Plasmon Resonances of an Infinite Grating of Silver Wires Coated with Dielectric Envelopes

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ABSTRACT

We study the scattering of the H-polarized plane wave by a freestanding periodic grating of nanowires made of infinite silver cylinders coated with dielectric shells. Reflection and transmission coefficients for such grating demonstrate interaction of different types of resonances including plasmon and grating ones.

Keywords: gratings, scattering, absorption, plasmon, nanowires.

1. INTRODUCTION

A growing interest of today's nanooptics is concentrated at the devices that use plasmonic effects on the nanometer scale, with wide range of applications. Truly nanoscale lasers were recently demonstrated in the form of random ensemble of spherical gold particles, each of tens of nanometers in diameter and coated with a spherical layer of dye-doped silica [1]. Because the output power of such lasers generally is commensurable with device size and as the noble-metal cores or shells have considerable losses, one can foresee that elementary nanolasers should be assembled in groups or arrays. Such an arrangement brings new physical phenomena in the form of structure resonances caused by the periodicity.

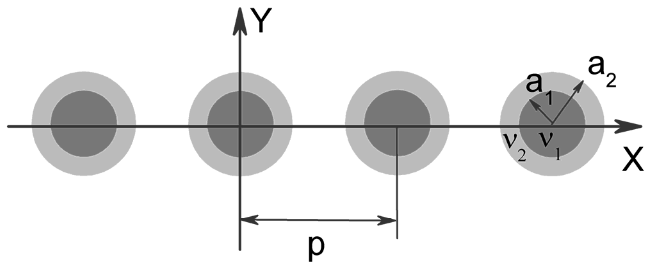

Fig. 1 The sketch of infinite periodic chain containing dielectric-coated silver circular cylinders.

Although the publications studying classical transmission and reflection of plane waves by the gratings of passive dielectric and metallic wires of the circular cross-section are numerous (for instance, see [2,3]), the first direct mention of the so-called grating resonances close to the Rayleigh anomalies occurred in [3]. In the further investigations of H-wave scattering by finite and infinite silver chains [4], a wideband enhanced reflection, at the frequencies between the plasmon resonance for one silver cylinder and the grating resonance at the wavelength equal to the period, was demonstrated. In general, by a proper tuning of these two resonances one can significantly broaden the band of the intensive reflection from the grating. More recently, a binary chain containing both dielectric and silver cylinders was investigated in [5]. Here, natural modes associated with the periodicity were studied and compared with similar modes of the pure dielectric lossless grating. This research showed that absorption dramatically affects these modes.

By the analogy to 3-D coated silver particle from [1], we investigate here a 2-D configuration consisting of a layered cylinder with a silver core and a dielectric coating, and assemble infinite number of them in a periodic grating where both plasmon and grating resonances are observed. The aim of this paper is investigation of impact of the coating on the plasmon resonances depending on physical and geometrical parameters. According to the surface nature of plasmon resonances, the scattering field intensifies on the outer boundary of noble metal, thus the dielectric media, in this geometry, overlaps with the field maximum.

For the silver nanowires, the values for refractive index of their material in the visible band are taken from [6] and interpolated using cubic splines.

2. PROBLEM FORMULATION

Consider a grating made of the parallel to the z-axis and periodic along the x-axis circular cylinders (wires) in free space – see Fig. 1. The period is p, each two-layered cylinder consists of the core of radius a_1 and refractive index ν_1 and the concentric coating of parameters a_2 and ν_2, respectively. We suppose that the electromagnetic field is time-harmonic $\sim \exp(-i\omega t)$ and does not vary along the z-axis, so the field analysis problem is 2-D one. Aiming at the study of plasmon impact on the considered grating, we investigate the scattering of the H-polarized plane wave incident normally from the upper half-space (along the negative direction of the y-axis). The function U, denoting the $Z_0 H_z$ component of the electromagnetic field, must satisfy the Helmholtz equation with appropriate wavenumber inside and outside of cylinders, the Sveshnikov radiation condition at infinity, the condition of local integrability of power, and the boundary conditions demanding continuity of the tangential field components at the cylinders' boundaries. The free space wavenumber is $k = \omega / c = 2\pi / \lambda$, where c is the free-space light velocity and λ is the wavelength, while inside the cylinders it is $k\nu_i$; besides, in this paper we will also use the normalized by period frequency, $\sigma = p / \lambda = kp / 2\pi$.

As we consider the normal incidence, then according to the Floquet theorem the scattered field is a periodic function of x with period p: $U(x, y) = U(x + p, y)$. Thus, the same conditions are required on each cylinder. In this case, we can investigate the field just within one cell of the grating and use the boundary conditions. The operator technique is applied, where construction of the composite T-matrix of two cylinders is performed similarly to [4]. Although the field in the cylinders' vicinity has the form of angular-exponent series with a linear combination of cylindrical functions in coefficients (inside the core the Bessel functions only and outside of cylinder the Hankel functions of the first kind only), such series are not a convenient tool for large values of the space variable $|y|$. Therefore, the Poisson summation formula is applied to cast the series into the exponentially convergent series in terms of the Floquet harmonics. Then the field function in the upper and lower half-spaces $|y| > d$, where the lines $|y| = d$ do not cross any of two cylinders, can be presented in the following way

$$U^+(x, y) = \sum_{s=-\infty}^{+\infty} f_s^+ e^{ik(\pi_s x + \tau_s y)}, \ y > d \ , \quad U^-(x, y) = \sum_{s=-\infty}^{+\infty} (\delta_0^s + f_s^-) e^{ik(\pi_s x - \tau_s y)}, \ y < -d \ . \tag{1}$$

where

$$\pi_s = s / \sigma, \qquad \tau_s = \sqrt{1 - \pi_s^{\ 2}} \tag{2}$$

The matrix equation for determining the scattered-field coefficients can be cast to the following form:

$$(I - J^{-1} \cdot T \cdot L \cdot J) \cdot P = T \cdot J^{-1} \cdot p^{inc} \tag{3}$$

with

$$L = [S_{m-n}(2\pi\sigma)] \ , \ S_l(x) = \sum_{p \neq 0} H_l^{(1)}(px) \tag{4}$$

$$p^{inc} = [(-1)^m] \ , \ J = [J_m(2\pi\sigma)\delta_n^m] \tag{5}$$

$$F^{\pm} = [(-i\pi_n \pm \tau_n)^m]/(\pi\sigma) \ , \ f^{\pm} = F^{\pm} \cdot J \cdot P \ , \tag{6}$$

here the matrix L consists of the lattice sums, which provide rapid way of the summation of the slowly converging Hankel function series, for details look in [7]. This matrix reflects the periodic nature of the grating as it depends only on the value p / λ. The vector p^{inc} represents the incident normal plane wave. The matrices $F^{\pm}$ transform the expansions in terms of the Hankel functions to the ones in terms of the Floquet series. The matrix T defines the aggregate *T*-matrix of two-layer cylinders in free space,

$$T = \left[-\frac{\nu_1^{-1} J_m(ka_1)\left(J_m'(k\nu_1 a_1) - t_m H_m'(k\nu_1 a_1)\right) - J_m'(ka_1)\left(J_m(k\nu_1 a_1) - t_m H_m(k\nu_1 a_1)\right)}{\nu_1^{-1} H_m(ka_1)\left(J_m'(k\nu_1 a_1) - t_m H_m'(k\nu_1 a_1)\right) - H_m'(ka_1)\left(J_m(k\nu_1 a_1) - t_m H_m(k\nu_1 a_1)\right)} \delta_n^m \right] \tag{7}$$

$$t_m = \frac{\nu_1 \nu_2^{-1} J_m(k\nu_1 a_2) J_m'(k\nu_2 a_2) - J_m'(k\nu_1 a_2) J_m(k\nu_2 a_2)}{\nu_1 \nu_2^{-1} H_m(k\nu_1 a_2) J_m'(k\nu_2 a_2) - H_m'(k\nu_1 a_2) J_m(k\nu_2 a_2)} \ , \tag{8}$$

The values of reflectance and transmittance can be obtained using (1) as follows:

$$R = \sum_{s<\sigma} \tau_s \, | f_s^+ |^2, \quad T = \sum_{s<\sigma} \tau_s \, | \delta_{0s} - f_s^- |^2, \tag{9}$$

and absorbance can be obtained from the Optical Theorem (power conservation) as $A = 1 - R - T$.

3. NUMERICAL RESULTS

The reliefs in Fig.2 show variation of the reflectance and absorbance of the grating with p = 400 nm for two kinds of gratings, the first is made of silver wires and the second of composite two-layer wires with the thickness of coating $a_2 = 20 + a_1$ and refractive index $\nu_2 = 1.4142$. In Fig. 2 (c) and (d), one can see the interaction of two kinds of resonances. The first, observed near the wavelength of $\lambda \approx 340\,\mathrm{nm}$, has plasmonic nature. Another appears at the wavelength slightly larger than the period, *p*. If merged together, they lead to a homogeneous wide-band reflectance (more results on this effect, with a deeper discussion, can be found in [4]).

The most interesting reliefs are presented in Figs. 2 (a) and (b). They show the reflectance of the grating made of two-layer cylinders, as a function of the wavelength and the radius of the silver core with a fixed shell

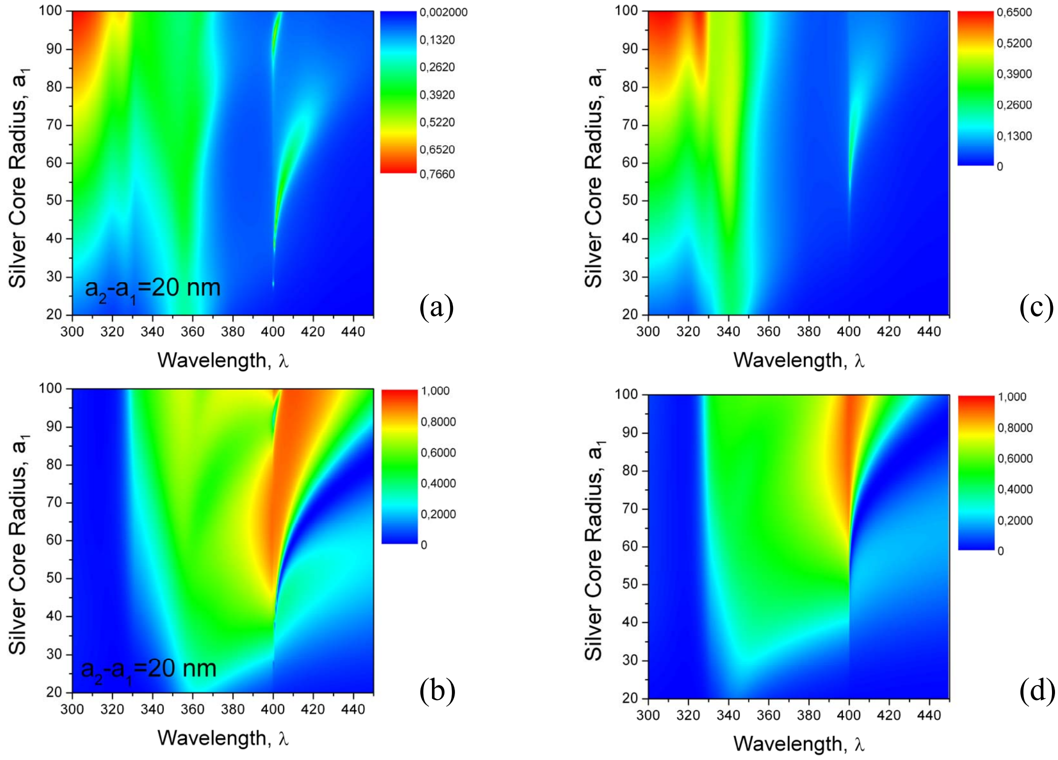

Fig. 2 Reflectance (b,d) and absorbance (a,c) for the grating with period $p = 400\,\mathrm{nm}$ *of two-layer cylinders with a silver core and a dielectric coating of the thickness* $a_2 = 20 + a_1$ *and refractive index* $\nu_2 = 1.4142$ *(a,b) and for the silver-wire grating (c,d).*

thickness. Comparing these reliefs with similar data for the silver-wire grating, one can see that in the band of interaction of two resonances the dielectric coating influence is in enhancing the reflectance where wavelength and the radius of silver core correspond to lower absorbance and decay one otherwise.

Fig. 3 shows reflectance and absorbance of the chain with period $p = 400\,\mathrm{nm}$ and with fixed radius of coating, $a_2 = 120\,\mathrm{nm}$, as a function of the wavelength and the radius of the silver core. Here we see the same wide band of high reflectance Fig. 3 (a), however for this geometry absorption shifts left and causes disturbance inside the band.

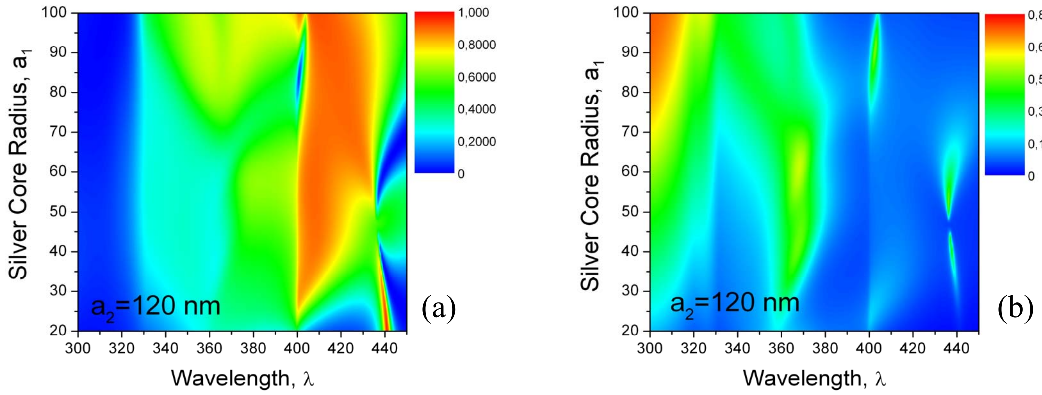

Fig. 3 Reflectance (a) and absorbance (b) of the grating with period $p = 400\,\text{nm}$ *and a two-layer wire: silver core with varying radius* a_1 *and dielectric coating with the radius* $a_2 = 120\text{nm}$ *and refractive index* $\nu_2 = 1.4142$.

4. CONCLUSIONS

We have presented some preliminary results of the resonance scattering and absorption of the H-polarized plane wave normally incident on the infinite grating made of circular silver nanowires coated with dielectric layers in concentric manner. They show the directions of optimization of such gratings, for obtaining enhanced scattering and/or absorption, by tuning the radii of the silver core and the dielectric coating.

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