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METHOD OF ANALYTICAL REGULARIZATION IN THE PROBLEM OF ELECTROMAGNETIC WAVE SCATTERING BY A THIN DIELECTRIC DISK

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GENERAL DESCRIPTION

This work is devoted to the development of efficient numerical-analytical method of solving the problems of diffraction of electromagnetic waves by a thin dielectric disk and to the study of characteristics of absorption and emission of localized sources in the presence of such a disk.

Timeliness of research. The problem of diffraction of electromagnetic waves by a thin disk is of great interest both from theoretical and practical points of view. It is associated with wide use of thin dielectric and metal disks as elements of various microwave and optical devices. Examples of such devices are circular printed antennas, where a thin metal disk is used as a part of an open resonator formed by a disk and a metal ground plane. Besides, thin metal disks are often replaced with high-contrast dielectric ones made of ceramic with high permittivity. Typically, the thickness of the dielectric disk is less than a quarter-wavelength in the material. The use of such dielectric disks broadens the operating range of antennas. Printed and dielectric-disk antennas are used as elements of cellular antenna arrays, in the ground-based and space communication systems, and in radar. Besides, thin dielectric disks cause a significant interest in radio-physics of the millimeter-wave and terahertz ranges and in micro and nano-optics. This is due largely to the use of semiconductor, crystal and polymeric thin flat microdisks as laser resonators with injection-type or photo-pumped active zones. The working frequencies of these devices are at terahertz and optical wavelengths. Such lasers are considered as the most promising sources of terahertz radiation. Another area where the problem of scattering of electromagnetic waves by a disk is of interest is the optics of colloidal systems. The reason for this is the use of nanosize disks of gold or silver as the metal cores of bio-molecular probes having properties of "recognition". Such probes are widely used in biosensing and genomics, and medical and optical coherent tomography.

To study the problems of scattering of electromagnetic waves by thin disks, researchers have developed various approaches and methods including asymptotic methods, direct methods of computational electromagnetics, and specialized numerical-analytical methods. However, asymptotic methods deliver only an approximate short-wave or long-wave solution of the problem in analytical form. Direct computational methods such as finite element method (FEM), boundary element method (BEM), the method of moments (MoM) and finite differences in time domain (FDTD) can obtain a numerical solution of the problem for the disk size comparable to the wavelength (in free space) with limited (and actually quite low) accuracy.

Thus, the task of building specialized numerical-analytical methods for solving the problems of diffraction by a thin dielectric disk and development of based on them algorithms having guaranteed convergence and providing the solution with controlled accuracy in a relatively short time is still timely and important.

Relation to R&D programs and projects. The research related to the thesis has been done in the framework of

1. Government R&D projects of IRE NASU: “Theoretical and experimental investigation of wave processes in the devices and components of microwave and millimeter-wave bands” (code Buksir-2, #01.00U006441, 2002-2006) and “Development and application of new methods of computational radio-physics, theoretical and experimental investigation of transformations of electromagnetic fields of the GHz and THz bands in the objects and media of anthropogenic and natural origin” (code Buksir-3, #01.06U011975, 2007-2011).
2. State Target Program "Nanotechnologies and nanomaterials," project "Fundamental mathematical and numerical study of optical electromagnetic fields of stand-alone and coupled microcavity lasers with nanosize active layers, wires and strips" (code Svitlo, #01.10U004737, 2010-2012).
3. Competitive program of NASU “Nanostructured systems, nanomaterials and nanotechnologies,” project “Micro and nanoscale electromagnetic modeling of optical fields

in resonators with active regions shaped as quantum layers, wires and dots" (code Porig, # 01.07U003983, 2007-2009).

4. Competitive project of the Ministry of Education and Science, Ukraine "Innovative numerical modeling of quasioptical focusing systems" (code Fokus, # 01.09U005351, 2009-2010).
5. Exchange program between NASU and TUBITAK, joint project «Innovative electromagnetic modeling of multielement quasioptical focusing systems for sub-mm and terahertz ranges» (#106E209, 2007-2009) with the Bilkent University, Ankara.
6. Training and Networking Programme «Advanced numerical modelling and design of dielectric lens antennas» of the Ministry of Higher Education and Research, France (2007-2009) with the Institute of Electronics and Telecommunications, University of Rennes 1, France.
7. Research and Networking Programme «NewFocus: New frontiers in millimeter and sub-millimeter wave integrated dielectric focusing systems» of the European Science Foundation, project «Scattering and focusing of electromagnetic waves by a thin dielectric disk» (2011) with the Institute of Electronics and Telecommunications, University of Rennes 1, France.

The aims and objectives of the study. The study aims at construction of three-dimensional model of the scattering of arbitrary electromagnetic waves by a thin dielectric disk, development of a numerical analysis algorithm based on analytical regularization, and systematic calculations of the characteristics of the disk scattering and absorption at the excitation with localized sources. The most interesting aspect of this work is to study the role of the position of the source in the excitation of the eigenmodes of the disk dielectric resonator. To achieve this goal it is necessary to consider the following tasks:

- develop a mathematical model of the three-dimensional scattering of an arbitrary vector electromagnetic field by zero-thickness perfectly electrically conducting and resistive disks;
- modify the developed mathematical model for the study of the wave scattering by a thin dielectric disk with losses:
- develop algorithms for calculating the power of radiation and absorption, as well as far-field radiation patterns for a localized source in the presence of dielectric disk;
- perform numerical study of the Purcell effect, which reduces, in terms of the classical electromagnetics, to the study of the power radiated by an elementary dipole in the presence of a disk.

The object of study is the phenomena of the scattering and absorption of time-harmonic electromagnetic waves by thin dielectric and metal disks.

The specific topic of research is the impact of perfectly conducting metal and dielectric disks, located in the near zone of a localized source, on the powers of radiation and absorption, and on the far-field radiation pattern.

Research Methods. The thesis has used the methods of the theory of dual integral equations in electromagnetics problems. In the modeling of thin resistive and dielectric disks we have used two-side generalized boundary conditions for thin layers combined with the edge condition. We have developed fast convergent algorithms of high accuracy. They are based on the reduction of these problems to the systems of linear algebraic equations using the following methods:

- The method of analytical regularization that is analytical inversion of the most singular parts of the dual integral equations operators and their reduction to integral equations of the Fredholm second kind on a semi-axis.
- The method of quadratures (Nystrom method) used to build a discrete model of the Fredholm second-kind integral equations.

Scientific novelty is determined by the following results obtained for the first time**:**

1. A mathematically grounded model for the three-dimensional scattering of the time-harmonic vector electromagnetic field by a thin dielectric disk has been built and reduced to the equivalent dual integral equations.
2. The method of analytical regularization for the reduction of dual integral equations to coupled integral equations of the Fredholm second kind has been developed, and a discrete model of these equations has been built, based on the interpolation method of quadratures (Nystrom method) and having guaranteed convergence.
3. If a dielectric disk is excited by arbitrarily located dipoles, the range of electrical radii (radius in terms of wavelength) has been identified, where the disk modes with fixed value of the radial index and different azimuth indices are radiating in most efficient manner.
4. The study of the power radiated by elementary dipoles in the presence of the dielectric disk and the absorbed power if the disk is lossy (Purcell effect) has been performed. It has been found that both characteristics exhibit resonant behavior, but the efficiency of radiation does not always behave in resonant manner.

Practical significance of obtained results is as follows:

1. The developed method and associated numerical algorithms can be used to calculate the radiation characteristics of sources in the presence of thin disks, and also the scattering of the other waves by them.
2. The created algorithms and software far exceed the known analogues in terms of efficiency and versatility. This allows their use as a core in computer-aided analysis and design of the basic characteristics of the devices whose key elements are thin disks under arbitrary-field excitation.
3. The obtained results broaden the understanding of the wave phenomena in the presence of disks and the effect of source location on the excitation of the whispering gallery modes in thin-disk dielectric resonators.
4. These results can be used to determine the optimal location of the source for the selective excitation of modes with a given number of variations of field along the disk azimuth and radius.

Personal contribution of the candidate. The main results presented in the thesis belong to the author. In the published with co-authors papers [2-4], this contribution is the derivation of basic equations, the development of computational algorithms, and the systematic calculation of characteristics of the powers of radiation and absorption and far-field patterns, and the interpretation of the numerical results. In the review paper [5], it is the derivation of basic equations and computation of the results that illustrate the variation of spontaneous emission and absorption rates of elementary radiator in the presence of a thin dielectric disk.

Dissemination of results. The results have been reported and discussed at the following scientific seminars: IRE NASU "Theory of diffraction and diffraction electronics" (headed by Prof. P.M. Melezhik), Department of Mathematics of the Kharkiv National University of Radio Electronics "Integral equations of electromagnetics" (headed by Prof. O.G. Nerukh), Department of Electronics and Electrical Engineering of the Bilkent University, Ankara (headed by Prof. A. Altintas), George Green Institute of Electromagnetics Research of the University of Nottingham (headed by Prof. T.M. Benson), Institute of Photonics and Electronics of the Academy of Sciences of the Czech Republic, Prague (headed by Prof. J. Ctyroky) and at the following international conferences and symposia:

- Advanced Optoelectronics and Lasers, Alushta (2003), Yalta (2005), Sevastopol (2010).
- Physics and Engineering of Microwaves, MM, and Sub-MM Waves, Kharkiv (2004).
- Days on Diffraction, St. Petersburg (2004, 2007).

- Mathematical Methods in Electromagnetic Theory, Kiev (2004), Kharkiv (2006), Kiev (2010), Kharkiv (2012).
- Antennas and Electromagnetics, Saint Malo, France (2005).
- Microwave and Optical Technologies, Fukuoka, Japan (2005).
- European Conf. on Antennas and Propagation, Edinburgh, UK (2007), Rome, Italy (2011).
- Theoretical and Computational Nanophotonics, Bad Honnef, Germany (2011).
- Micro and Nano Photonic Materials and Devices, Trento, Italy (2012).

Publications. Materials of dissertation have been published in 21 scientific papers, including 5 papers in professional technical journals [1-5] and 16 papers at international conferences, most important of them being [6-15].

The structure and scope of the thesis. The work consists of introduction, 5 chapters, conclusions, and the list references. Its full size is 152 pages. The thesis contains 40 pictures, 2 of them on separate pages. The list of references used occupies 13 pages and contains 110 entries.

THESIS ESSENTIALS

The introduction grounds the timeliness of the chosen topic, formulates goals and objectives of the study, and presents the general characteristics of the thesis.

The first chapter is an overview of literature published around the topic of dissertation. It provides also general information about various applications of thin metal and dielectric disks in microwave and optical devices, surveys the known methods of solving the problems of scattering of electromagnetic waves by thin disks, and lists mathematical models and methods used in the thesis.

The problems of wave diffraction by thin disks have been attracting the attention of researchers for many years both from the theoretical and the practical sides. Various approaches and methods have been applied for their analysis. The section presents an overview of asymptotic, direct-numerical and specialized numerical-analytical methods. Particular attention is given to a group of methods having mathematical justification and leading to numerical algorithms with guaranteed convergence. Emphasized is important role played by the works of Y.V. Gandel[1], V.G. Sologub[2] and A.N. Khizhnyak[3] in connection to the problem of wave diffraction by a zero-thickness perfectly electrically conducting (PEC) disk.

The timeliness of the development of specialized numerical-analytical method of solving the problem of electromagnetic wave scattering by a dielectric disk is grounded; it should be based on the solution of a coupled pair of integral equations for each azimuth order of the field function, in conjunction with analytical regularization.

The second chapter is devoted to reducing the problem of scattering of an arbitrary time-harmonic (depending on time as $e^{-i\omega t}$) electromagnetic field by a thin dielectric disk to the system of the Fredholm integral equations in the space $L^2(0,\infty)$. As auxiliary problems, considered and regularized are the problems of scattering of arbitrary electromagnetic field by a zero-thickness PEC disk and a resistive disk.

We begin from the formulation of the boundary-value problem for finding the components of the electromagnetic field scattered by the disk of radius a and thickness τ. It involves a set of homogeneous Maxwell differential equations for the total field (sum of the incident and the scattered fields) and the generalized (effective) boundary conditions in the plane of the central section of the disk,

[1] Y.V. Gandel, "Integral equations of some axially symmetric problems of wave diffraction," PhD Thesis, Kharkiv State University, 1971.
[2] V.G. Sologub, "Short-wave asymptotics of the solution to the problem of wave diffraction by a circular disk," USSR. J. Computational Mathematics and Mathematical Physics, vol. 12, no 3, pp. 388-412, 1971.
[3] A.N. Khizhnyak, "Plane wave diffraction by a thin disk," USSR Acoustic J., vol. 25, no 6, pp. 929-933, 1989.

$$\left[\vec{E}_{tg}^{+}+\vec{E}_{tg}^{-}\right]=2Z_0 R\cdot\vec{n}\times\left[\vec{H}_{tg}^{+}-\vec{H}_{tg}^{-}\right],\qquad Z_0\left[\vec{H}_{tg}^{+}+\vec{H}_{tg}^{-}\right]=-2Q\cdot\vec{n}\times\left[\vec{E}_{tg}^{+}-\vec{E}_{tg}^{-}\right].$$

Here Z_0 is the free-space impedance, $\vec{n}$ is the unit normal vector to the disk. Also required is that the scattered field components satisfy the Silver-Mueller condition of radiation at infinity and certain conditions on the behavior of the components of electric and magnetic currents at the disk edge (or only electric current on a resistive or PEC disk). In the case of dielectric and resistive disks they are as follows:

$$j_r(\rho,\varphi)\underset{\rho\to1}{\simeq}\mathrm{O}\left(1-\rho^2\right)^{1/2},\qquad j_\varphi(\rho,\varphi)\underset{\rho\to1}{\simeq}\mathrm{O}\left(1\right).$$

This disk thickness τ is assumed to be small, so that $k\tau<<1$, k being the free-space wavenumber. It enters the generalized boundary conditions as a parameter in the expressions for the so-called electric and magnetic resistivities, namely

– in the case of a dielectric disk with relative dielectric permittivity ε_r and magnetic permeability μ_r of the disk material and the intrinsic impedance $Z=(\mu_r/\varepsilon_r)^{1/2}$, this is

$$R=iZ\,/\,2\,\mathrm{ctg}\left(\sqrt{\varepsilon_r\mu_r}\,k\tau\,/\,2\right),\qquad Q=iZ^{-1}\,/\,2\,\mathrm{ctg}\left(\sqrt{\varepsilon_r\mu_r}\,k\tau\,/\,2\right);$$

– in the case of a resistive disk with finite electric conductivity σ, this is $R=(Z_0\sigma\tau)^{-1}$ and $Q=\infty$;

– in the case of a zero-thickness PEC disk, this is $R=0$ and $Q=\infty$.

Then the boundary-value problem for the Maxwell equations reduces to the following equivalent set of coupled dual integral equations for unknown functions, that are the images of the jump and average values of the normal to the disk components of the scattered field in the spectral space of the Bessel-Fourier transforms, $u_m^{sc,\pm}(\kappa)$ and $v_m^{sc,\pm}(\kappa)$:

$$\begin{cases}\int\limits_0^\infty \bar{H}_m(\kappa\rho)\begin{pmatrix}\gamma(\kappa)\left(u_m^{sc,-}(\kappa)+u_m^{in,-}(\kappa)\right)+2R\,ka\,u_m^{sc,-}(\kappa)\\ ika\left(v_m^{sc,+}(\kappa)+v_m^{in,+}(\kappa)\right)+2R\,i\,\gamma(\kappa)v_m^{sc,+}(\kappa)\end{pmatrix}d\kappa=\bar{0} & (\rho<1)\\ \int\limits_0^\infty \bar{H}_m(\kappa\rho)\begin{pmatrix}ika\,u_m^{sc,-}(\kappa)\\ -\gamma(\kappa)v_m^{sc,+}(\kappa)\end{pmatrix}d\kappa=\bar{0} & (\rho>1)\end{cases}$$

$$\begin{cases}\int\limits_0^\infty \bar{H}_m(\kappa\rho)\begin{pmatrix}\gamma(\kappa)\left(v_m^{sc,-}(\kappa)+v_m^{in,-}(\kappa)\right)+2Q\,ka\,v_m^{sc,-}(\kappa)\\ -\left(ika\left(u_m^{sc,+}(\kappa)+u_m^{in,+}(\kappa)\right)+2Q\,i\,\gamma(\kappa)u_m^{sc,+}(\kappa)\right)\end{pmatrix}d\kappa=\bar{0} & (\rho<1)\\ \int\limits_0^\infty \bar{H}_m(\kappa\rho)\begin{pmatrix}ika\,v_m^{sc,-}(\kappa)\\ \gamma(\kappa)u_m^{sc,+}(\kappa)\end{pmatrix}d\kappa=\bar{0} & (\rho>1)\end{cases}$$

Here $\gamma(\kappa)=\sqrt{(ka)^2-\kappa^2}$ is the complex-valued function such that $\mathrm{Re}(\gamma(\kappa))\geq0$, $\mathrm{Im}(\gamma(\kappa))\geq0$, $\bar{H}_m(\kappa\rho)$ is the matrix kernel of the vector Hankel transform,

$$\bar{H}_m(\kappa\rho)=\begin{pmatrix}J'_{|m|}(\kappa\rho) & mJ_{|m|}(\kappa\rho)/(\kappa\rho)\\ mJ_{|m|}(\kappa\rho)/(\kappa\rho) & J'_{|m|}(\kappa\rho)\end{pmatrix}.$$

Further the obtained coupled dual integral equations are reduced to the Fredholm second-kind integral equations via the following operations:

1) Reduction of vector (i.e. coupled) dual integral equations to scalar ones by integration in radial coordinate; introduction of the constants of integration (coupling constants).
2) Analytical regularization of scalar dual integral equations via static-part inversion using scalar Hankel integral transforms and inverse Abel transform.
3) Derivation of additional equations for the coupling constants from the requirements on the current components behavior at the disk edge.

In the end of the chapter, presented are the Fredholm second-kind integral equations for finding the unknown functions for each type of problems: dielectric, resistive and PEC disks.

The third chapter deals with derivation of the spectral images of the normal-to-disk field components excited by the given sources in free space: horizontal and vertical electric and magnetic dipoles located on the axis of the disk at some distance from it and ring electric and magnetic currents. Presented are also expressions arising from the Graf addition theorem for cylindrical functions that help find images of the field components of arbitrarily oriented and located electric and magnetic dipoles. Further, the images of the field components of obliquely incident plane E-and H-polarized (relatively to the axis of the disk) electromagnetic waves are derived that involve delta-functions. Eventually the expressions are given for the images of the field components of the complex-point sources simulating the field of the wave beam with directed radiation.

The fourth chapter is devoted to building an efficient numerical algorithm for solving the integral equations of the Fredholm second kind on semi-axis, studying its rate of convergence by the example of the scattering of the field of an on-axis horizontal electric dipole by a zero-thickness PEC disk, and solving the problems of diffraction of (i) a complex-point source field by a PEC disk and (ii) field of an axial ring of electric current by a resistive disk.

At first, we present the proposed interpolation scheme of discretization of the Fredholm integral equations using the PEC disk as an example. This scheme allows reducing the problem of finding the unknown functions to the solution of a set of linear algebraic equations, which can be written, in operator notations, as $(I+M)\cdot X=Y$. The scheme consists of the following stages:

1) Introduction of parameter $N>ka+1$ and replacement of the infinite integration interval with the truncated interval (0, *N*).
2) Discretization of the truncated integral equations using Nystrom method with Gauss interpolation formulas.
3) Recovery of unknown functions on the whole semi-infinite interval.

After that we present the results of computation of the relative errors in the total power radiated by an on-axis horizontal electric dipole in the presence of a zero-thickness PEC disk. We consider two types of errors: the relative error of truncation of the interval of integration (Fig. 1) and the relative error of the interpolation-type discretization (Fig. 2).

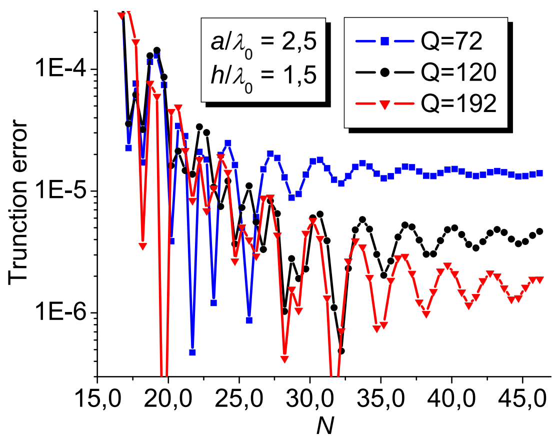


Figure 1 – Relative error as a function of the integration-interval truncation parameter

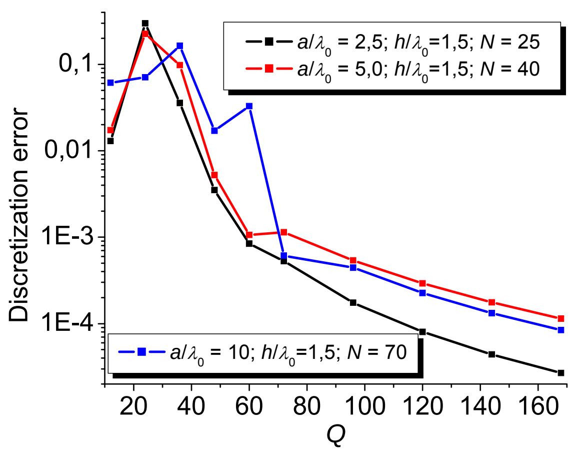


Figure 2 – Relative error as a function of the discretization (interpolation) order

The errors have been computed as a normalized difference in the radiation powers calculated with truncation parameters N and $N + \Delta$ and fixed order Q of the interpolation formula (or those calculated with Q and $Q + \Delta$ at fixed N) normalized to the absolute value of power calculated at the parameter value N (or interpolation order Q, respectively). The figures show that the error of computation reduces if the truncation parameter N and the order of the interpolation formula Q are taken greater.

The fifth chapter is devoted to the study of physical characteristics of the waves emitted by the elementary sources in the presence of a thin dielectric resonator. First, we consider the modal eigenvalue problem for a thin dielectric layer as an open waveguide. Then, we consider the problem of the scattering of the field of on-axis horizontal electric dipole by a dielectric disk. Finally, we study the problem of radiation of a similar electric dipole located near the edge of the dielectric disk.

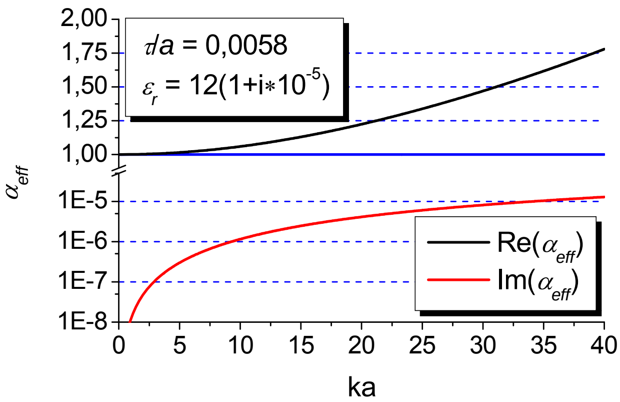


Figure 3 – Effective refractive index of the principal TM-waves of a dielectric layer as a function of the normalized frequency

As one can see, by performing this study we quantify the Purcell effect for a thin-disk resonator - modification of the rate of spontaneous emission of a point source in the presence of such a resonator. As known, in the framework of classical electromagnetics this characteristic coincides with a change in the average-over-period power radiated by a time-harmonic dipole. This makes the results of our study relevant not only to classical electromagnetics, but also to nuclear physics.

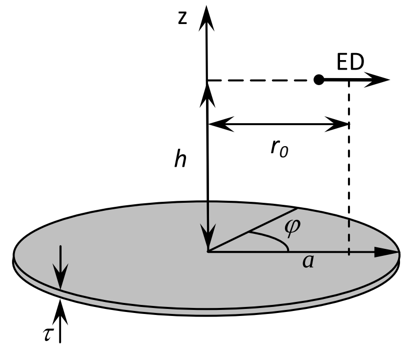


Figure 4 – Off-axis horizontal electric dipole above a thin dielectric disk

Here, the problem of finding the complex constants of propagation of electromagnetic waves guided by a thin dielectric layer of the thickness τ is important for correct interpretation of the further numerical results. It is solved by the method of separation of variables in the homogeneous Maxwell equations with generalized boundary conditions of Chapter 1, at the central section of the layer. As a result, we derive analytical expressions for the eigenvalues of propagation constants and the corresponding effective refractive indices as a function of the normalized frequency and electric and magnetic resistivities. In Fig. 3, we show the plots of the real and imaginary parts of effective refractive index corresponding to the fundamental TM-wave of a thin dielectric layer as a function of the normalized frequency.

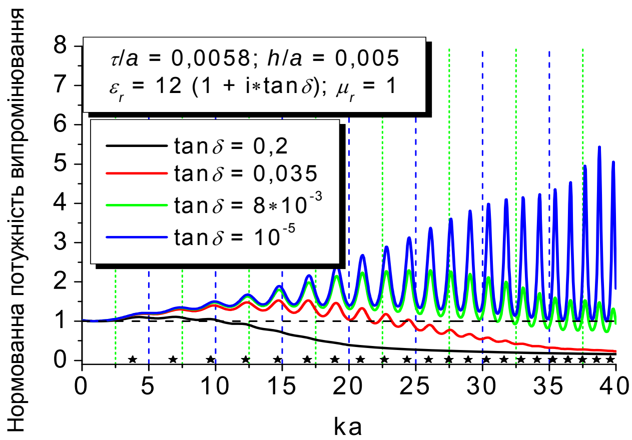


Figure 5 – Dependence of the normalized radiation power on the normalized frequency

Then the radiation of a horizontal electric dipole in the presence of a thin dielectric disk (Fig. 4) in the case of on-axis location is studied $(r_0 = 0)$. Here, the incident and the scattered fields have only one variation along the azimuth. Therefore solution to the problem needs consideration of only two families of equations corresponding to the indices $m = \pm 1$. The plots in Fig. 5 show the results of calculation of the normalized radiation power as a function of the normalized frequency in the case of a lossy disk. This value exhibits resonant behavior. It is visible that the frequencies at which the resonance peaks are observed are close to the frequencies of natural modes of dielectric disk (marked with asterisks in the figure) found using the model of effective refractive index. As known, if applied to a circular thin-disk cavity, such a model leads to approximate characteristic equations $J_{|m|}(\alpha_{eff} ka) = 0$ for each azimuth order *m*; in the given case $|m| = 1$.

Further in this chapter, the plots are presented of the normalized power P_{abs} absorbed in the disk and of the efficiency of radiation, calculated by the formula $\eta = P_{rad} / (P_{rad} + P_{abs})$. It is shown that if the dipole is located on the axis, the radiation efficiency does not demonstrate resonance peaks, especially for small values of the loss tangent of disk material, and the efficiency of radiation is almost always close to 100%. However, for greater losses there is a significant reduction of radiation efficiency if the electric size of the disk gets larger. Fig. 6 shows normalized far-field radiation patterns of the on-axis dipole in the presence of a dielectric disk at three resonance frequencies: $ka = 9.8974$; $ka = 12.46$, and $ka = 20.95$. They demonstrate that at relatively small values of electric size of the disk the in-resonance radiation is concentrated in the disk plane; with increasing the size of the disk the radiation spreads to other directions, and for even larger values of *ka* the most intensive radiation occurs in the direction close to normal.

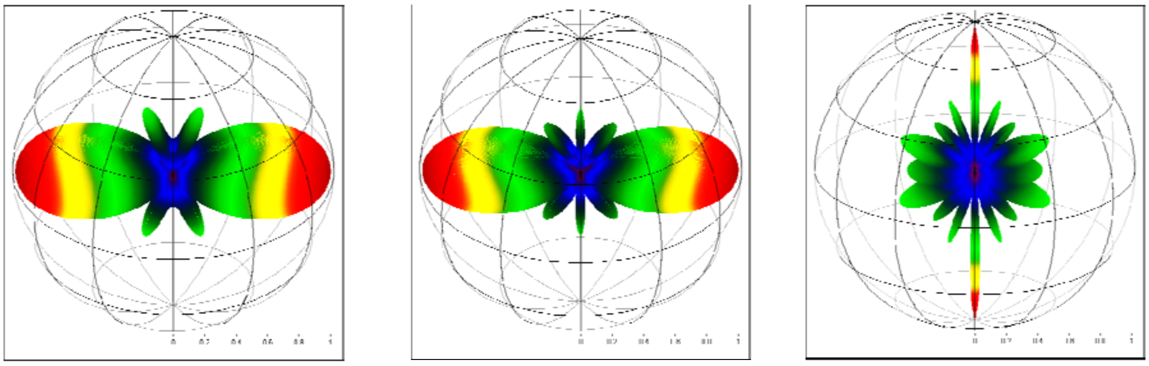


Figure 6 – Normalized radiation patterns at the resonance frequencies $ka = 9.8974$; $ka = 12.46$ and $ka = 20.95$, respectively

It has been further considered the problem of the radiation of a horizontal electric dipole located near the edge of the disk, for the distance from the disk axis equal to $0.8a$, $0.9a$ and $1.0a$. Unlike the on-axis dipole case, here to solve the problem we have to consider more than two families of equations. In this case, the number of azimuth indices, which should be taken into account depends on at least two parameters: the value of kr_0 that is the normalized to the wavelength in free space distance from the dipole to the disk axis and the value of $ka\sqrt{\varepsilon_r'}$ that is the normalized to the wavelength in material radius of the disk. What is very important, in this case the natural modes of the dielectric disk with several filed variations along the radius and well as along the azimuth can be excited (denoted using the indices *n* and *m*, respectively). These are the so-called whispering-gallery modes. In Fig. 7 and Fig. 8, we show the dependences of the normalized radiation power and the absorption power on the normalized frequency for a horizontal electric dipole located just above the edge of the disk. Disk parameters are as follows: relative permittivity $\varepsilon_r = 12(1 + i \cdot 10^{-5})$, thickness $\tau = 0.0058a$, and dipole elevation $h = 0.005a$.

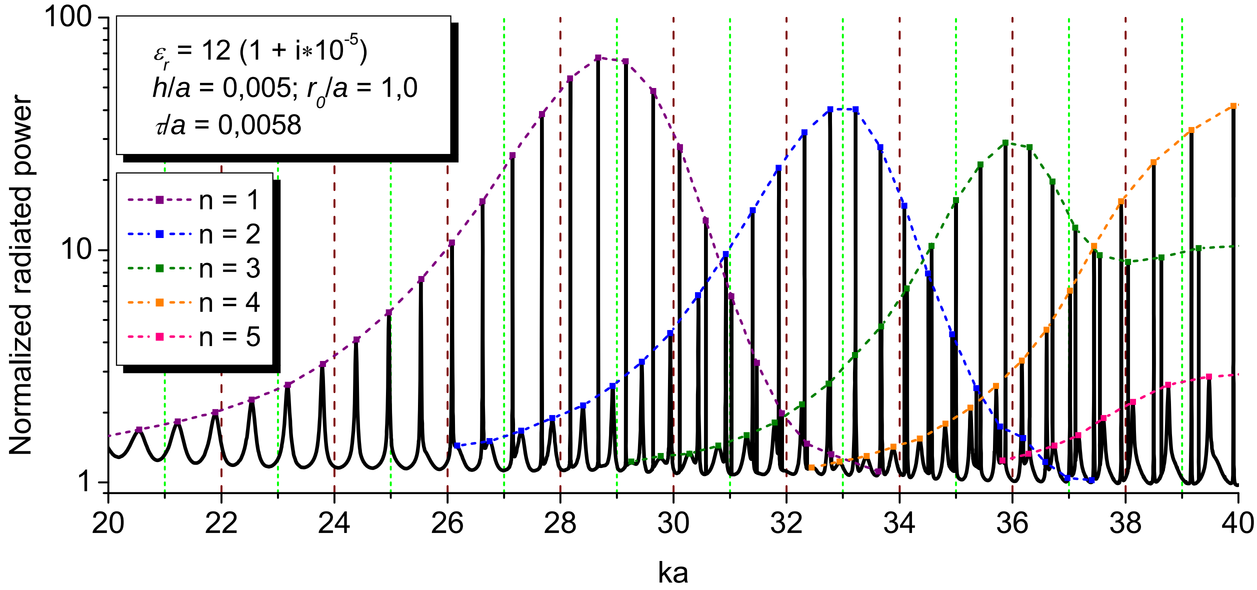


Figure 7 – The normalized radiation power as a function of the normalized frequency

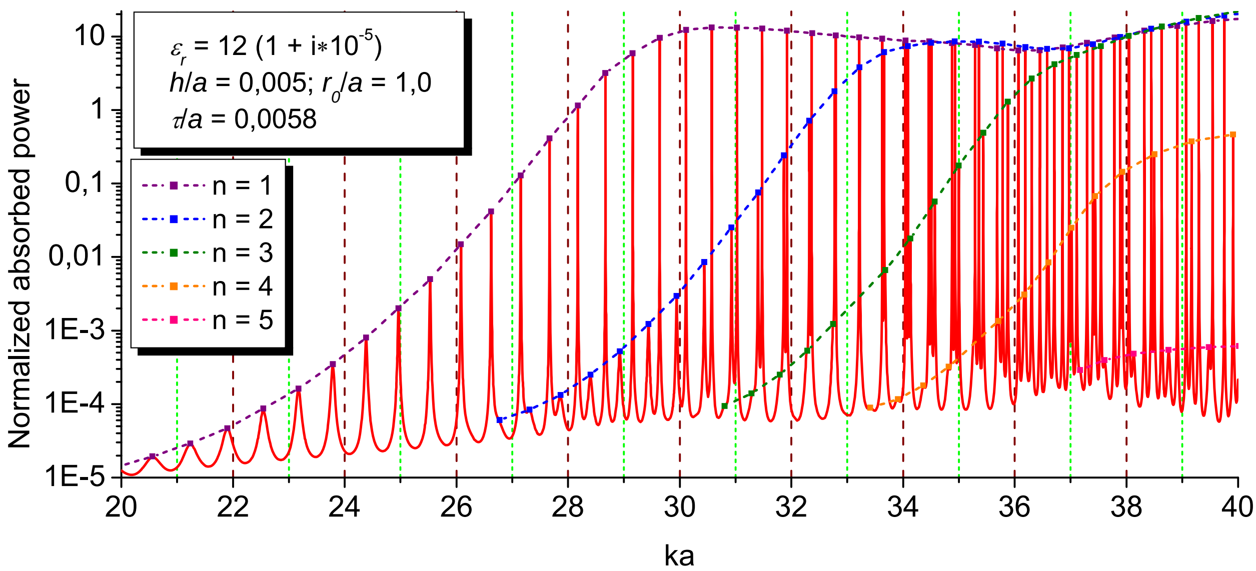


Figure 8 – The normalized power of absorption as a function of the normalized frequency

As already mentioned, these dependences can be interpreted as modification in the spontaneous emission rate (radiative decay rate) and the absorption rate (non-radiative decay rate) of an atomic dipole in the presence of thin disk as an open resonator– i.e. the Purcell effect.

For each family of modes of the same radial index *n*, we have plotted an envelope curve of the maximum values of the powers of radiation and absorption (dotted curves in Fig. 7 and. 8). From these figures, one can see that the most pronounced are the resonance spikes at the frequencies of the first family of radial ($n = 1$) indices having different varying numbers of field variations along the azimuth. However, if the frequency increases, the resonance spikes corresponding to the next families in *n* (second, third, etc.), i.e. the whispering-gallery modes of higher orders, appear on the plots of powers.

Fig. 9 demonstrates the plots of efficiency of the same electric dipole radiating near the edge of the disk as a function of the normalized frequency. They show that the efficiency of radiation is close to 100% almost everywhere except the values that correspond to the natural frequencies of the whispering-gallery modes (at higher frequencies) that are effectively excited in the disk however do not contribute to radiation.

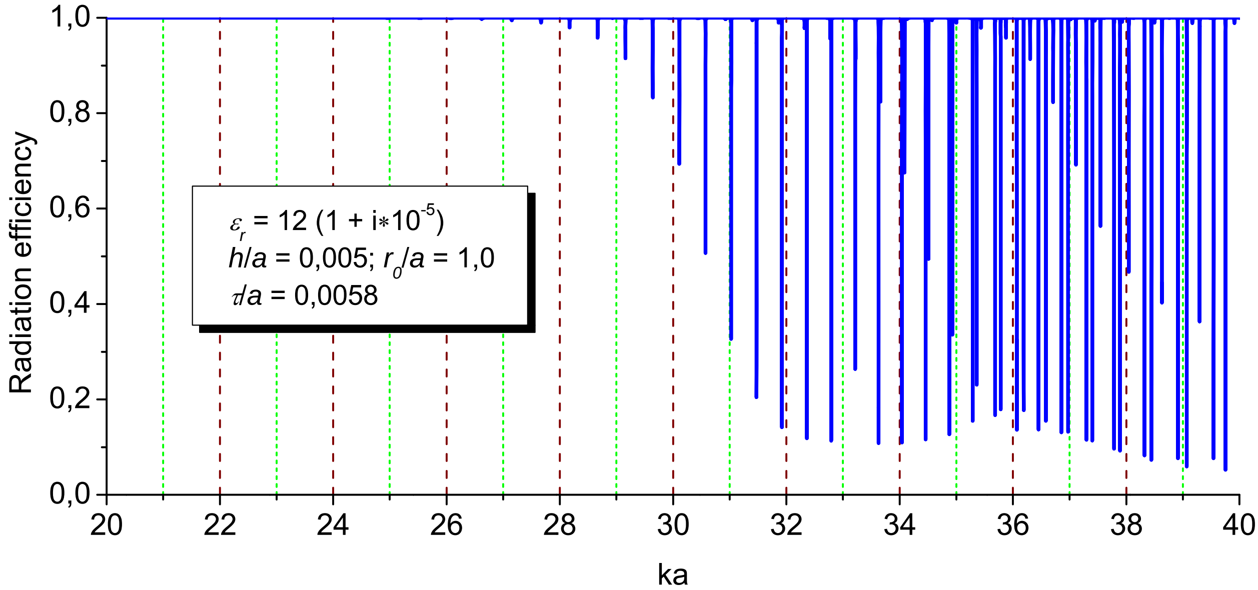
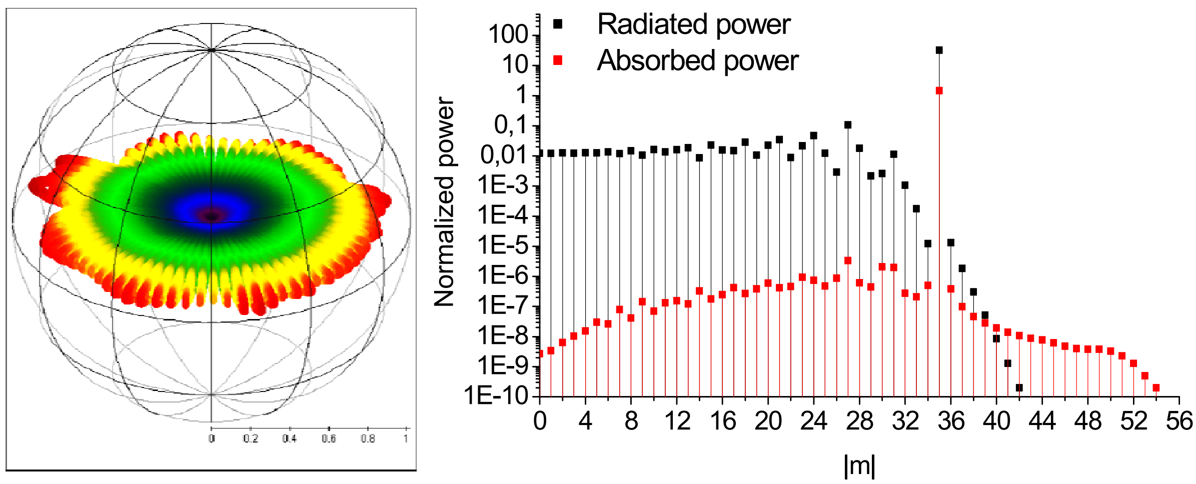


Figure 9 – The efficiency of radiation as a function of the normalized frequency

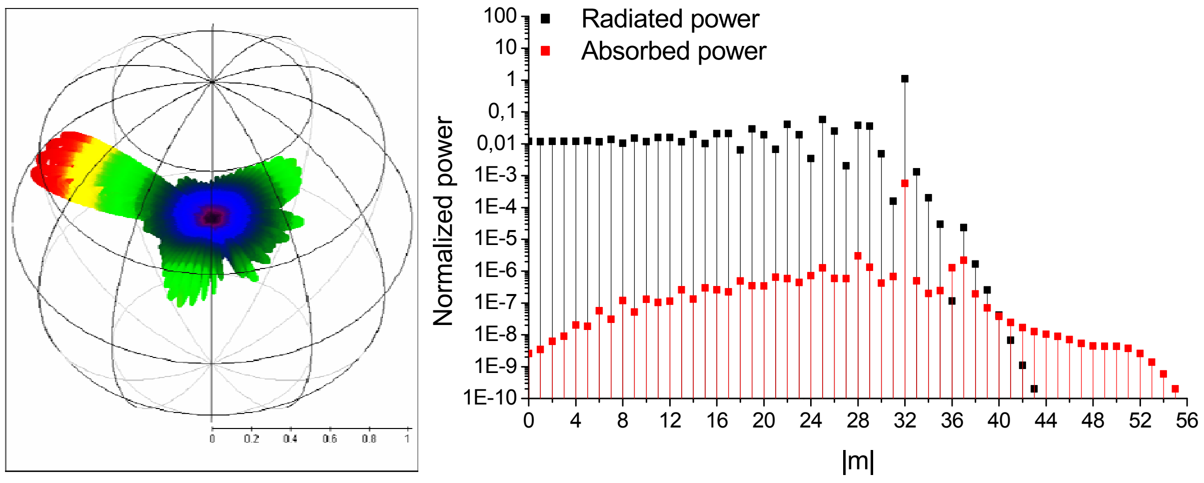
In Fig. 10, we present the normalized far-field radiation pattern (a) and the distribution of the power of emission and absorption between the total-field harmonics with different azimuth indices *m* at the resonance frequency $ka = 28.6708274$. This corresponds to the whispering-gallery mode with indices $m = 35$ and $n = 1$ as certified by the number of almost equal radiation beams in the disk plane (70). As one can see, the largest contribution to both the power of emission and that of absorption comes from the harmonic with azimuth index $m = 35$, which agrees with the number of beams. Also from this figure one can estimate the number of the values of the azimuth indices *m* that should be taken into account in the numerical solution of the scattering problem. In this case, to find the normalized powers of radiation and absorption with accuracy of 10^{-10}, one has to compute 107 field harmonics in *m*.



(a) (b)

Figure 10 – The radiation pattern (a) and the distribution of the emitted and absorbed powers on the azimuth index *m* (b) at the resonance frequency $ka = 28.6708274$ of the whispering-gallery mode with indices $n = 1$ and $m = 35$

Fig. 11 shows the data similar to that shown in Fig. 10, however at the frequency of $ka = 29.44068$. This frequency corresponds to the natural whispering-gallery mode of the same dielectric disk with indices $n = 2, m = 32$.



(a) (b)

Figure 11 – The same as in Fig. 10 however at the resonance frequency $ka = 29.44068$ of the whispering-gallery mode with indices $n = 2$ and $m = 32$

It should be noted that a thin dielectric disk at this frequency displays quite noticeable far-field collimation properties, i.e. behaves as a quasioptical lens. Indeed, from the radiation pattern one can see that the maximum radiation field has several lobes orientated along almost the same direction in the disk plane that links the disk center and the point opposite to the source location.

At the end of this chapter, we present a comparison of the results of calculation of the far-field radiation patterns for the on-axis electric dipole in the presence of PEC and dielectric disks obtained using the developed method and those obtained using the commercial electromagnetic simulation code FEKO – the algorithm based on boundary element method. We note that the corresponding results agree with graphical accuracy (difference less than 10^{-4}). Besides, the numerical experiments have shown a significant advantage of the developed algorithm, which is much less demanding to computer time and memory. For example, to calculate the radiation characteristics using the code FEKO for a dielectric disk with $\varepsilon_r = 12(1 + i \cdot 10^{-5})$ at $ka = 9$, one needs about 15 hours on a desktop computer compared to 2 seconds using the algorithm developed in this work.

CONCLUSIONS

In the dissertation, an important problem of today's radio physics has been solved: an efficient numerical-analytical method has been developed to study the effects of the scattering and absorption of electromagnetic waves by a thin dielectric disk. Besides, the numerical analysis with guaranteed accuracy has been performed of the characteristics of the absorption and emission of the fields of localized sources in the presence of the disk. Thus we have investigated the Purcell effect – modification of the rate of spontaneous emission and absorption for an atomic dipole located near an open resonator in the form of a thin dielectric disk.

The main scientific and practical results are as follows:

1. A rigorous mathematical model of the scattering of time-harmonic electromagnetic fields by a thin dielectric disk has been built. The model is based on the boundary-value problem for Maxwell's equations with generalized boundary conditions, radiation condition, and the local finiteness of energy condition and reduces to the equivalent set of dual integral equations.
2. The method of analytical regularization has been developed for the reduction of dual integral equations to coupled Fredholm second-kind integral equations on semi-axis. The method is based on the analytical inversion, using the Abel, Hankel, and Fourier transforms, of the main part of the dual integral equations. A discrete model of these equations has been built based

on the interpolation method of quadratures (Nystrom method) having guaranteed convergence.

3. The expressions for the normal to the disk field components have been derived for the obliquely incident on the disk plane waves and the following localized sources:
   - electric and magnetic dipoles;
   - electric and magnetic ring currents;
   - wave beams generated by the complex electric and magnetic dipoles and complex Huygens element.
4. It has been shown that results obtained by the method developed in the framework of the proposed model are consistent with the calculations using commercial computer-aided design codes and greatly surpass the latter in performance and reliability.
5. The study of the characteristics of the power radiated by the localized sources in the presence of a dielectric disk and the power of absorption in the event of a disk with losses has revealed that the resistances of radiation and absorption exhibit in-resonance growth, but the effectiveness of radiation can have only resonant drops.
6. The efficiency of excitation of resonance modes of a dielectric disk with various azimuth indices has been determined in the case of the dipole displaced from the disk axis. With increasing frequency the curves of the radiated and absorbed powers show a series of peaks that correspond to the modes with a fixed azimuth index and consistently varying radial index. This behavior can not be captured, even approximately, by the so-called "Purcell factor" valid only for the dipole radiating in a closed cavity.
7. The approximate method of effective refractive index widely used in calculations of the resonance frequencies of thin dielectric disks has been confirmed from the viewpoint of the theory of Maxwell's equations.

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