Numerical Optimization of a TARA-like Shielded Parabolic Reflector

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Abstract **The problem of monochromatic electromagnetic wave diffraction by a perfectly electrically conducting (PEC) surface of rotation located in free space is investigated. The problem is reduced to sets of hypersingular and singular integral equations, which are solved by the method of discrete singularities using interpolation type quadrature formulas. From the solutions of these sets of equations the electric field and the far-zone patterns are obtained. The method presented has guaranteed convergence for a non-axially symmetric primary field. Using this method the Transportable Atmospheric Radar system (TARA) is considered.**

Keywords: diffraction, singular and hypersingular integral equations, interpolation type quadrature formulas

I. INTRODUCTION

Among the works on the diffraction of electromagnetic waves by perfectly conducting rotation surfaces the papers [1, 2, 3] are of particular note since, by analytically taking into account the axial symmetry of a perfectly conducting rotation surface, the three-dimensional diffraction problem is reduced to solving a set of one-dimensional integro-differential equation systems. This significantly decreases the order of the discrete model matrix when compared to an arbitrary three-dimensional diffraction problem which typically leads to a two-dimensional system of equations.

The radiation pattern of the large parabolic reflectors of the Transportable Atmospheric Radar system (TARA), developed at Delft University of Technology, has been simulated in [4], using the method of moments. However the authors use the method of moments with a Rao-Wilton-Glisson(RWG) basis function, and therefore have to deal with large matrix size. Moreover, the convergence of their method is not proven rigorously. In the present paper the problem is numerically simulated using interpolation-type quadrature formulas with theoretically proved convergence. The calculation time of the present method is several hundreds of times less than in [4].

II. DIFFRACTION BY OPEN ROTATION SURFACE

Let us consider the diffraction by a PEC surface of rotation. Represent the total field as a sum of incident and scattered fields: $\vec{E}^{tot} = \vec{E} + \vec{E}^0$. The scattered field satisfies Maxwell's equations and a Sommerfeld radiation condition outside the surface, the Meixner edge condition, and PEC boundary conditions on the surface of rotation. The problem is solved using the rigorous theory of singular and hypersingular integral equations (IEs)[5-7].

We reduce the problem of electromagnetic wave diffraction by a rotation surface to a set of one-dimensional hypersingular and singular IEs with variable coefficients. These IEs are solved by the Nystrom-type method, using interpolation-type quadrature formulas [8]. From the solutions of these IEs the scattered electric field is obtained.

Almost all calculation time in this method involves calculation of the Modal Green Function (MGF). A lot of effort has been put into evaluating MGFs [9-18]. Each of these authors presents a method to calculate the MGF in some domain of parameters. However all these methods depend on wavenumber, and the convergence rate decreases if the wave number increases. In the quasi-optic case the MGF is an integral with a rapidly oscillating integrand and therefore these methods work slowly. We develop a formula, which does not depend on wavenumber and has an exponential convergence.

III. SERIES FOR MGF CALCULATION

In the quasi-optical range the unknown current densities have many oscillations on the reflector surface. This calls for high order discretizations. To reduce the calculation time it is necessary to develop faster numerical methods to calculate the matrix elements. The calculation of the MGF (1) placed in the IEs and its first and second derivatives takes almost all the calculation time in the methods [5-7,9-18].

Let C be a rotation surface contour. If t is the integration variable on C, we will use cylindrical coordinates notations like $\rho_0 = \rho_0(t),\ z_0 = z_0(t)$, while for the observation point the notations will be $\rho,\ z$ and we define the MGF function:

$$S_M = \int_0^{2\pi} \frac{\exp(-ikL)}{L} \cos(M\psi)\, d\psi, \tag{1}$$

where $$L = \sqrt{\rho^2 + \rho_0{}^2 - 2\rho\rho_0 \cos\psi + (z - z_0)^2} \tag{2}$$

Substitute the integration variable in (1):

$$u = \sqrt{\rho^2 + \rho_0{}^2 - 2\rho\rho_0 \cos\psi + (z - z_0)^2} \tag{3}$$

After computations, we have:

$$S_M = 4C \cdot \int_{-1}^{1} \frac{\exp(-i\Omega x) f(u(x))}{\sqrt{1-x^2}} dx, \qquad (4)$$

where

$C = \exp(-ik(L_{\max} + L_{\min})/2), \ \Omega = k(L_{\max} - L_{\min})/2$

$f(u) = T_M(g(u)) / \sqrt{(u + L_{\min})(L_{\max} + u)}$

$u(x) = [L_{\min}(1-x) + L_{\max}(1+x)]/2$

$g(u) = \left(\rho^2 + \rho_0^2 + (z-z_0)^2 - u^2\right)/2\rho\rho_0$

$L_{\min} = \sqrt{(\rho-\rho_0)^2 + (z-z_0)^2}$, $L_{\max} = \sqrt{(\rho+\rho_0)^2 + (z-z_0)^2}$

Further we consider the case $L_{\min} \neq 0$. If $L_{\min} = 0$ then the kernels in the IE have finite limits, which are calculated using the asymptotic behavior of the MGF [12].

Note that the function $f(x) = f(u(x))$ is infinitely differentiable and has not any oscillations. Therefore it can be interpolated with low interpolation order $f(x) \approx f_{n-1}(x)$. Any infinitely differentiable function can be expressed in the form of

$$f(x) = \sum_{p=0}^{\infty} a_p T_p(x), \ x \in [-1,1] \qquad (5)$$

$$a_p = \frac{2-\delta_{p,0}}{\pi} \int_{-1}^{1} \frac{T_p(x) f(x)}{\sqrt{1-x^2}} dx. \qquad (6)$$

where $T_k(x)$ is the Chebyshev polynomial of the first kind.

Using the formula [19, p. 850]:

$$\int_{-1}^{1} e^{-i\Omega x} \frac{T_n(x)}{\sqrt{1-x^2}} dx = \pi(-i)^n J_n(\Omega) \qquad (7)$$

we obtain the expression for MGF in the form of series:

$$S_M = 4C\pi \sum_{p=0}^{\infty} (-i)^p a_p J_p(\Omega) \qquad (8)$$

Because of the infinite differentiability of the function $f(x)$, the components of series (8) tend to zero with an exponential rate.

Let us evaluate the number of terms we need to calculate in the series (8). This problem is equal to the definition of the degree of the interpolation polynomial which interpolates the function $f(x)$ with satisfactory relative error. Because of the factor $g(x) = 1/\sqrt{(u(x) + L_{\min})}$ the function f has large derivatives in the case of small $L_{\min}$. To find the degree of the interpolation polynomial, which interpolates the factor g, we find it exactly for function $\tilde{g}(x) = 1/(u(x) + L_{\min})$. We have proved that an interpolation polynomial of degree $p : |b_p| < \varepsilon$ interpolates $\tilde{g}(x)$ with relative error ε, where

$$b_p = \frac{4}{3L_{\min} + L_{\max}} \frac{1}{\sqrt{1-a^2}} \left(\frac{\sqrt{1-a^2} - 1}{a} \right)^p \qquad (9)$$

For sufficient error it is found necessary to calculate first $n = p + M + 5$ items in the series (8).

To calculate the coefficients a_p we use quadrature formulas [8].

$$a_p = \frac{2-\delta_{p,0}}{n} \sum_{m=0}^{n-1} T_p(t_m^n) f_{n-1}(t_m^n), \qquad (10)$$

where $t_p^n = \cos(\pi(2p+1)/2n)$, $p = 0,1..n-1$ - the roots of the first kind Chebyshev polynomial. Because of the infinite differentiability of the integrand in (6) the quadrature formula (10) has an exponential convergence.

Note that $J_p(\Omega)$ quickly tends to zero if $p > \Omega$. We prove that it is enough to take $p = \Omega \frac{e}{2} + 5$.

The use of the Fast Fourier Transform (FFT) can decrease the calculation time of a_p in (10). We use the FFT to calculate

$$\tilde{a} = F_{2n} \tilde{f}, \qquad (11)$$

where $F_{2n} = \left\{ \exp\left(-i \frac{pk}{2n} 2\pi\right) \right\}_{p,k=0}^{2n-1}$, $\tilde{f} = \left(f, \underbrace{0 \ldots 0}_{n} \right)$, $\{f_p\}_{p=0}^{n-1} = \{f_{n-1}(t_p^n)\}_{p=0}^{n-1}$. The coefficients a_p are expressed through $\tilde{a}_p$ in the following way:

$$a_p = \frac{1+\delta_{p,0}}{n} \operatorname{Re}\left(\exp\left(-i \frac{2\pi}{n}\right) \tilde{a}_p \right) \qquad (12)$$

The series for the first and second derivatives of the MGF are derived in an analogous manner. However we need to find an interpolation polynomial degree, which interpolates the second and third powers of the factor $g(x)$. For this purpose we use the expression [19, p. 380];

$$\int_0^{\pi} \frac{\cos(n\psi)}{1 - 2a\cos\psi + a^2} = \frac{\pi a^n}{1-a^2}, \ |a| < 1 \qquad (13)$$

In the case of small $L_{\min}$ ($L_{\min} \cdot k < 2, \sqrt{\rho \cdot \rho_0} \cdot k > 2$) it is found better to use the series presented in [12, p.236].

IV. TRANSPORTABLE ATMOSPHERIC RADAR (TARA)

Consider the real-life reflector antenna TARA. One of the major antenna design requirements is a very low level of the sidelobes around 90 degrees- less than -70 dB. The TARA is used for studying atmospheric phenomena such as clouds, precipitations and clear air turbulence. It consists of a parabolic reflector and a conical shield. The shield decreases the radiation sidelobes in the direction that is orthogonal to the axis of rotation and near to it. Fig. 1 shows the cross-sectional geometry of TARA. The parabolic reflector has a diameter $D = 2f$ of 33 wavelengths (3 meters) and the shield has a width L of 22 wavelengths (2 meters) and angle of inclination φ_0 of 30 degrees. For modeling the TARA in the transmitting case the feed is simulated using a Complex Huygens Element (CHE) in the focus of the parabolic reflector. For modeling the TARA in the receiving case we consider plane wave diffraction by a shielded paraboloidal reflector.

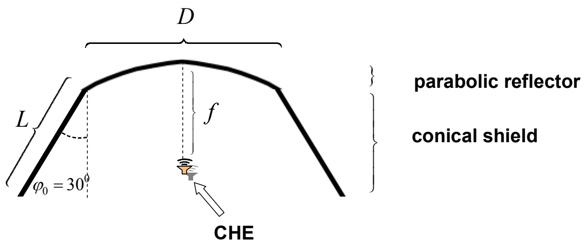

Fig. 1. The cross-sectional geometry of TARA

The CHE is a convenient simplified model of a realistic corrugated-horn or horn-lens antenna. Its field function has some parameter "*b*" that is formally the imaginary part of the source location point.

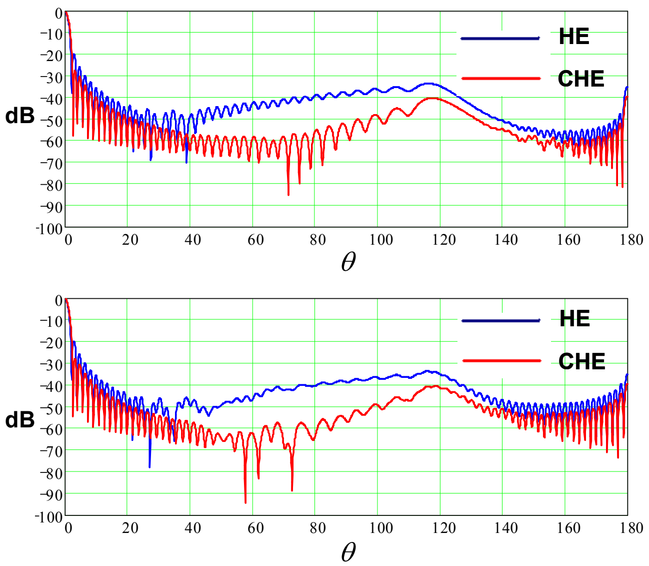

Fig. 2. The total far-zone radiation patterns of a stand-alone paraboloidal reflector illuminated by HE and CHE with the optimal parameter in the H-plane (top) and in the E-plane (bottom)

If $b = 0$ then the field function coincides with the field of the classical Huygens Element (HE) which consists of orthogonal to each other elementary electrical and magnetic dipoles. As known, HE has fixed directivity. If b is increased, then the directivity of such a modified source can be made larger and, correspondingly, the reflector edge illumination lower. Therefore such a feed is convenient for simulating the incident fields in the modeling of reflector antennas.

In Fig. 2 we demonstrate the far-zone radiation patterns of the TARA parabolic reflector without conical shield illuminated by the CHE with optimal parameter "b" (that which gives the largest directivity) and by the classical HE in the E- and H- planes. One can see that the paraboloidal reflector illuminated by the CHE has lower sidelobes than the one illuminated by the HE.

In Fig. 3 we visualize the near-zone field of the parabolic reflector illuminated by the HE in the E- and H-planes on a logarithmic scale. All patterns are normalized by the same value.

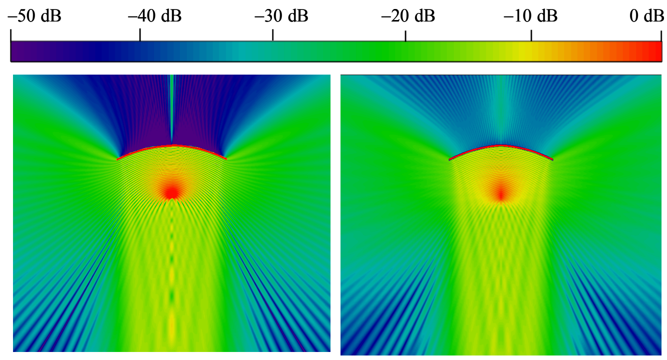

Fig. 3. The near field of the paraboloid illuminated by a HE for $D/f = 2$ and $D = 33\lambda$ in the H-plane (left) and the E-plane (right)

In Fig. 4 we visualize the near-zone field of the parabolic reflector illuminated by the CHE with the optimal parameter in the E- and H-planes in logarithmic scale. All patterns are normalized by the same value.

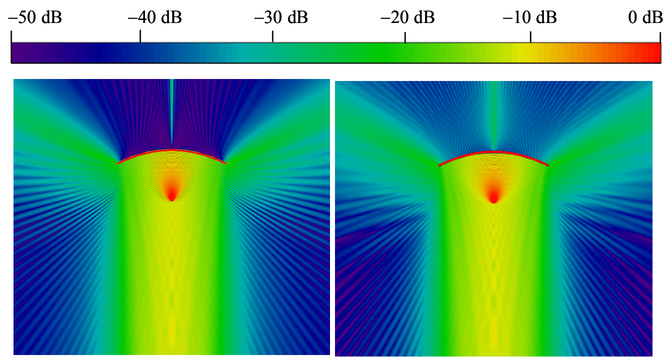

Fig.4. The near field of the paraboloid illuminated by a CHE with the optimal parameter $kb = 2.37$ for $D/f = 2$ and $D = 33\lambda$ in the H-plane (left) and the E-plane (right)

The method considered needs contour smoothness. But our numerical experiments show that the far-zone patterns for the actual TARA contour and for a TARA contour smoothed by splines are almost equal.

In Fig. 5, the H- and E-plane far-zone radiation patterns of a TARA paraboloidal reflector without the conical shield and the full TARA system are compared on a logarithmic scale. One can see that in the direction which is orthogonal to the axis of rotation (90 degrees), and near to it, the TARA radiation pattern has sidelobes lower than for the parabolic reflector alone by some 20 to 30 dB. The CHE here was taken in such a way that it provided the maximum directivity for the stand-alone parabolic reflector.

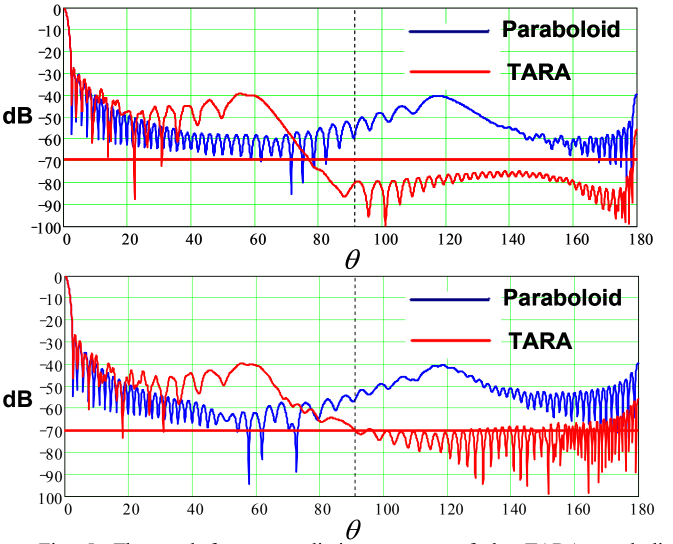

Fig. 5. The total far-zone radiation patterns of the TARA parabolic reflector without the conical shield and full TARA system in the H-plane (top) and in the E-plane (bottom)

In Fig. 6 we show the near-zone field of the full TARA illuminated by the optimal CHE in the E- and H-planes on a logarithmic scale. The $|\vec{E}^{tot} / \vec{E}^{0}|$ of the full TARA illuminated by the plane wave in the E- and H-planes is shown in Fig. 7.

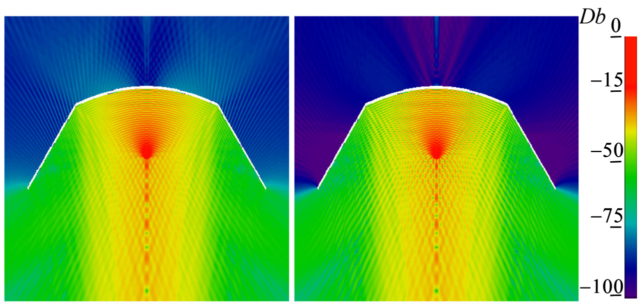

Fig.6. The near-field of the full TARA illuminated by the CHE with the optimal parameter in the H-plane (left) and the E-plane (right)

In Fig. 7 we can see an interesting phenomenon that escapes geometrical-optics descriptions. In addition to the main focal spot (area of the field concentration) there is another split "focus" near the paraboloid top. The latter areas of the field concentration appear because of the combined diffraction by the conical shield and paraboloidal reflector. The distance between the geometric focus of the paraboloid and the splitting focus is $dist = 15.51\lambda$ (the distance between the top of paraboloid and the splitting focus is 0.99λ).

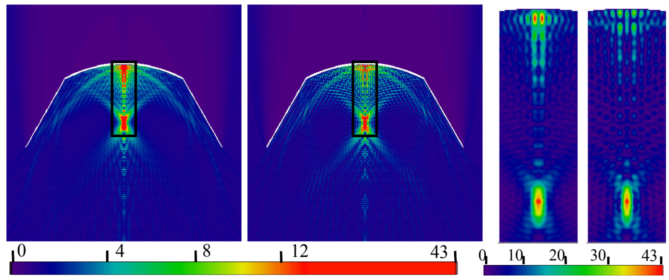

Fig.7. The near-field of the full TARA illuminated by the CHE with the optimal parameter in the H-plane (left) and the E-plane (right)

Let us now place the Huygens element in the splitting focus location and compare the far-field patterns of the shielded paraboloidal reflector TARA and a stand-alone paraboloidal reflector illuminated by this element in Fig. 8.

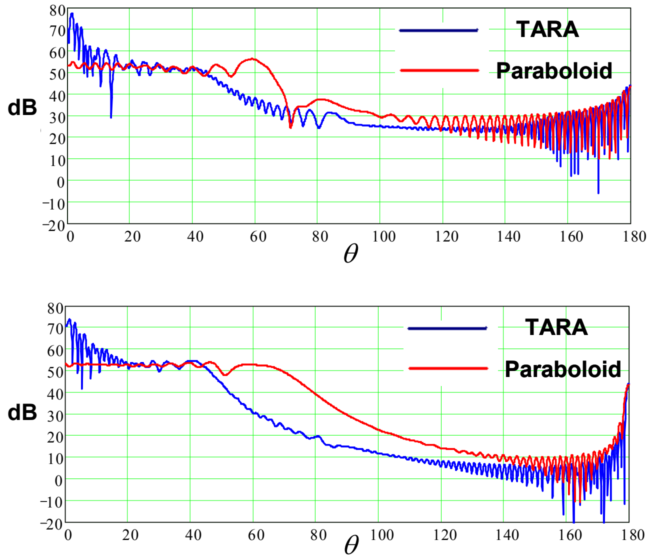

Fig. 8. The total far-zone radiation patterns of the shielded paraboloidal parabolic and a stand-alone paraboloidal reflector illuminated by a HE placed in the splitting focus location in the H-plane (top) and in the E-plane (bottom)

The TARA directivity can be improved, and the length of the conical shield can be reduced by half a meter, by changing the shield inclination φ_0 from 30 deg to 5 deg. In Fig. 9 we compare on a logarithmic scale the far-field pattern for a shielded paraboloidal reflector with an inclination angle $\varphi_0 = 5^0$, $L = 1.5m$ with the real TARA($\varphi_0 = 30^0$, $L = 2m$).

From Fig. 9 it follows that the shielded paraboloidal reflector with $\varphi_0 = 5^0$ satisfies the major antenna design requirement on sidelobes referred to above.

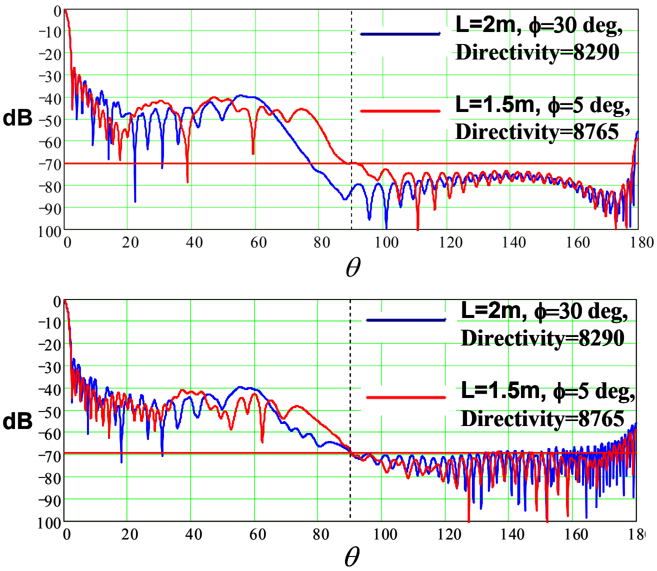

Fig. 9. The total far-zone radiation patterns of the TARA in the H-plane (top) and in the E-plane (bottom) for $\varphi_0 = 5^0, L = 1.5m$ and $\varphi_0 = 30^0, L = 2m$

It should be noted that the directivity for the shielded paraboloid with $\varphi_0 = 5^0$, $L = 1.5m$ is larger than for TARA ($\varphi_0 = 30^0$, $L = 2m$).

CONCLUSION

This paper improves on the method reported in [5-7] to use it for two-reflector quasi-optical antennas. For this purpose we derived an exact series form for the MGF. The terms in the series tend to zero with an exponential rate and calculation time doesn't depend on the wavenumber value. The use of these series meant that we were able to investigate the electrically large TARA using a PC. To model the TARA feed we use a CHE with an optimal parameter. Because of this it is necessary to calculate only one system of 1-D hypersingular and singular integral equations in the method [5-7]. This system corresponds to the first azimuthal harmonics of the current density components.

We consider the far-field of TARA and the calculation time is hundreds of times faster than an MLFMA-MoM algorithm [4]. Because of the efficiency of the method, we were able to investigate the near-field of TARA in transmitting and receiving cases.

An interesting physical fact was observed that escapes geometrical-optics descriptions: In addition to the main focal spot (area of the field concentration) there is another split "focus" near the paraboloid top.

In addition, elementary numerical optimization of the TARA paraboloid plus shield antenna has been performed. It showed that the directivity can be improved and the length of the conical shield can be reduced by a half meter by changing the shield inclination from 30 deg to 5 deg.

Using the rigorous theory of integral equations with a Nystrom-type discretization offers many opportunities, not only in reflector antenna design. One can find a lot of applications in the diffraction by dielectric bodies, and eigenvalue problems for dielectric bodies. Therefore as a future step we propose to further improve the method of [5-7] to investigate dielectric bodies of revolution.

REFERENCES

[1] A. G. Davydov, E. V. Zakharov, and Y.V. Pimenov, "Numerical analysis of fields in the case of electromagnetic excitation of unclosed surfaces," *J. Communic. Technol. Electronics*, vol. 45, no l.2, pp. 247-259, 2000.

[2] A. G. Davydov , E. V. Zakharov , Y. V. Pimenov, . "Numerical decision method of the electromagnetic wave diffraction by unclosed surfaces problem" *Dokl. Akad. Nauk SSSR*, 1984, 276, no.1, pp.96-100.

[3] A. Berthon, R. Bills, "Integral analysis of radiating structures of revolution" *IEEE Trans. Antennas Propagat.* 1989, vol. 37, no 3,. pp. 159-170.

[4] A. Heldring, J.M. Rius, L. P. Ligthart, and A. Cardama, "Accurate numerical modeling of the TARA reflector system," *IEEE Trans. Antennas Propagat.*, vol. 52, no 7, pp. 1758-1766, 2004.

[5] V. S. Bulygin, Y. V. Gandel, A. I. Nosich "Near field focusing and radar cross section for a finite paraboloidal screen" *European conference of Antennas and Propagation (EuCAP-2011),* Rome, 2011, pp 3836-3840

[6] V.S. Bulygin, A.I. Nosich, Y.V. Gandel, "Nystrom-type method in three-dimensional electromagnetic diffraction by a finite PEC rotationally symmetric surface," *IEEE Trans. Antennas and Propagation,* 2011, submitted (#AP-1106-0686)

[7] M.V. Balaban, E.I. Smotrova, O.V. Shapoval, V.S. Bulygin, A.I. Nosich, "Nystrom-type techniques for solving electromagnetics integral equations with smooth and singular kernels," *J. Numerical Modeling: Electronic Networks, Devices and Fields*, 2012, submitted (#JNM-11-0081).

[8] Y. V. Gandel, S. V. Eremenko, T. S. Polyanskaya, "Mathematical problems of the Method of Discrete Currents", KhNU Press,1992 (in Russian).

[9] M.R. Barclay and W.V.T. Rusch, "Moment-method analysis of large, axially symmetric reflector antennas using entire-domain functions," *IEEE Trans. Antennas Propagat.*, vol. 39, no 4, pp. 491-496, 1991.

[10] .W. Gander and W. Gautschi, "Adaptive quadrature—Revisited," *BIT Numer. Mat.*, vol. 40, no. 1, pp. 84–101, 2000.

[11] A. A. K. Mohsen and A. K. Abdelmageed, "A fast algorithm for treating EM scattering by bodies of revolution," *Int. J. Elect. Commun.*, no. 3, pp. 164–170, 2001.

[12] E. N. Vasilyev, *Excitation of Bodies of Rotation*, Moscow: Radio i Svyaz, 1987 (in Russian).

[13] K. Abdelmageed, "Efficient evaluation of modal Green's functions arising in EM scattering by bodies of revolution," *Prog. Electromagn. Res.*, vol. 27, pp. 337–356, 2000.

[14] R.D. Graglia, P.L.E. Uslenghi, R. Vitiello, U. D'Elia, "Electromagnetic scattering for oblique incidence on impedance bodies of revolution," *IEEE Trans. Antennas Propag*, vol. 43, pp. 11–26, Jan. 1995.

[15] W. M. Yu, D. G. Fang, T. J. Cui "Closed form modal Green's functions for accelerated computation of bodies of revolution" *IEEE Trans. Antennas Propag*, vol. 56, No 11, pp. 3452–3461, Jan. 2008.

[16] D.R.Wilton, N.J. Champagne "Evaluation and integration of the thin wire kernel" *IEEE Trans. Antennas Propag*, vol. 54, No 4, pp. 1200–1206, Apr. 2006.

[17] D. Greenwood, J.-M. Jin "Finite-element analysis of complex axisymmetric radiating structures" *IEEE Trans. Antennas Propag*, vol. 47, No 8, pp. 1260–1266, Aug. 1999.

[18] S.D. Gedney, R. Mittra "The use of the FFT for the efficient solution of the problem of electromagnetic scattering by a body of revolution" *IEEE Trans. Antennas Propag*, vol. 38, No 3, pp. 313–322, Mar. 1990.

[19] Gradshteyn N.S. and Ryzhik "Table of Integrals, Series, and Products", Moscow:Fizmatgiz, 1962, 1100 p.