Electromagnetic Properties of a Dielectric Slab with an Embedded Silver Nanowire Grating

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Abstract— Scattering of a plane wave from a silver nanowire grating embedded into a dielectric slab is investigated using Maxwellian formalism, to determine and distinguish effects caused by the structure periodicity and the presence of silver wires which entail plasmon resonances. Comparison of two polarizations is provided. Common use of the slab and the grating gives an opportunity to maintain the effectiveness of the plasmon resonance while the variation of the slab refractive index allows managing the wavelength of the plasmon resonance.

Index Terms—**Plasmon resonance; scattering; grating**

I. Introduction

Plasmonic effects on the nanometer scale attract growing interest in nanooptics due to wide range of applications. Recently nanoscale lasers were demonstrated in the form of random ensembles of spherical gold particles, each of tens of nanometers in diameter and coated with a spherical layer of dye-doped silica [1]. Because the output power of such lasers generally is proportional to the device size and as the noble-metal cores or shells have considerable losses, one can foresee that elementary nanolasers should be assembled in clusters or arrays. Here, periodical arrangement brings new physical phenomena in the form of structure resonances caused by the periodicity. In this paper we offer an investigation of a 1-D periodical problem of silver nanowires embedded in a dielectric slab.

A theoretical model of a grating of infinite dielectric cylinders placed periodically in free space and illuminated by a plane wave has been investigated as a fundamental electromagnetic problem since the 1950s – see, for instance, [2,3]. The high-Q resonances that occur near the Rayleigh anomalies were found numerically some time ago as can be concluded, for instance, from Figs. 2 and 3 in [3], however no attempt was done to explain them before approximate analytical analysis was performed in [4].

In [5], we have found that a quantum-wire grating possesses specific grating modes in addition to the more conventional natural modes of a single dielectric cylinder perturbed by the presence of other cylinders. These modes display counter-intuitive properties: (i) the resonance Q-factor gets higher if the grating gets sparser, i.e. for the larger ratios of period to cylinder radius and (ii) mode near-field is a characteristic wave standing along the grating and stretching over many periods in the normal to the grating direction [6]. Infinite extent of the structure, being idealisation of real scatterers, leads to good agreement with the scattering by a finite grating of many wires, as it was shown for a grating of silver cylinders [7]. Additionally the silver material brings the plasmon resonances, which are very different by their quality-factors and linkage to the particular frequency. Similar results have been published on the resonance sctattering and absorption by the gratings of silver nanostrips [8-10].

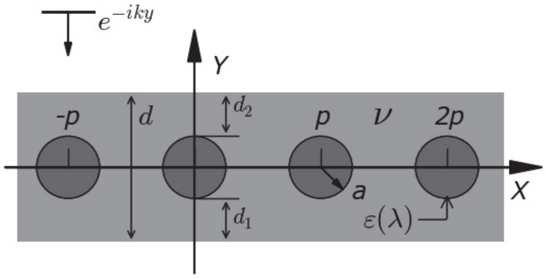

Fig.1 The cross-sectional geometry of a silver wire grating embedded in a dielectric slab.

In practical applications, silver wire nanogratings are frequently embedded in dielectric slabs. A slab with embedded nanowire grating of circular cylinders (Fig. 1) obviously possesses the above mentioned properties plus can support the slab modes, now perturbed by the nanowires. Further we build an accurate model of such structure to reveal the impact of the physical and geometrical parameters of the slab on the grating modes. A similar wave-scattering problem for purely dielectric structure was investigated in [11,12], however here we will concentrate our study of the scattering properties of the composite structure on the interaction between the resonances caused by the plasmon modes and the grating modes in dependence on the slab refractive index. We will also emphasize the possibility of enhancement of the latter mode electromagnetic properties by changing geometrical parameters of the slab.

II. Formulation and basic equations

Consider a grating of the parallel to the *z*-axis and periodic along the *x*-axis circular nanowires. The silver grating period is p, each nanowire is a cylinder of radius a, and its refractive index $\varepsilon^{1/2}(\lambda)$ is taken from the Johnson and Christy work [13]. This grating is embedded in a dielectric slab, whose refractive index is ν and thickness is $d = d_1 + d_2 + 2a$, where $d_{1,2}$ are distances between the grating and the upper and lower slab surfaces. Suppose that the

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electromagnetic field is time-harmonic $\sim \exp(-i\omega t)$ and does not vary along the *z*-axis, hence the field analysis problem is two-dimensional. Illumination by a plane wave incident from the upper half-space (in the negative direction of the *y*-axis) is investigated.

The function U, denoting the H_z component of the electromagnetic field in the *H*-polarization case and the E_z in the *E*-polarization case, must satisfy the Helmholtz equation with appropriate wavenumber inside and outside of cylinders, the Sveshnikov radiation condition at infinity, the condition of local integrability of power, and the boundary conditions demanding continuity of the tangential field components at cylinder boundaries [5-12]. The free-space wavenumber is $k=\omega/c=2\pi/\lambda$, where c is the free-space light velocity and λ is the wavelength, while inside the cylinders it is $k\varepsilon^{1/2}(\lambda)$ and inside the slab $k\nu$. Besides, in this paper we will also use the normalized frequency as the ratio of period to wavelength, $\sigma = p/\lambda = k\nu p/2\pi$.

The field inside the slab but outside the grating ($a<|y|<a+d_{1,2}$) and in the outer space ($|y|>d/2$) can be represented as Floquet series of the incoming harmonics $\{a_l^i e^{-i\tau_l\kappa^i y+i\pi_l k^i x}\}$ and the outgoing ones $\{b_l^i e^{i\tau_l\kappa^i y+i\pi_l k^i x}\}$, where $\pi_l = l/\sigma$, $\tau_l = i(\pi_l^2-1)^{1/2}$, and the index i corresponds to the domain so that $\kappa^1 = k$ in free space and $\kappa^2 = k\nu$ in the slab. Briefly the field and the incident wave can be written as a vector,

$$U^i = \begin{pmatrix} a^i \\ b^i \end{pmatrix}, \qquad U^{incident} = \begin{pmatrix} a^{incident} \\ 0 \end{pmatrix}. \tag{1}$$

The total scattering matrix A that characterises a slab with a grating has a block-type nature and can be presented as

$$\begin{pmatrix} t \\ 0 \end{pmatrix} = A\begin{pmatrix} a^{incident} \\ r \end{pmatrix}, \qquad A = \begin{pmatrix} A^{II} & A^{I} \\ A^{III} & A^{IV} \end{pmatrix}. \tag{2}$$

Here $r=\{r_l\, e^{i\tau_l\kappa y+i\pi_l kx}\}$ and $t=\{t_l\, e^{-i\tau_l\kappa y+i\pi_l kx}\}$ are the vectors of the reflected and transmitted Floquet harmonics, respectively. Then the reflectance and transmittance are expressed as

$$R(\sigma)=\sum_{|i|<\sigma}\tau_i\,|r_i|^2, \qquad T(\sigma)=\sum_{|i|<\sigma}\tau_i\,|\delta_i^0+t_i|^2. \tag{3}$$

The matrix A is built by successive multiplication of the matrices that characterise the transmission of the incident field (1) throughout each elementary scatterer,

$$A = F^1 S^1 G S^2 F^2, \tag{4}$$

where F^i are the transfer matrices for the upper and lower interfaces of the slab.

They can be expressed using well-known Fresnel matrices as

$$F^i = \begin{pmatrix} T^+ - R^-\left(T^-\right)^{-1}R^+ & R^-\left(T^-\right)^{-1} \\ -\left(T^-\right)^{-1}R^+ & \left(T^-\right)^{-1} \end{pmatrix}, \tag{5}$$

Further, S^i are the matrices of the phase shifts inside the slab,

$$S^i = \begin{pmatrix} S^+ & 0 \\ 0 & S^- \end{pmatrix}, \qquad S^{\pm}=\{e^{\pm ik\tau_s\nu_S d_i}\}, \tag{6}$$

and the matrix G is the transfer matrix of the field (1) through the grating. It can be obtained similarly to [12]. To build the vector of the reflected Floquet harmonics, we use equation (2),

$$r = -(A^{IV})^{-1}A^{III}a^{incident}, \tag{7}$$

$$t = A^{II}a^{incident} + A^{I}r. \tag{8}$$

Note that as our mathematical approach is similar to [8] then the resulting matrix equation (7) is the Fredholm second kind equation.

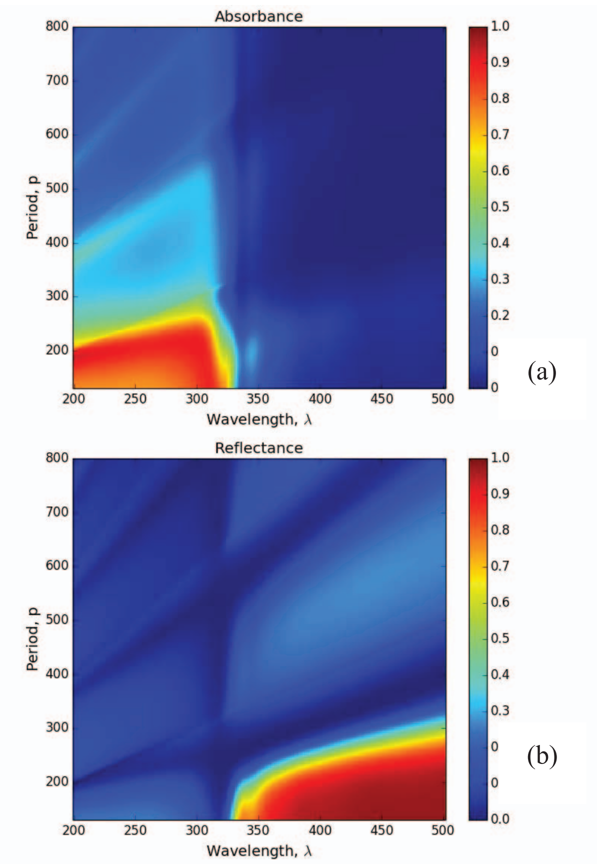

Fig.2 The reliefs of absorbance (a) and reflectance (b) from the dielectric slab $\nu = 1.2$ with embedded silver grating of cylinders with radii a= 60 nm depending of the wavelength and grating period from p=180nm to p = 800 nm. Slab width equal to the grating period. E-polarization.

III. Numerical results

Previous investigation of the silver-wire grating in free space has shown the presence of two different effects in its

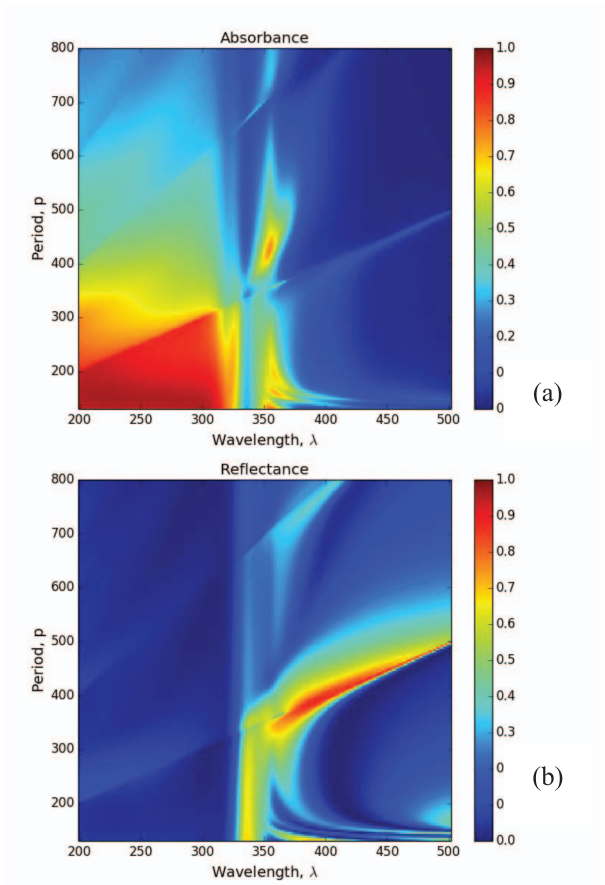

Fig.3 The reliefs of absorbance (a) and reflectance (b) from the dielectric slab $\nu = 1.2$ with embedded silver grating of cylinders with radii a= 60 nm depending of the wavelength and grating period from p=180nm to p = 800 nm. Slab width equal to the grating period. H-polarization.

electromagnetic properties: the plasmon resonances aligned to the specific wavelength λ=348 nm (in the H-polarization case) and the grating resonances that appear close to the wavelength equal to the structure period (in the both polarizations). When the silver grating is sparse and the period does not coincide with plasmon resonance their interaction is minimal. For such cases (e.g. radius is equal to 20 nm and period to 450 nm) the maximum reflectance might be achieved at the level of 0.15. Putting the silver grating in a dielectric slab gives opportunity to enhance its electromagnetic properties.

The reliefs of absorbance and reflectance for the structure with parameters a=60 nm, $p=d=2a+d_1+d_2$, and $d_1=d_2$, and refractive index $\nu = 1.2$ are shown in Fig.2, 3. For the E-polarization, the plasmon resonances do not exist however other resonances are present, apparently explained by the effect of slab modes, for example, on the relief of absorbance Fig.2 (a) near the wavelength λ=350 nm. The H-polarization is more fruitful for plasmonic effects. The absorbance values for both polarizations behave similarly for the wavelength λ<350 nm because of increased bulk losses in silver. At the plasmon wavelength for some values of the structure period it may reach the values close to 0.9. The reflectance also shows a high efficiency plasmon resonance at a slightly shifted wavelength. However the maxima of reflectance can be observed if the plasmon-mode resonance interacts with the resonance on the first-order grating mode.

IV. CONCLUSION

We have built an accurate mathematical model for the dielectric slab with an embedded silver nanowire grating. The structure has been investigated for reflectance and absorbance in the visible spectrum. The grating and the plasmon mode resonances have been visualised and explained. Dependences of the reflectance and absorbance on the structure period have been studied.

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