Plasmon and Structure Resonances in the Scattering of Light by a Periodic Chain of Silver Nanocylinders

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ABSTRACT
In this paper, we study the scattering of the H-polarized plane wave by an infinite periodic chain of silver circular cylinders standing in free space. The scattering problem is reduced to a matrix equation with favourable features using the method of partial separation of variables. In the visible-light band, we demonstrate an interesting interaction of two types of resonances: the localized surface plasmons of the nanosize cylinders and a specific grating-type resonance near the wavelength equal to the chain period.
Keywords: scattering, silver grating, plasmon resonances.

1. PROBLEM FORMULATION

A chain of circular cylinders lying parallel to the *z*-axis and periodic along the *x*-axis is considered – see Fig. 1.

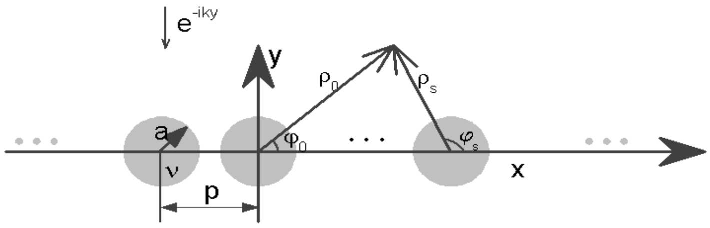

Fig. 1 Infinite periodic chain of silver circular cylinders illuminated by a normally incident plane wave.

The period of the chain is p and the radius of each cylinder is a . The material of cylinders is silver and they sit in the air. We assume that the electromagnetic field is time-harmonic ($\sim e^{-i\omega t}$) and does not vary along the *z*-axis, so that the field analysis problem is two-dimensional. Aiming at the study of plasmonic behavior, we consider the scattering of the H-polarized plane wave incident normally on the chain from the upper half-space (along the negative direction of the *y*-axis) and introduce a periodic function $U(x,y)=U(x+p,y)$, which denotes the H_z component of the electromagnetic field. The requirement that the function is periodic is the direct consequence of the invariance of the system with respect to the translation by an integer number of periods along the *x*-axis. The accurate formulation demands that U satisfies the Helmholtz equation with different coefficients inside the cylinders and outside of them. The values for the refractive index of silver in the visible band are taken from [1] and interpolated using cubic splines. The field components tangential to each cylinder's boundary must be continuous. Besides, U must satisfy the condition of the local power finiteness and the Sveshnikov condition of radiation at infinity [2]. The task is to find reflectance and absorbance of the chain, as the fractions of power of the incident wave that are reflected to the upper half-space and absorbed in the chain respectively.

2. BASIC EQUATIONS

We reduce the problem to a single period and treat it using the method of partial separation of variables. Thus we represent the total field inside a cylinder and the incident field as series of the Bessel functions with unknown coefficients, and the scattered field as a double series with Hankel functions of the first kind. The latter choice is dictated by the radiation condition. For finding the unknown coefficients, we use the boundary conditions on the contour of cylinder. Here, the use of addition theorems and a summation of the series in terms of Hankel functions are necessary. Direct calculation of such sums, in view of extremely slow convergence, is not efficient and needs a special approach. For the acceleration, we have used the lattice sum technique (see [3]). Excluding coefficients of the internal field brings us to an infinite set of linear equations for the coefficients of the external field. This set is a Fredholm operator equation of the second kind. Therefore the equation obtained guarantees rapid convergence, so that the accuracy of several digits is provided by a small truncation order of the matrix. Details of the derivations can be found in [4].

The coefficients of the external field decay rapidly enough and we may, using the Poisson summation formula, rewrite it in terms of the Floquet harmonics as generalized (both propagating and attenuating) along the *y*-axis plane waves. The reflectance and transmittance are defined as the squares of the absolute values of the amplitudes of zero Floquet harmonics in the corresponding half-space. According to the low of energy conservation, the absorbance can be obtained by the subtraction of the reflectance and transmittance from 1.

3. NUMERICAL RESULTS

In Fig. 2, we show the dependences of the reflectance and absorbance on the wavelength for a silver-cylinder chain with $p = 350$ nm. Here, we see the interaction of two kinds of resonances. The first that is observed near the wavelength of 340 nm has plasmonic nature; this is the only distinct resonance for small-radius cylinders ($a = 10 – 30$ nm, $a << p$) and it splits to several peaks if the chain becomes denser. For larger cylinders we see the appearance of the resonance that is connected to the periodicity of the chain. It sits at the wavelength slightly larger than the period, *p*. If these two resonances merge together, the resulting dependences display a wide-band reflectance at the level of 0.8. In Fig. 3 the period is larger ($p = 450$ nm), therefore the distance between two characteristic resonant wavelengths is larger, and the band of high reflectivity is even wider. The absorbance also displays a resonant behaviour, with minima/maxima correlating to the maxima/minima of the reflectance. Note also that if the wavelength is larger than the period, then the reflection and transmission take place on the zero Floquet harmonic, otherwise two higher harmonics, +1 and -1, also give their contribution.

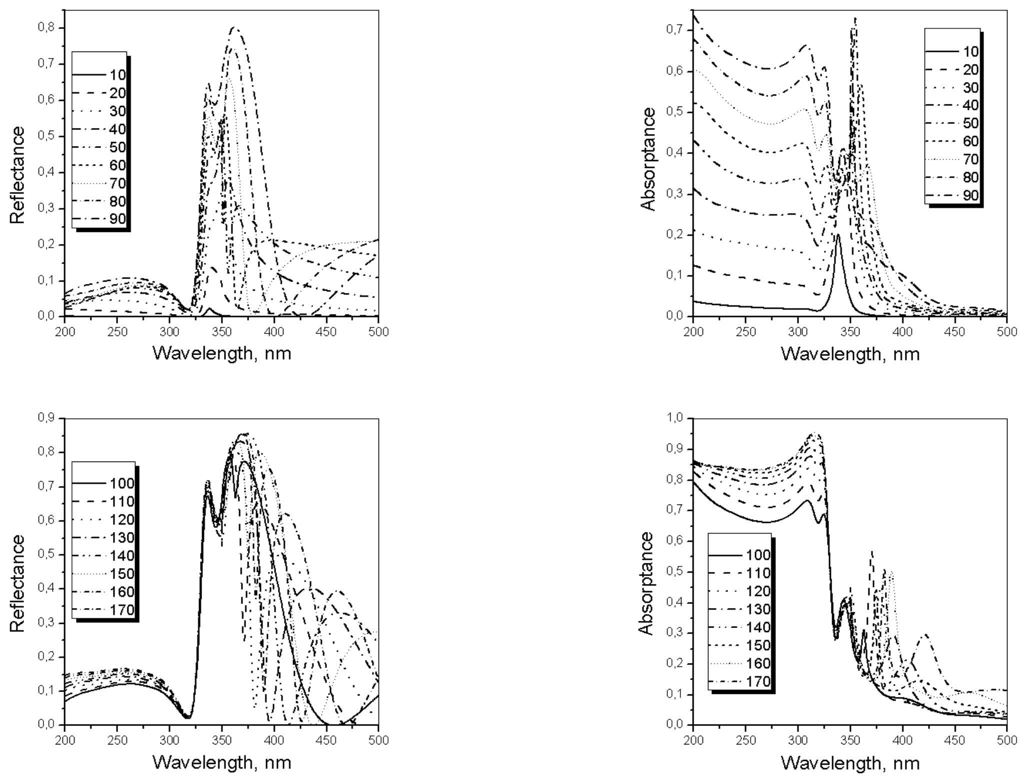

Fig. 2. Reflectance and absorbance of a normally incident plane wave for a periodic chain of silver cylinders. The distance between cylinders (p) is 350 nm and the radii (a) vary from 10 nm to 170 nm (see the insets).

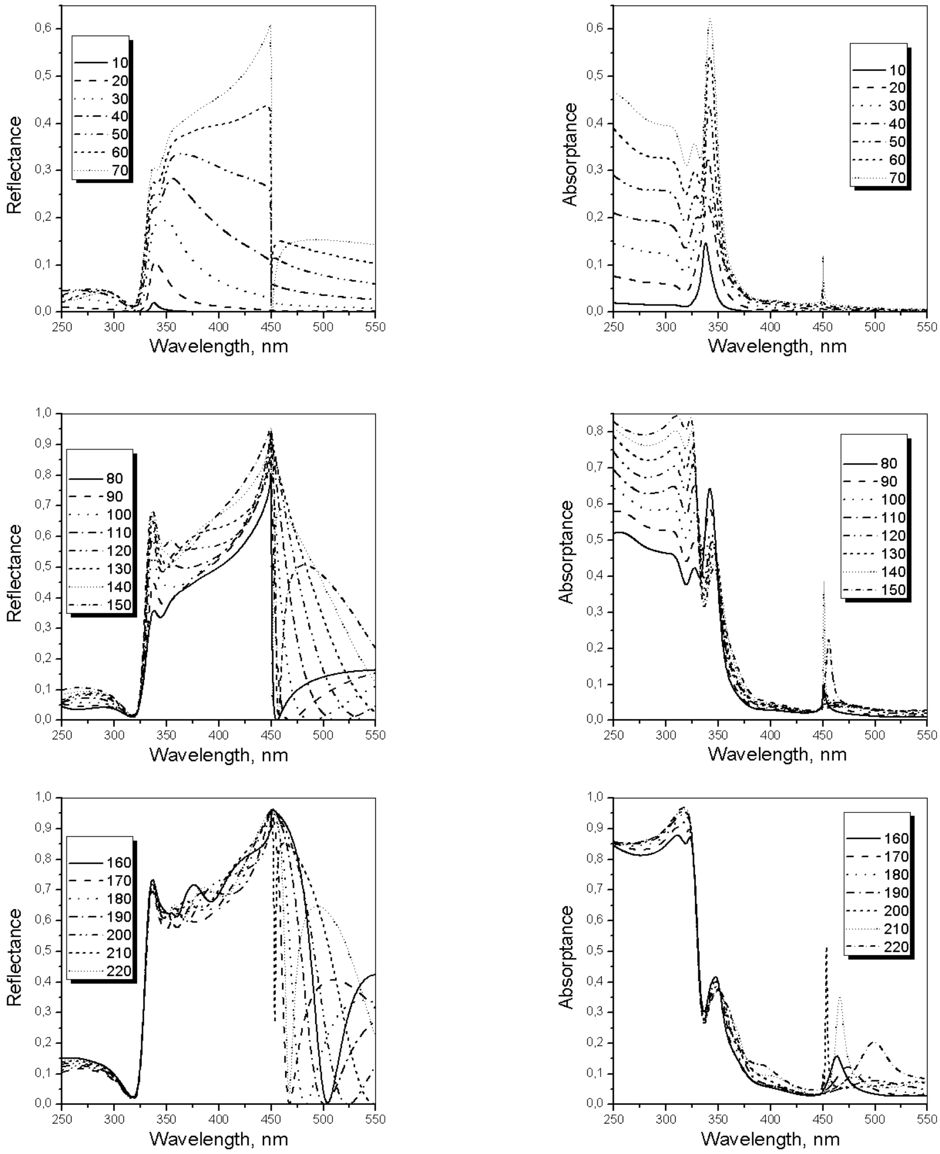

Fig. 3. The same as in Fig. 2, however for the chain with period p = 450 nm and the cylinders radii a varying from 10 nm to 220 nm.

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