Linear Modeling of Lasing Thresholds in a VCSEL-like Cavity with an Active Region

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Abstract

Optical mode frequencies and associated linear thresholds of lasing for a simplified VCSEL model consisting of a quantum well placed between two distributed Bragg reflectors are investigated as a specific electromagnetic-field eigenvalue problem with "active" imaginary part of the quantum well refractive index. Intuitively predictable lowering of the thresholds within the stop-bands of reflectors is quantified in accurate manner.

Key words: eigenproblem, laser, microcavity, quantum well, threshold, VCSEL

1. Introduction

Many advanced light-emitting devices of today's photonics rely on single or multiple quantum wells (QWs) embedded in epitaxially grown semiconductor microcavities. The area of their applications relates to the physics fundamentals such as spontaneous emission rate manipulation and investigation of quantum optical phenomena, and also to the design of optoelectronic devices such as light emitting diodes with narrower and more directional emission and vertical cavity surface emitting lasers (VCSELs) [1], photonic-crystal membrane lasers [2], etc.

VCSEL-type QW-equipped semiconductor microcavities usually confine a high-intensity optical field by using two (top and bottom) distributed Bragg reflectors (DBRs) composed of quarter-wavelength multilayers grouped in periodic pairs (Fig. 1).

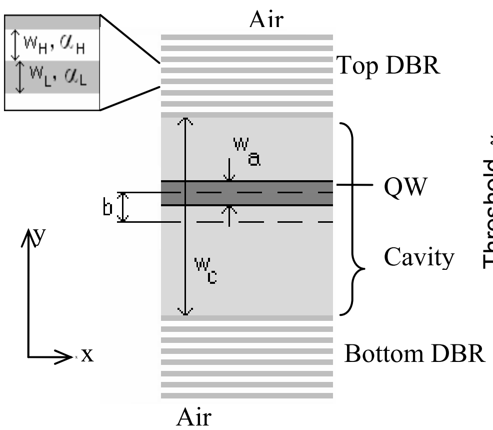

Fig. 1 Sketch of the VCSEL-type layered structure

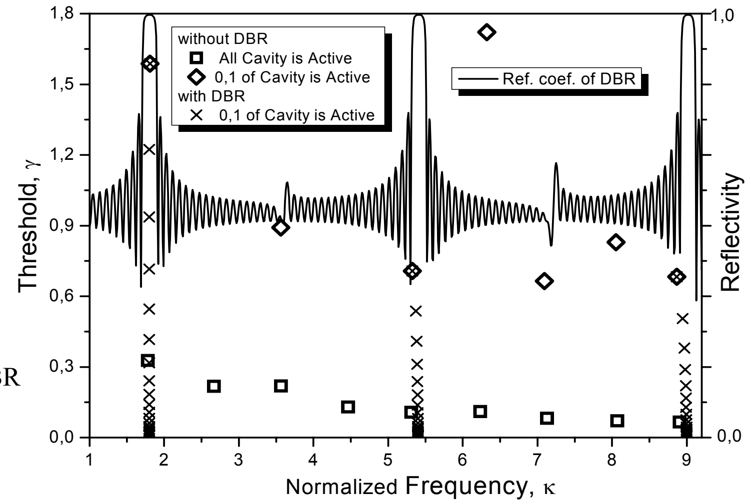

Fig. 2 LEP eigenvalues and reflectivity of one DBR sandwiched between two halfspaces with refractive indices of the cavity and free space.

Over recent years, much attention has been concentrated on making use of the microcavity effect in semiconductor optical devices to improve their performance in several respects, such as suppression of noise and low divergence of output beams. The lasing wavelength for a VCSEL is determined by the optical properties of the mirrors and the cavity. The threshold current for the onset of lasing, however, is known to be sensitive mainly to the shape and placement of the current-injection electrodes. An understanding of the relationship between the active region location and width and the cavity natural-mode field is important in order to achieve ultralow room-temperature thresholds. However, little attention has been given to the direct analysis of the dependence of thresholds on the microcavity and DBRs geometry and reflectivity. Instead, passive cavities characterized with Q-factor have been studied. Another point, which has been often neglected, is the discreteness of the lasing modes.

In this work, we report the results of the modelling of the lasing spectra and associated thresholds for the modes in a laterally uniform, i.e., one-dimensional (1-D) model of the VCSEL-type layered microcavity equipped with a QW. It is based on the modified formulation of the electromagnetic-field eigenvalue problem, specifically tailored for the analysis of emission of non-attenuated light, i.e. lasing. This model can be straightforwardly extended to multiple QWs and membrane-type cavities clad with noble-metal layers.

2. Problem formulation

In mathematical sense, the lasing modes can be viewed as source-free solutions to the Maxwell equations. We look for the non-attenuating time-harmonic electromagnetic field $\sim e^{-ikct}$ (with $k = \mathrm{Re}\, k > 0$ and c for the light velocity) inside and outside a multilayer dielectric medium schematically depicted in Fig. 1. It consists of a cavity whose width is w_c, sandwiched between two DBRs that may be identical or different from each other.

Further we assume that the electromagnetic field does not vary along the axes x and z and therefore can be characterized by a scalar function U, which is, for instance, the E_z component. Off the layer interfaces, this function must satisfy the Helmholtz equation, $[\Delta + k^2 \nu^2(y)]U(y) = 0$. Here, a step-like function $\nu(y)$ is assumed 1 outside the cavity and DBRs, i.e. in the lower and upper half-spaces are supposed filled with air; inside the layered structure it takes the values corresponding to the refractive indices of semiconductor cavity, α_c, and the high and low-index materials of the DBR pairs, α_H and α_L. All α_s $(s = c, H, L)$ are assumed real, i.e. we neglect absorption. The index $\nu = \mathrm{Re}\,\alpha_c - i\gamma$ is, however, assumed complex-valued with a negative imaginary part inside the QW region that corresponds to the active (or, equivalently, negative-loss) material. The optical field tangential components must satisfy the continuity conditions at the layer interfaces and obey the outgoing wave radiation condition at infinity ($|y| \to \infty$). Keeping in mind source-free solutions to this boundary-value problem, we seek its eigenvalues as pairs of real-valued parameters, (κ_n, γ_n). The first of them is the normalized frequency of lasing, $\kappa = kw_c$ and the second is the associated threshold material gain. Note that the gain per unit length, frequently met in the Fabry-Perot cavities descriptions, is obtained as $g = k\gamma$. This formulation is different from the "classical" formulation of the eigenvalue problem for an open cavity with the complex-valued frequency or wavenumber k being the eigenvalue parameter. Note that this is still a linear problem, however it takes into account the presence of the active region and enables one to extract the threshold gains, in terms of γ, in addition to the natural-mode wavelengths. This is more appropriate to the analysis of lasing than merely determining the Q-factors ($Q_s = |\mathrm{Re}\, k_s / \mathrm{Im}\, k_s|$), and can be qualified as building a "warm" model of laser. Such a model was systematically introduced in [3] and in greater detail in [4], however similar ideas have been also expressed, e.g., in [2] and [5].

3. Basic equations

For a generic DBR-based microcavity with a QW shifted by arbitrary distance b from the centre, the lasing eigenvalue problem (LEP) can be reduced to the following equation:

$$e^{-i2\kappa(\alpha_c - i\gamma)(w_a/w_c)} = R_1 R_2, \quad R_{1,2} = \frac{R_{DBR} e^{i\kappa\alpha_c[1 \pm (2b \mp w_a)/w_c]} + R_a}{1 + e^{i\kappa\alpha_c[1 \pm (2b \mp w_a)/w_c]} R_{DBR} R_a}, \quad R_a = \frac{i\gamma}{2\alpha - i\gamma}, \tag{1}$$

where R_{DBR} is the Fresnel reflection coefficients of DBR.

In the case when the DBR is absent, the solutions of the LEP are given by the roots of (1), where reflection coefficients R_{DBR} are replaced with the ones corresponding to the reflection from the interface between the cavity and the free half-space, $R_0 = (\alpha_c - 1)/(\alpha_c + 1)$. Asymptotically, i.e. for $n \gg 1$, they behave as

$$\kappa_n = \frac{\pi n}{\alpha_c}, \quad \gamma_n = \frac{\ln[(\alpha_c + 1)/(\alpha_c - 1)]}{\kappa_n (w_a / w_c)}. \tag{2}$$

Equation (1) is quite simple; the theory of complex variables tells that its solutions form at most a discrete set of points on the plane (κ, γ). The reflectivity of DBRs can be obtained by using the classical transfer-matrix approach. In fact, with the transfer matrices one may also obtain a determinantal equation corresponding to the whole cavity plus DBRs system, instead of (1); these two approaches are equivalent to each other. Here the main idea is to transform the complex amplitudes of the incident wave field at the interfaces with the aid of transmission matrices and shift them in phase with the aid of corresponding diagonal matrices between the interfaces [6]. The general form of these matrices is as follows:

$$\tau(\alpha, \beta) = \frac{1}{2}\begin{pmatrix} (1 + \alpha/\beta) & (1 - \alpha/\beta) \\ (1 - \alpha/\beta) & (1 + \alpha/\beta) \end{pmatrix}, \qquad \Delta(\varepsilon d \kappa) = \frac{1}{2}\begin{pmatrix} e^{-i\varepsilon d\kappa} & 0 \\ 0 & e^{i\varepsilon d\kappa} \end{pmatrix} \tag{3}$$

The transfer matrix for the whole layered structure can by obtained as a product of partial transfer matrices, $\tau(\varepsilon_{i-1}, \varepsilon_i)$ and $\Delta(\varepsilon_i d_i \kappa)$, namely

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \tau(\varepsilon_0, \varepsilon_1) \cdot \Delta(\varepsilon_1 d_1 \kappa) \cdot \tau(\varepsilon_1, \varepsilon_2) \cdot \ldots \cdot \Delta(\varepsilon_{n-1} d_{n-1} \kappa) \cdot t(\varepsilon_{n-1}, \varepsilon_n) \quad (4)$$

This is the matrix linking together the waves outside of the layered structure as

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = P \begin{pmatrix} a_n \\ 0 \end{pmatrix} \qquad R = b_0 / a_0 = p_{21} / p_{11}, \qquad T = a_n / a_0 = 1/ p_{11}. \quad (5)$$

The reflection coefficient is the ratio of the complex amplitudes of the outgoing (i.e., reflected) and the incoming (i.e., incident) waves in the 0-th domain, and the transmission coefficient is the ratio of the outgoing (i.e., transmitted) wave after the layered structure in the *n*-th domain and the incoming (i.e., incident) one before it. From the relations (5), one can derive as an alternative to (1) the characteristic equation, $p_{11}(\kappa, \gamma) = 0$.

4. Lasing spectra and thresholds

We have considered the LEP for the cavity made of a GaAs caity ($\alpha_c = 3.53$) located in free space (standard Fabry-Perot cavity) and between two identical DBRs having low and high index layers as $\alpha_L = 3.08$ and $\alpha_H = 3.53$, respectively. Usually the DBR layer thicknesses are taken as quarter-wavelength of the lasing mode to provide the maximum reflectivity. We have chosen the relative widths of the DBR layers as $w_L / w_c = 0.2795$ and $w_L / w_c = 0.2495$ so that the lowest rejection band is close to the frequency of the mode E_1.

In the computations, we start from the initial values taken from the asymptotic expressions (2) where we assume that $w_a = w_c$ (whole cavity is active). Then we iteratively compute eigenvalues for a sequence of cavity configurations with progressively narrower QWs until we reach the cavity where the QW occupies just a tenth part of its width and sits exactly in the middle. This corresponds to $b = 0$ in (1), and in such case the characteristic equation splits into two independent equations, one for the even modes (whose field functions are symmetric with respect to the cavity centre) and another for the odd modes (whose field functions are anti-symmetric).

If we change cavity and QW parameters with small enough steps, the two-parameter Newton iterative method meets no problems. Fig. 2 shows the locations of the lowest nine LEP eigenvalues on the plane (κ, γ) for three different configurations. The lower set of hollow squares depicts them for the simplest case of the uniformly active Fabry-Perot cavity sitting in free space. As could be anticipated, the thresholds are comparatively low. Shrinking the active region to one-tenth of the cavity width (the cavity in this case is still in air) shifts the thresholds up. They jump to the values illustrated by the diamonds in the same figure. Note that some of them (those of the even-mode family having numbers 2 and 4) obtain so high thresholds that they do not fit the figure. The fundamental mode E_0 has initially a very high threshold in the Fabry-Perot cavity and its frequency strongly depends on the number of DBR layers. Therefore matching this frequency with a DBR rejection band is a difficult task

The introduction of two identical DBRs consisting of 2M or 2M+1dielectric layers with low and high refractive indices on the both sides of the cavity is able to essentially reduce the thresholds of the modes, whose frequencies get inside the rejection bands of DBRs. The crosses in Fig. 2 show the LEP eigenvalues for the same cavity equipped with a QW occupying 0.1 of the cavity but sandwiched between two DBRs with an even number of layers. They reduce exponentially if M is increased. Note that the modes whose frequencies happen to be out of the DBR rejection band, do not feel the effect of the DBRs because their fields are not reflected back to the cavity in a complete enough manner. Therefore their thresholds remain around the initial levels and sometimes even may go up.

A more detailed picture of the behaviour of LEP eigenvalues when the number of DBR layer pairs is increased is given in Figs. 3 and 4 for the mode E_5. The filled circles correspond to the complete DBRs (i.e., with even number of layer pairs, so that the outermost layer has higher index), while the filled squares are for the DBRs having the outer pair incomplete (i.e., with an odd number of layers, so that the outermost layer has lower index). Calculation showed that a presence of outermost layer of lower index decreases the efficiency of DBR to the value provided by the reflector having roughly ten pairs less without it. The same is approximately valid for the thresholds in our cavity with an active region.

The frequency of the mode E_5 also shows a much larger deviation from the limit value if the DBR has the outermost layer of lower refractive index than in the case of complete DBRs without that layer.

These effects are actually what is intuitively expected from the use of DBRs. Note, however, that it is only the LEP formulation which enables us to extract the thresholds in addition to the natural-mode frequencies. Note also that LEP remains a linear electromagnetic problem that involves only the Maxwell equations with associated boundary and radiation conditions.

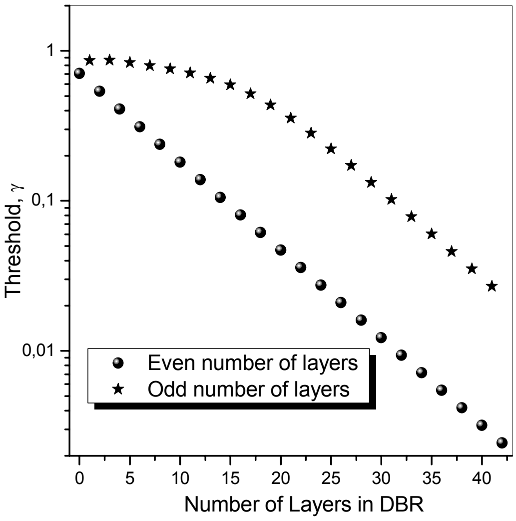

Fig. 3 Thresholds of the mode E_5 in the cavity with a centered QW occupying 0.1 of its width as a function of the number of layers in symmetric DBRs

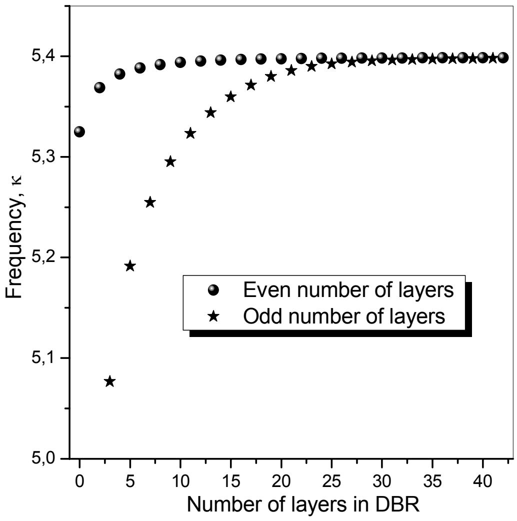

Fig. 4 Frequencies of the mode E_5 in the cavity with a centered QW occupying 0.1 of its width as a function of the number of layers in symmetric DBRs

5. Optical theorem for a VCSEL-type cavity

A very instructive insight into the nature of lasing is obtained from the Optical Theorem (OT) applied to the LEP. In the time-harmonic plane wave scattering, the OT links the total extinction cross-section of a scatterer with the amplitude of the forward-scattered field in the far zone (e.g. see [7]). The most general form of such relation is also known as the Complex Poynting Theorem. For the complex-valued k, this is

$$(1/2)\oint_S \vec{E}\times\vec{H}^* ds = -(1/2)\int_V (\vec{j}^{e^*}\vec{E}+\vec{j}^m\vec{H}^*)dv + (i/2)\int_V (k^*\varepsilon^* Z_0^{-1}\,|\,\vec{E}\,|^2 - k\mu Z_0\,|\,\vec{H}\,|^2)dv, \quad (6)$$

where the right-hand part is the total outward flux of the Poynting vector, averaged over the period of oscillations, through the boundary S enclosing a volume V containing all scatterers, $\varepsilon = \nu^2$ and μ are the relative permittivity and permeability, respectively, $\vec{j}^e$ and $\vec{j}^m$ are the given electric and magnetic currents, respectively, and the asterisk means complex conjugation.

Application of (6) to the field $\{\vec{E}_s, \vec{H}_s\}$ of the s-th lasing mode in the cavity with the active-region domain V_a implies that $\vec{j}^e = \vec{j}^m = 0$ and the eigenfrequency k_s is real-valued, i.e. $\operatorname{Im} k_s = 0$; additionally, we shall assume that $\mu = 1$. On the extraction of the real part of the obtained expression, we obtain the OT for lasers:

$$(1/2)Z_0 \operatorname{Re}\oint_S \vec{E}_s \times \vec{H}_s^* ds = \alpha_a \gamma_s k_s \int_{V_a} |\,\vec{E}_s\,|^2 dv \quad (7)$$

The left-hand part of (7) stands for what can be called the mode radiation cross-section and the right-hand part is the mode gain cross-section due to the power generated in the active region. Hence, for the s-th mode having the wavenumber k_s, the radiation losses are balanced by the "anti-losses", provided that the material gain equals γ_s. Computations show that on finding (κ_s, γ_s) from (1) this identity is satisfied with machine precision.

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