{M}\sum_{q=1}^{Q_m} \begin{Bmatrix} T_{pj,qm}, q\neq p, m\neq j \\ R_{pj,pj}, q=p, m=j \end{Bmatrix} V_{qm}e^{i\gamma_{qm}h_{qm}z_m} \tag{44}$$

Domain D_* involved in the proof of uniqueness and derivations of power conservation and reciprocity relations, must be tailored to suit the branching geometry. So, it is to be bounded by a circular curve terminated by M straight lines normal to each branch. Note that due to the Green's function behaviour, far-field scattering patterns $\Phi_{pj}^{\pm m}(\theta)$ have nulls in grazing directions along every branch of the junction.

Power conservation and reciprocity relations now take account of several channels carrying power to infinity.

5) 2-D periodic open waveguides. Let us now discuss a relative class of problems associated with not regular but regularly-periodic (with period l along z-axis) open waveguides. Such a guide can be formed by any periodic grating, such as grooves in a substrate, parallel bars, periodic material interfaces, etc. (Fig. 7e). Of course, whether or not such a guide can support any nondecaying mode of propagation across the grooves or bars, depends on the parameters of the particular problem. However, in principle, they can [26]. As it was pointed out in

[19], this fact must be taken into account when studying the problems of wave scattering from localized objects near a grating. Of course, the analysis can be partially simplified by assuming that the medium housing the scatterer, or the grating itself, is lossy and therefore no nondecaying guided mode can exist. This assumption was used, e.g., in [27] to treat a circular cylinder over a lossy sinusoidal halfspace. Besides, certain types of gratings, like a flat perfectly-conducting strip grating in free space, do not support guided waves at all. Such kind of structure was considered in the papers [28-30] on modeling open resonators formed by curved screens and strip gratings. In both cases this is enough to validate the use of the Sommerfeld radiation condition. However, if it is a guided mode itself which is incident from infinity along the grating, one must modify this condition to ensure uniqueness.

The treatment is similar to that used for regular open waveguides. The needed modifications include the generalized integral transformation of the Fourier type, but with transform function $F(h)$ periodically dependent on z, z'. This function is not available explicitly, like for regular dielectric-layered structure of Section 3. However, due to the validity of the Floquet expansions, for many particular gratings it may be reduced to the Fredholm matrix equations of the 2-nd kind. For example, this is true for gratings of plane strips on stratified substrates, of circular and polygonal bars, of echelette type, and others. Then, once more, the operator theory may be applied (see [31]) resulting in far field behaviour similar to (24). One important difference appears in the nature of analytic continuation of transform functions: the complex domain is here more complicated. It is the Riemann surface of the function $\sum_{n=-\infty}^{\infty} g_n(h)$, with $g_n^2 = k^2 - (h + 2\pi n/l)^2$, having infinite number of square-root branching points.

6) 3-D periodic open waveguides. Example of such kind of a waveguide is given by a periodic structure of metallic circular rings, a helical metal strip on a dielectric core, etc. (see Fig. 7f). Excitation of these waveguides by point sources was treated in a mathematically rigorous manner by Sologub [32], assuming a lossy surrounding medium. However, if considering the problems of guided mode conversion and scattering from localized obstacles, one should again modify the radiation condition at infinity. A thorough investigation reveals that transform functions are to be studied in the even more complicated Riemann surface of $\sum_{n=-\infty}^{\infty} Ln\, g_n(h)$. However, in this surface their analytic properties are essentially the same as for regular guides, so the corresponding expressions (36)-(39) are still valid.

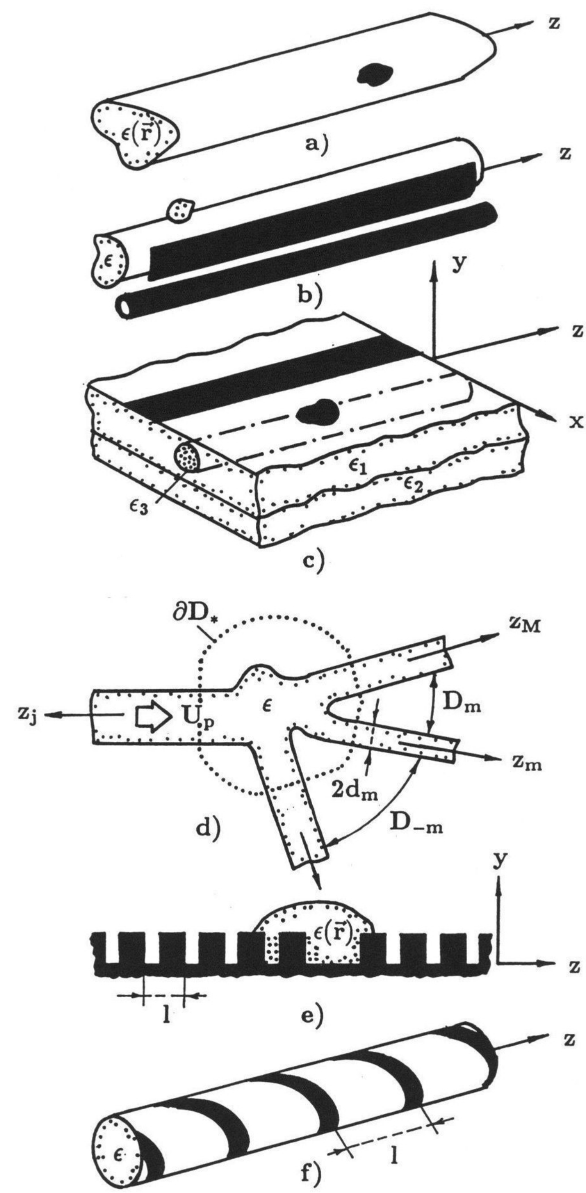

Figure 7. Examples of more complicated open waveguides containing localized inhomogeneities

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