Scattering of the Plane Wave from a Periodically Perforated Dielectric Slab

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Abstract: An infinite grating of identical dielectric or metal cylinders or strips illuminated by a plane wave has high-quality optical resonances near the Rayleigh anomalies [1-16]. These resonances are present for any periodic 2-D structures and have striking properties as their Q-factors increase if the grating period becomes larger. Besides, in such a resonance the domain of high-intensity scattered field occupies a very wide domain along the grating. In this paper we investigate a configuration where we have a dielectric slab and an infinite grating of air holes inside it. This structure is closer to practical applications in nano and micro-optics and our study shows the presence of the grating resonances having complicated behaviour.

In this work we investigate a dielectric slab (Fig. 1) of the refractive index ν_S with an embedded into it dielectric grating of circular cylinders that is placed at the distances d_1 and d_2 from the upper and lower boundaries, respectively. The grating period is p and its cylinders have radius a and refractive index $\nu = 1$ or in fact are the air holes. In computations, we have taken the slab refractive index to be $\nu_S = 1.4142$, i.e. small to eliminate excessive impact of the slab modes.

In the preceding studies we have investigated the scattering of a plane wave normally incident on a passive periodic structure of lossless dielectric circular cylinders in free space, parallel to the *z*-axis and periodic along the *x*-axis. The reflectance of the plane wave from such a structure has shown the presence of specific grating resonances near the Rayleigh anomalies [7,13]. These resonances possess interesting feature as they become sharper if the grating becomes sparser. A sample relief of the reflectance of the plane wave normally incident on such a grating is shown in Fig. 2. Here the grating resonances are seen as bright ridges that become sharper with the growth of the period and eventually merge with the Rayleigh anomalies, i.e. the lines $\sigma = 1, 2, \ldots$, where $\sigma = p/\lambda$ is the dimensionless frequency and λ stands for the wavelength in free space. Additionally the scattered field at a grating-resonance frequency forms a standing wave along the grating that stretches far away from it in the normal direction

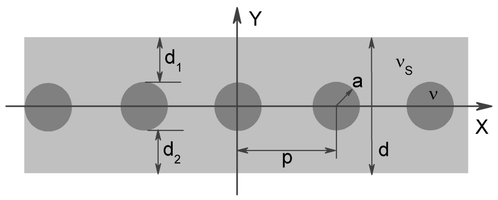

Fig. 1 Cross-sectional geometry of a periodically perforated dielectric slab

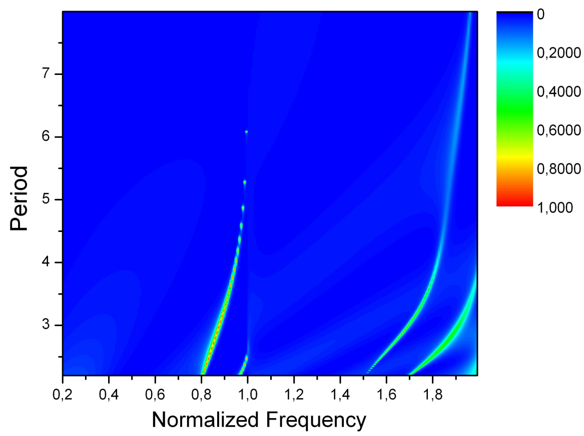

Fig.2 The reliefs of reflectance of the H-polarized normally incident plane wave from the grating of circular dielectric cylinders in free space $\nu_S = 1.0$ and $\nu = 1.4142$, as a function of the normalized frequency and the period-to-radius ratio.

[8]. Although there is infinite number of natural modes near each Rayleigh anomaly, only few first ones can be observed for realistic material and geometrical parameters of the resonator. If the grating elements are not magnetic, then the Q-factors of the grating resonances for the H-polarization are higher than for the E-polarization.

We solve the perforated-slab scattering problem using the method of separation of variables and transfer matrices. Firstly,

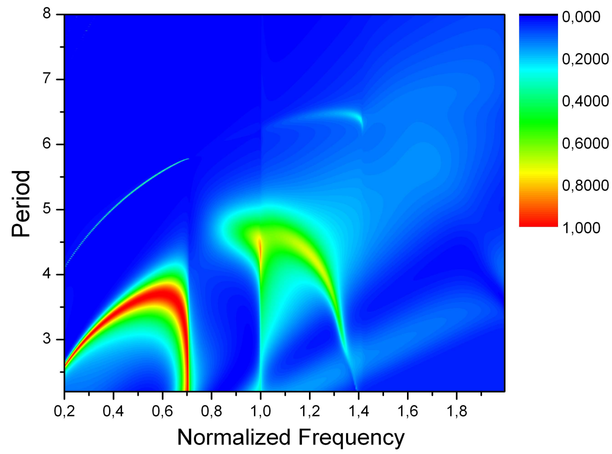

Fig.3. The same as in Fig. 2 for a lossless dielectric slab with a grating of circular air holes placed in the middle with $d_1 = d_2 = 0.05a$; slab refractive index is $\nu_S = 1.4142$

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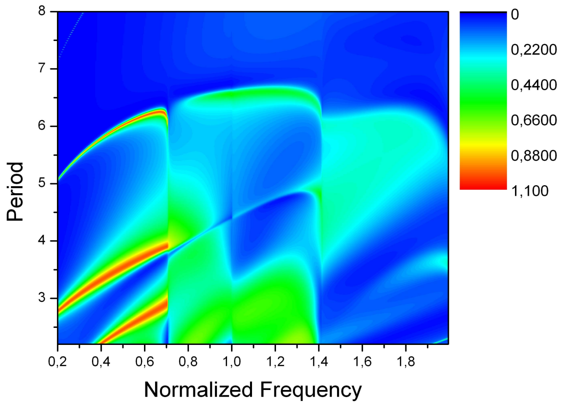
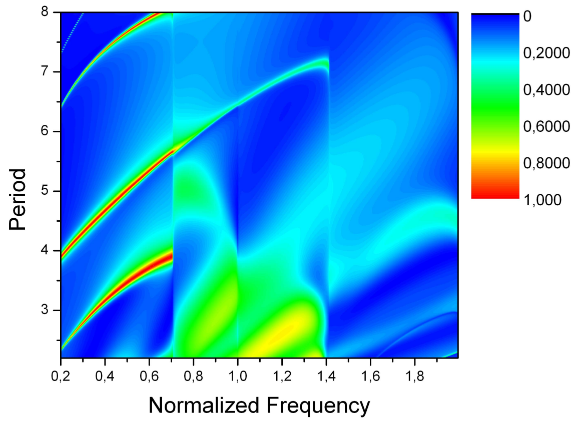
we construct the scattering matrices of the grating; thereto we represent the filed inside a hole and the incident one using the Fourier expansions with Bessel functions in coefficients, while the scattered field series involve the Hankel functions of the first order. The order is chosen according to the time dependence and the radiation condition. In analytical derivations, among several mathematical operations infinite summation of the Hankel functions appears.

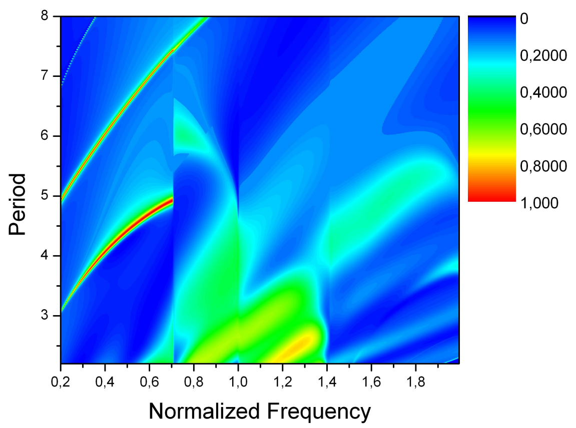
Direct calculation of such sums, in view of their extremely slow convergence, is not efficient. For acceleration we use the lattice-sums technique [17]. Exclusion of the coefficients of the internal field brings us to an infinite matrix equation for the Fourier coefficients of the external field. This matrix equation is the Fredholm equation of the second kind. As a result the algorithm built on solving the truncated equation has good stability and provides high accuracy for rather small truncation order of the matrix. The Fourier coefficients of the scattered field have to be converted to the amplitudes of the Floquet harmonics, which are the elements of the grating scattering matrix. The planar boundaries of the slab are represented using the diagonal Fresnel matrixes. Compiling both kinds of matrices we build the scattering matrix of the whole structure. The overall scheme of the solution coincides with one explained in [13,18].



(a)



(b)



(c)

Fig.4. The reliefs of reflectance of the H-polarized normal incident plane wave on the dielectric slab ($\nu_S = 1.4142$) with periodic air holes ($\nu = 1.0$), where $d_1 = 0.05a$ and $d_2 = 0.45a$ for (a), $d_2 = 0.95a$ for (b), and $d_2 = 1.45a$ for (c).

In Fig 3, we present the relief of the reflectance of a perforated dielectric slab depending on the perforation period and the normalized frequency. Here one can see that besides of the free-space Rayleigh anomalies, new set of anomalies appears at the frequencies $\sigma = n/\nu_S$, $n = 1, 2...$. Besides, unlike the grating of cylinders in free space, here the most intensive resonance in the strip $\sigma < 0.7$ does not exist for large enough values of period; instead it seems to vanish in the Rayleigh anomaly $\sigma = 1/\nu_S \approx 0.7$ (branch point) at a finite value of period-to-radius ratio. Similar dynamics is demonstrated by the higher order resonance visible as a thin curve at larger values of period. The resonances in the strip between the first branch point at $\sigma \approx 0.7$ and the second one at $\sigma \approx 1.41$ show similar behaviour, however they are weaker.

We have also investigated the impact of the slab width on the resonances associated with periodic perforations. We looked at the three different geometries, where the distance between the grating of air holes and upper boundary, d_1, stays the same but the slab width varies. In Fig. 4, three reliefs of the reflectance depending on the perforation period and the normalized frequency are shown for identical material parameters however different geometries: for (a) $d_1 = 0.05a$ and $d_2 = 0.45a$ therefore the width of slab is $d = 1.5p$; for (b) $d_2 = 0{,}95a$ and $d = 2p$; and for (c) $d_2 = 1.45a$ and $d = 2.5p$. Comparing Figs. 3 and 4, we can see that now the number of bright resonances in the strip $\sigma < 0.7$ is at least three, and each of them is accompanied by the almost total reflection in a narrow range. One should note that for wider slabs the grating resonances are more intensive and have larger Q-factors.

In conclusion we may say that a perforated slab possesses high-quality resonances whose nature is strongly connected to the periodicity of perforation. They may appear for any value of the normalized frequency between the branch points and are also in complicated interplay with the resonances of the homogeneous slab perturbed by the perforations.

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