Near Field Focusing and Radar Cross Section for a Finite Paraboloidal Screen

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***Abstract*— A problem of monochromatic electromagnetic wave diffraction by a perfectly electrically conducting (PEC) surface of rotation located in a free space is investigated. The problem is reduced to sets of hypersingular and singular integral equations, which are solved by the method of discrete singularities, using interpolation type quadrature formulas. From the solutions of these sets the electric field expression and the far –zone patterns are obtained. The presented method has a guaranteed convergence for none axially symmetric primary field.**

I. Introduction

Based on solving of integral equations methods are used in numerical modeling of three-dimensional arbitrary electromagnetic wave diffraction by an arbitrary PEC surface for a long time [1,2]. However, such methods are in need with considerable computer resources, because deals with big-size matrices, and still the question of their convergence is open.

The Nystrom method means that unknown smooth functions approximate by polynomials. Because of this, the numerical algorithms, using this method, have high-order convergence. However, they are used only in two-dimensional diffraction problems [3-5]. Using the Nystrom method, we developed a fast numerical algorithm with guaranteed convergence for three-dimensional diffraction by the rotation surface, deals with the small-size matrices.

Among the works on the of electromagnetic waves diffraction by perfectly conducting rotation surfaces should mark the papers [6,7,8]. Because of analytical taking into account of axial symmetry of perfectly conducting rotation surface, three-dimensional diffraction problem is reduced to the one-dimensional integro-differential equations systems solving (although for three-dimensional diffraction problems is typical the two-dimensional ones), that significantly decreases the order of discrete model matrix.

The methods developed only for some specific three-dimensional problems with reflectors of simple shape (disc, sphere, cone, circular cylinder) and only in the case of primary fields small set is rigorously justified and have guaranteed convergence. Let list them below. The electromagnetic wave diffraction by PEC circular disk is considered for example in [9,10], by PEC circular cylinder is consider in [11]. In [12] the axial-symmetric electromagnetic wave diffraction by PEC finite cone is solved, using the method of analytical regularization. The plane electromagnetic wave diffraction by PEC sphere with a hole is solved in [13] in the case of plane wave propagate parallel to its rotation axis.

In the paper the arbitrary electromagnetic wave diffraction by arbitrary PEC rotation surface reduce to systems of one-dimensional hypersingular and singular integral equations with variable coefficients for the first time. Numerically these equations are solved, using interpolation-type quadrature formulas that provide a high convergence order.

II. Diffraction by Rotation Surface

A. Problem formulation

A problem of arbitrary monochromatic electromagnetic wave $(\vec{E}^0,\vec{H}^0)$ diffraction by a PEC surface of rotation S located in a free space is investigated. The total electromagnetic field $(\vec{E}^{tot},\vec{H}^{tot})$ is presented in the form of primary $(\vec{E}^0,\vec{H}^0)$ and scattering field $(\vec{E},\vec{H})$ sum. The scattering electrical and magnetic field satisfy Maxwell equations, Sommerfeld radiation condition, Meixner edge condition and PEC boundary condition on the surface of rotation.

The dependence of the fields on time is $e^{i\omega t}$. Choose cylindrical coordinates ρ,φ,z . The surface S is created by rotation some contour C around the axis z . It is convenient to choose curvilinear orthogonal coordinates q,τ,φ in which the surface S has the parameterization

$$S: q = q_0\,,\ \tau\in[-1,1]\,,\ \varphi\in[0,2\pi]\,. \tag{1}$$

Dependence of cylindrical coordinates from the introduced curvilinear coordinates is expressed, using the formula

$$\rho = \rho(q,\tau)\,,\ \ z = z(q,\tau) \tag{2}$$

The Lame coefficients of the coordinate system q,τ,φ is

$$l_q = \sqrt{(\rho_q')^2+(z_q')^2}\ ,\ l_\tau = \sqrt{(\rho_\tau')^2+(z_\tau')^2}\ ,\ l_\phi = \rho\,.$$

The curvilinear coordinate orts is $(\vec{q}^{\,0},\vec{\tau}^{\,0},\vec{\varphi}^{\,0})$:

$$\vec{\nu}^{\,0} = (\vec{x}^{\,0}\rho_\nu'\cos\varphi + \vec{y}^{\,0}\rho_\nu'\sin\varphi + \vec{z}^{\,0}z_\nu')/l_\nu\,,\ \ \nu = q,\tau \tag{3}$$

$$\vec{\varphi}^{\,0} = -\vec{x}^{\,0}\sin\varphi + \vec{y}^{\,0}\cos\varphi \tag{4}$$

The points on the surface S have such cylindrical coordinates: $\rho = \rho(t) = \rho(q_0, t), \quad z = z(t) = z(q_0, t)$. If t is the integration variable, we will use notations like $\rho_0 = \rho_0(t),\ z_0 = z_0(t)$, $h_\tau = l_\tau(q_0, t)$, $t \in [-1,1]$, while for the observation point we will use notations as $\rho := \rho(\tau),\ z := z(\tau)$, $\tau \in [-1,1]$. The rotation surface, curvilinear orts, an integration point X , an observation point Y and the primary field $(\vec{E}^0, \vec{H}^0)$ are demonstrated in Fig.1:

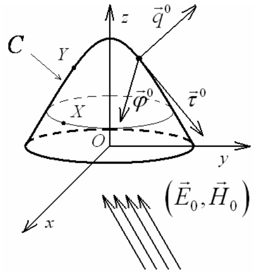

Fig. 1 Problem formulation

In term of introduced notations, the PEC boundary condition is equal to:

$$E_\tau + E_\tau^0\Big|_S = E_\varphi + E_\varphi^0\Big|_S = 0 \qquad (5)$$

Our purpose is to find the scattering field $(\vec{E}, \vec{H})$.

B. The systems of hypersingular and singular integral equations

The electric field vector $\vec{E}$ can be expressed in terms of the electromagnetic scalar and vector potentials as:

$$\vec{E} = -grad\Phi - i\varpi \vec{A} \qquad (6)$$

Using expression for electrical field throw scalar and vector potentials (6), the PEC boundary condition (5) are written as:

$$\lim_{X \to Y}[i\omega A_\tau(X) + (1/l_\tau)\partial\Psi / \partial\tau(X)] = E_\tau^0(Y)\,,\ Y \in S\,, \qquad (7)$$

$$\lim_{X \to Y}[i\varpi A_\varphi(X) + (1/\rho)\partial\Psi / \partial\varphi(X)] = E_\varphi^0(Y)\,,\ Y \in S\,, \qquad (8)$$

where $\vec{A}(Y)$ is the vector electromagnetic potential and Ψ is the scalar electromagnetic potential.

The current density components j_τ and j_φ components and the primary field $\vec{E}^0$ components on surface S are represented in terms of the Fourier series on the surface S as follows:

$$j_\nu(t, \psi) = \sum_{M=-\infty}^{\infty} j_\nu^M(t) e^{iM\psi}\ ,\ \nu = \tau, \varphi \qquad (11)$$

$$E_\nu^0(t, \psi) = \sum_{M=-\infty}^{\infty} E_\nu^{0(M)}(t) e^{iM\psi}\ , \nu = \tau, \varphi \qquad (12)$$

Introduce unknown smooth functions $u^M(t)$ and $w^M(t)$ in conformity with the edge condition:

$$\rho(t) j_\tau^M(t) = u^M(t)\sqrt{1-t^2}\ ,\ j_\varphi^M(t) h_\tau(t) = w^M(t) / \sqrt{1-t^2} \quad (13)$$

For going to the limit in (7), (8) we need to introduce the following integral operators:

hypersingular integral operator understood in the sense of Hadamard finite part

$$(Au)(\tau) = (1/\pi)\int_{-1}^{1} u(t)\sqrt{1-t^2}\,/(\tau - t)^2\, dt \qquad (14)$$

the singular integral operators with the different weights:

$$(\Gamma u)(\tau) = (1/\pi)\int_{-1}^{1} u(t) / ((\tau - t)\sqrt{1-t^2})dt\,, \qquad (15)$$

$$(\Gamma^{-1} u)(\tau) = (1/\pi)\int_{-1}^{1} u(t)\sqrt{1-t^2}\,/(\tau - t)dt\,, \qquad (16)$$

integral operators with logarithm kernels:

$$(L^I u)(\tau) = (1/\pi)\int_{-1}^{1} \ln|\tau - t|\, u(t)\sqrt{1-t^2}\, dt\,, \qquad (17)$$

$$(L^{II} u)(\tau) = (1/\pi)\int_{-1}^{1} \ln|\tau - t|\, u(t) / \sqrt{1-t^2}\, dt\,, \qquad (18)$$

and integral operators with smooth kernels:

$$(K^I u)(\tau) = (1/\pi)\int_{-1}^{1} K(\tau, t) u(t)\sqrt{1-t^2}\, dt \qquad (19)$$

$$(K^{II} u)(\tau) = (1/\pi)\int_{-1}^{1} K(\tau, t) u(t) / \sqrt{1-t^2}\, dt\,. \qquad (20)$$

We will notate the integral operators with smooth kernels (19,20) and kernels of this operators, using the same symbol.

If we go to the limit in (7)-(8) then we obtain the set of one-dimensional hypersingular and singular integral equations (HSIE and SIE) systems with variable coefficients:

$$\begin{pmatrix} a_{11}A + b_{11}\Gamma^{-1} + c_{11}^M L^I + K_{11}^{M(I)} & b_{12}^M \Gamma + c_{12}^M L^{II} + K_{12}^{M(II)} \\ b_{21}^M \Gamma^{-1} + c_{21}^M L^I + K_{21}^{M(I)} & c_{22}^M L^{II} + K_{22}^{M(II)} \end{pmatrix} \bullet$$

$$\bullet \begin{pmatrix} w^M(\tau) \\ u^M(\tau) \end{pmatrix} = \begin{pmatrix} 4ik\rho^3 E_\tau^{0(M)}(\tau) h_\tau(\tau) / Z \\ 4ik\rho^3 E_\varphi^{0(M)}(\tau) / Z \end{pmatrix},\ M \in \mathbb{Z} \qquad (21)$$

where the variable coefficients are

$$a_{11}(\tau) = -2\rho^2\,, b_{11}(\tau) = -\rho'\rho\,, \qquad (22)$$

$$c_{11}^M(\tau) = (k^2\rho^2 + M^2)(\rho'^2 + z'^2) - (z'^2 + 3\rho'^2)/4\,, \quad (33)$$

$$b_{12}^M(\tau) = 2iM\rho^2\,, c_{12}^M(\tau) = -iM\rho'\rho\,, b_{21}^M(\tau) = 2iM\rho\,, \quad (24)$$

$$c_{21}^M(\tau) = iM\rho'\,,\ c_{22}^M(\tau) = -2M^2\rho + 2k^2\rho^3 \qquad (25)$$

the smooth kernels are

$$K_{11}^M(\tau,t)=\rho^3[\partial^2 S_M/\partial t\partial\tau - k^2(\rho_0'\rho' S_M^+ + z_0' z' S_M)] - \\ -a_{11}(\tau)/(\tau-t)^2 - b_{11}(\tau)/(\tau-t) - c_{11}^M(\tau)\ln|\tau-t| \quad (26)$$

$$K_{12}^M(\tau,t)=\rho^3[k^2\rho_0\rho' S_M^- - iM\cdot\partial S_M/\partial\tau] - \\ -b_{12}^M(\tau)/(\tau-t) - c_{12}^M(\tau)\ln|\tau-t| \quad (27)$$

$$K_{21}^M(\tau,t)=\rho^3\left[(iM/\rho)\partial S_M/\partial t - k^2\rho_0' S_M^-\right] - \\ -b_{21}^M(\tau)/(\tau-t) - c_{21}^M(\tau)\ln|\tau-t| \quad (28)$$

$$K_{22}^M(\tau,t)=\rho^3\left[(M^2/\rho)S_M - k^2\rho_0 S_M^+\right] - c_{22}^M(\tau)\ln|\tau-t| \quad (29)$$

$$S_M(q,\tau,t)=S_M=\int_{-1}^{1}\cos(M\psi)e^{-ikL}/L d\psi, \quad (30)$$

$$L=\sqrt{\rho^2+\rho_0{}^2-2\rho\rho_0\cos\psi+(z-z_0)^2},\ Z=\sqrt{\mu/\varepsilon}. \quad (31)$$

Successively satisfying the first integral equation in the system (21) at the zeros of the second kind Chebyshev polynomial $\{t_{0j}^n\}_{j=0}^{n-2}=\{\cos(j+1)\pi/n\}_{j=0}^{n-2}$ and the second integral equation in (21) at the zeros of the first kind Chebyshev polynomial $\{t_k^n\}_{k=0}^{n-1}=\{\cos(2k+1)\pi/(2n)\}_{k=0}^{n-1}$, using the interpolation type quadrature formulas [14] we obtain a set of linear algebraic equations with unknowns $u_{n-2}^{(M)}\left(t_{0l}^n\right)$ and $w_{n-1}^{(M)}\left(t_k^n\right)$. Convergence of approximate solutions to exact one if $n\to\infty$ is guaranteed according to [15].

C. *Electric field and far-field scattering patterns expressions*

Representing in terms of the Fourier series the scalar and vector potentials we obtain expressions for electrical field using the solutions $u_{n-2}^M(t_0)$ and $w_{n-1}^M(t)$ of the system (21):

$$E_\nu(\rho,z,\varphi)=\sum_{M=-\infty}^{\infty}E_\nu^M(\rho,z)e^{iM\varphi},\ \nu=\rho,\varphi,z \quad (32)$$

$$E_\nu^M(\rho,z)=(iZ/4\pi k)[\int_{-1}^{1}Q_{\nu\tau}^M(\rho,z,t)u^M(t)\sqrt{1-t^2}dt \\ +\int_{-1}^{1}Q_{\nu\varphi}^M(\rho,z,t)w^M(t)/\sqrt{1-t^2}dt],\ \nu=\rho,\varphi,z, \quad (33)$$

$$Q_{\rho\tau}^M=\frac{\partial^2 S_M}{\partial\rho\partial t}-k^2\rho_0' S_M^+,\ Q_{\rho\varphi}^M=\rho_0 k^2 S_M^- - iM\frac{\partial S_M}{\partial\rho}, \quad (34)$$

$$Q_{\varphi\tau}^M=\frac{iM}{\rho}\frac{\partial S_M}{\partial t}-k^2\rho_0' S_M^-,\ Q_{\varphi\varphi}^M=\frac{M^2}{\rho}S_M-k^2\rho_0 S_M^+, \quad (35)$$

$$Q_{z\tau}^M=\partial^2 S_M/\partial z\partial t-k^2 z_0' S_M,\ Q_{z\varphi}^M=(-iM)\partial S_M/\partial z. \quad (36)$$

The far-field scattering pattern components are defined as follows:

$$F_\eta(\theta,\varphi)=\lim_{R\to\infty}E_\eta(R,\theta,\varphi)Re^{ikR},\ \ \eta=\theta,\varphi \quad (37)$$

Using the expressions (32)-(36) for electric field, we obtain the far-field scattering pattern components expressions:

$$F_\nu(\theta,\varphi)=\sum_{M=-\infty}^{+\infty}e^{iM\varphi}F_\nu^M(\theta),\ \nu=\theta,\varphi \quad (38)$$

$$F_\nu^M(\theta)=(iZ/4\pi k)[\int_{-1}^{1}U_{\nu\tau}^M(\theta,t)u^M(t)\sqrt{1-t^2}dt+ \\ +\int_{-1}^{1}U_{\nu\varphi}^M(\theta,t)w^M(t)/\sqrt{1-t^2}dt], \quad (39)$$

$$U_{\varphi\tau}^M=(-k^2\rho_0'/2i)(f^{M+1}-f^{M-1}), \quad (40)$$

$$U_{\varphi\varphi}^M=(-k^2\rho_0/2)(f^{M+1}+f^{M-1}), \quad (41)$$

$$U_{\theta\tau}^M=k^2[z_0' f^M\sin\theta-\rho_0'\cos\theta(f^{M+1}+f^{M-1})/2], \quad (42)$$

$$U_{\theta\varphi}^M(\theta,t)=k^2\rho_0\cos\theta(f^{M+1}-f^{M-1})/2i. \quad (43)$$

Denote the far-field scattering pattern:

$$F(\theta,\varphi)=\sqrt{|F_\theta(\theta,\varphi)|^2+|F_\varphi(\theta,\varphi)|^2} \quad (44)$$

III. PARABOLIC REFLECTOR ILLUMINATED BY A PLANE ELECTROMAGNETIC WAVE.

Consider a paraboloidal reflector with the focal distance f and the diameter D illuminated by a plane electromagnetic wave (Fig.2).

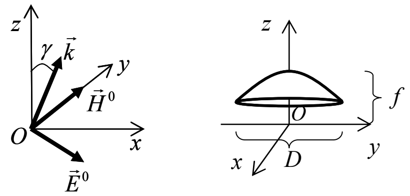

Fig. 2 Parabolic reflector illuminated by the electromagnetic plane wave

The plane wave electric field expression is

$$\vec{E}^0(\rho,\varphi,z)=Z\vec{m}e^{-i(\vec{k},\vec{R})}, \quad (45)$$

where $\vec{R}=(\rho\cos\varphi,\rho\sin\varphi,z)$, $\vec{k}=k(\sin\gamma,0,\cos\gamma)$, $\vec{m}=(\cos\gamma,0,-\sin\gamma)$ .Using (3,4) and (45) we obtain E_τ^0 and E_φ^0 azimuth harmonics:

$$E_\tau^{0(M)}=e^{-ikz\cos(\gamma)}Z[(1/2)\rho_\tau'\cos(\gamma)[(-i)^{M+1}J_{M+1}(T)+ \\ +(-i)^{M-1}J_{M-1}(T)]-z_\tau'\sin(\gamma)(-i)^M J_M(T)]/l_\tau, \quad (46)$$

$$E_\varphi^{0(M)}=-e^{-ikz\cos(\gamma)}Z\cos(\gamma)(1/2i)\cdot \\ \cdot[(-i)^{M-1}J_{M-1}(T)-(-i)^{M+1}J_{M+1}(T)],\ T=k\rho\sin(\gamma) \quad (47)$$

In Fig.3 and Fig.4 we show the normalized backscattering cross section $\sigma_B/\pi D^2$, $\sigma_B=4\pi|F(\pi+\gamma,0)|^2/Z^2$ and total scattering cross section $\sigma/\pi D^2$, $\sigma=\int_0^\pi\int_0^{2\pi}F(\theta,\varphi)^2\sin(\theta)d\theta d\varphi/Z^2$ (respectively) as a function of the normalized frequency in the case of the orthogonal incidence ($\gamma=0$), $f/D=0.5$ and $f/D=0.25$.

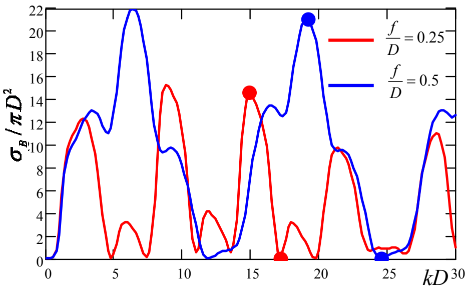

Fig.3 Paraboloidal reflector $f/D=0.5$ and $f/D=0.25$ normalized backscattering cross section in the case of the orthogonal incidence ($\gamma=0$).

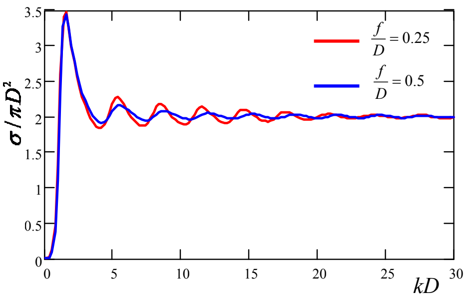

Fig.4 Paraboloidal reflector $f/D=0.5$ and $f/D=0.25$ normalized total scattering cross section in the case of the orthogonal incidence ($\gamma=0$).

In Fig. 3 we mark backscattering cross section local minimum and maximum. In the case of $f/D=0.25$ $\sigma_B/\pi D^2=14.7$ in $ka=14.9$ and $\sigma_B/\pi D^2=0.01$ in $ka=17.2$. In the case of $f/D=0.5$ $\sigma_B/\pi D^2=21.1$ in $kD=19.2$ and $\sigma_B/\pi D^2=0.073$ in $ka=24.6$

Let consider the normalized backscattering cross section as a function of the plane wave incident angle γ in the marked in Fig. 3 the local $\sigma_B/\pi D^2$ maximums $kD=14.9$, $f/D=0.25$ and $kD=19.2$, $f/D=0.5$ (Fig.5) and the local $\sigma_B/\pi D^2$ minimums $kD=17.2$, $f/D=0.25$ and $kD=24.6$, $f/D=0.5$ (Fig.5,6).

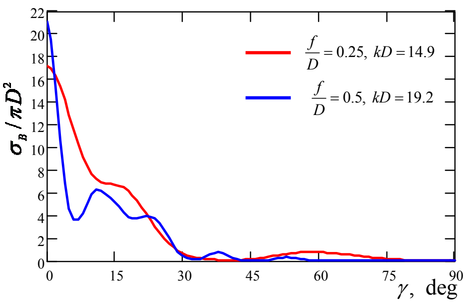

Fig.5. Paraboloidal reflector $f/D=0.5$ and $f/D=0.25$ normalized backscattering cross section as a function of the angle γ .

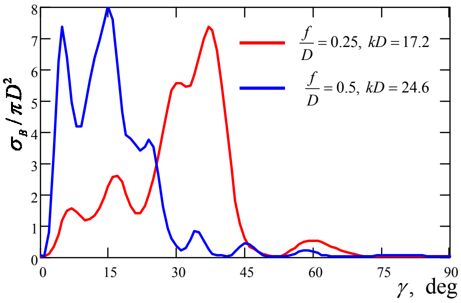

Fig.6 Paraboloidal reflector $f/D=0.5$ and $f/D=0.25$ normalized backscattering cross section as a function of the angle γ

In Fig. 7 we show total electric field module in the near-zone of the paraboloidal reflector $f/D=0.5$ illuminated by the orthogonal incident plane wave in $kD=19.2$.

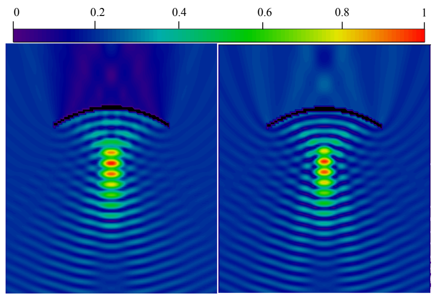

Fig. 7. The near fields of the paraboloidal reflector $f/D=0.5$ illuminated by the plane wave in $kD=19.2$ .in the E-plane (left) and the H-plane (right)

In Fig. 8 we show total electric field in the near-zone of the paraboloidal reflector $f/D=0.25$ illuminated by the orthogonal incident plane wave in $kD=14.9$.

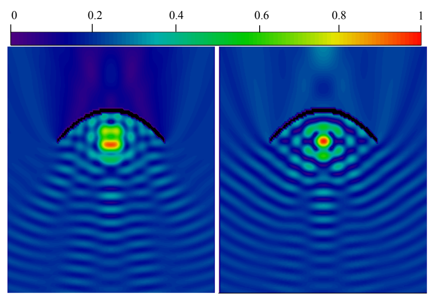

Fig. 8. The near fields of the paraboloidal reflector $f/D=0.25$ illuminated by the plane wave in $kD=14.9$ .in the E-plane (left) and the H-plane (right)

Let consider the field behavior in the paraboloid geometric focus. The electric field module in the focus is $E^{foc}=|\vec{E}(0,0,0)|$. The $E^{foc}/|\vec{E}^0|$ as a function of f/D is shown in Fig. 9 in the case of $D=5\lambda,\ 10\lambda,\ 15\lambda$, $\gamma=0$

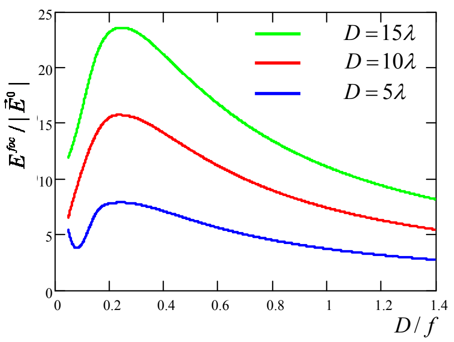

Fig. 9. The electric field module in the paraboloid geometric focus as a function of f / D in the case of $D = 5\lambda,\ 10\lambda,\ 15\lambda$, $\gamma = 0$

One can see that the biggest electric field module in the focus we obtain in the case of $f / D \approx 0.25$.

The $E^{foc} / |\vec{E}^0|$ as a function of kD is shown in Fig. 10. in the case of $f / D = 0.25, 0.5$, $\gamma = 0$

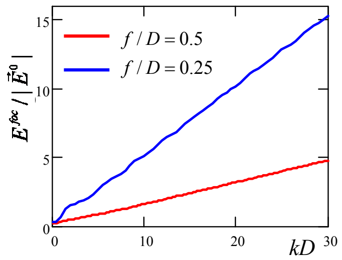

Fig. 10. The electric field module in the paraboloid geometric focus as a function of normalized frequency in the case of $f / D = 0.25, 0.5$, $\gamma = 0$.

The $E^{foc} / |\vec{E}^0|$ as a function of the angle γ is shown in Fig. 11 in the case of $D = 10\lambda$, $f / D = 0.25, 0.5$ in logarithm scale.

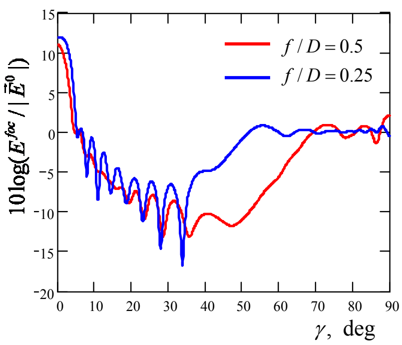

Fig. 11. The electric field module in the paraboloid geometric focus as a function of the angle γ in the case of $f / D = 0.25, 0.5$, $D = 10\lambda$.

IV. Conclusions

Summarizing, we have presented an efficient and convergent numerical method for the analysis of an arbitrary PEC rotationally symmetric screen illuminated by an arbitrary incident electromagnetic field. The presented method is the most efficient in the case of Fourier series primary field rapidly converges or consists of a finite number of harmonics. For example in the case of considered in the paper PEC surface illumination by plane electromagnetic wave propagating parallel to rotation axis problem, it is necessary to solve only one system of one-dimensional hypersingular and singular integral equations. Because of current density singularity on the surface edge is taken in account analytically and unknown smooth functions approximate by polynomials, the method have high-order convergence.

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