Modification of the Power Radiated by an Electrical Dipole in the Presence of a Thin Dielectric Disk

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Abstract: We consider the problem of electromagnetic waves emission by a horizontal electrical dipole in the presence of a thin high-contrast dielectric disk. To find the solution of the problem we use analytical-numerical scheme which is based on the dual integral equations and the method of analytical regularization. We present the normalized radiation patterns and total radiated power for different values of geometrical and material parameters.

Electromagnetic wave scattering by thin dielectric disks is interesting for many reasons: such a disk is met as a part of printed antennas; it is used as a simplified model of a tree leave [1]; thin few-micron radius disks are used as resonators of semiconductor lasers with ultralow thresholds [2]. Moreover, this scattering problem is of great interest for the optical community because it can be used to study the spontaneous emission in disk-shape nanosize dielectric particles. As known, the change in the power radiated by a dipole (or any other source) in the presence of a dielectric particle is equivalent to the modification of the spontaneous emission rate of a quantum-mechanical atom in the same particle [3].

Many analytical high-frequency approximations and direct computational methods have been used for the scattering analysis. However, high-frequency methods fail to reproduce fine details of the wave field and have uncertain domains of applicability. Direct computational methods meet other problems like large-size matrices to be inverted, low convergence of solution, and hence huge computational time.

Therefore in this study we use a rigorous numerical method, which is fast and accurate for any disk size and needs moderate computer resources; this method has been presented in [4].

Consider the problem of diffraction of the electromagnetic field emitted by a horizontal electrical dipole from a dielectric disk of radius a and thickness τ (Fig. 1). Introduce dimensionless cylindrical coordinates $(\rho = r/a, \varphi, \zeta = z/a)$ with origin in the center of the disk. Assume that the dipole is located at the height d above the disk center $(\zeta_0 = d/a)$ Denote total field as a sum of the scattered and the incident fields: $E = E_{in} + E_{sc}$, $H = H_{in} + H_{sc}$, where the incident field corresponds to the dipole in the free space. The scattered field has to satisfy the homogeneous Maxwell equations out of the sources and the disk, and the following generalized boundary conditions at the disk median section [5]:

$$(E_{tg}^+ + E_{tg}^-) = 2Z_0 R \cdot \vec{n} \times (H_{tg}^+ - H_{tg}^-),$$
$$Z_0(H_{tg}^+ + H_{tg}^-) = -2Q \cdot \vec{n} \times (E_{tg}^+ - E_{tg}^-).$$

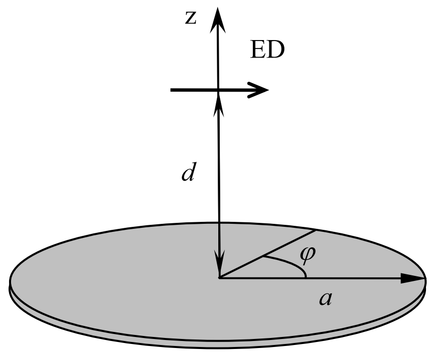

Figure 1. Problem geometry

Here, Z_0 is the free-space impedance, and R and Q are the electric and magnetic resistivities which are given by $R = iZ/2\cot(\sqrt{\varepsilon_r\mu_r}k\tau/2)$, $Q = R/Z^2$ in case of $k\tau \ll 1$ and $|\varepsilon_r\mu_r| \gg 1$, Z is the relative impedance of the disk material, $k = \omega/c$ is the wavenumber, ε_r and μ_r are the relative permittivity and permeability, respectively. On the rest part of the disk plane the components of the total field are continuous. Also, the components of the scattered field must satisfy the 3-D radiation condition and the condition of local integrability of power [6].

To find the solution of the scattering problem, we use the method of analytical regularization [9] applied to the dual integral equations (IEs) [7,8] in the Fourier-Hankel spectral domain. It enables us to obtain the Fredholm second kind IEs for the unknowns that are the images of the jumps and the average values of the normal to the disk field components. This guarantees the existence of solution, which can be found numerically using any reasonable disctretization. We truncate the interval of integration form $(0,\infty)$ to $(0,N)$, where $N > ka + 1$ and apply the Nystrom method with the Gauss-type higher-order quadratures to solve it numerically.

Some of preliminary results of computations are presented in Figs. 2 to 4. Fig. 2(a) shows the dependences of the total power radiated by the elementary electrical dipole in the presence of thin dielectric disk for three different types of high-contrast dielectric materials with $\varepsilon_r = 10 + i$; $100 + i$; $1000 + i$ and $\mu_r = 1$. The power is normalized by the same quantity for the

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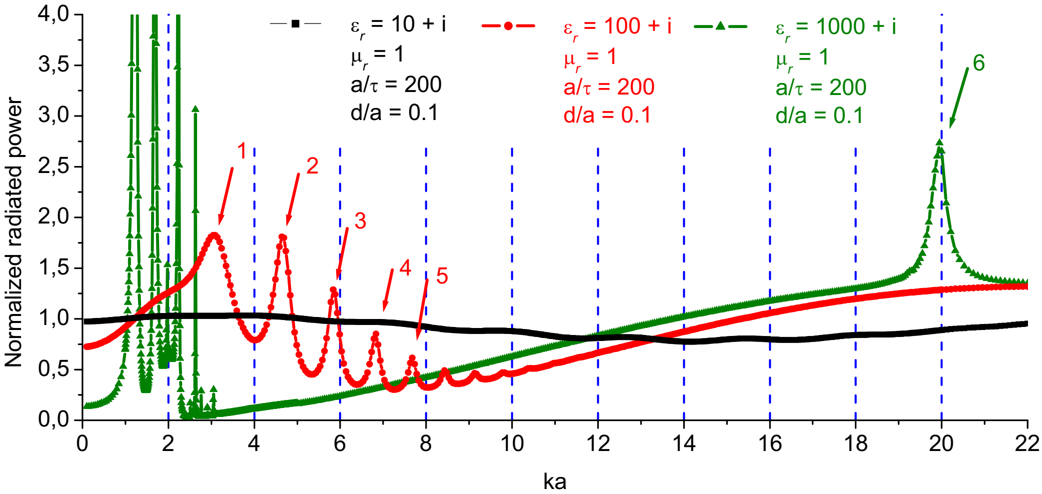
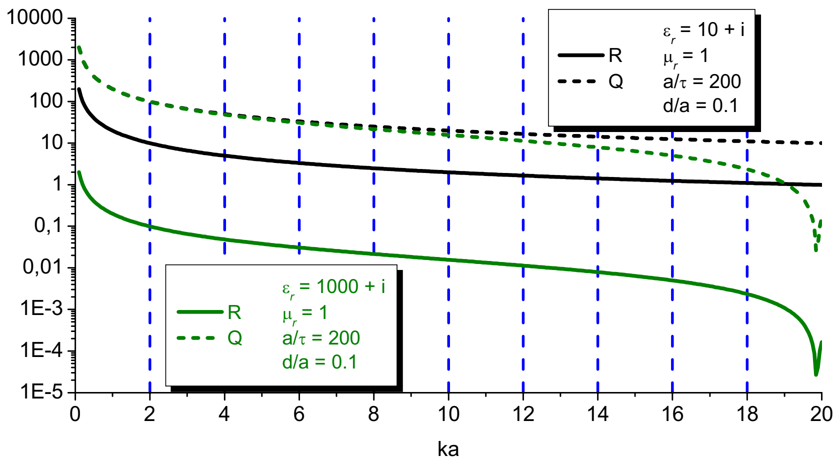
elementary electrical dipole in free space. One can see resonance nature of the normalized radiated power. That correspond to two different resonance types which are became more clear for the materials with low dielectric losses ($\tan\delta = 0.01$ and 0.001). First type of resonances (marked by 1, 2, 3, 4 and 5 in Fig 2(a)) are radial resonances in the dielectric disk. Radiation patterns of the incident and total fields at these points are presented in Fig. 3(a)-3(e).

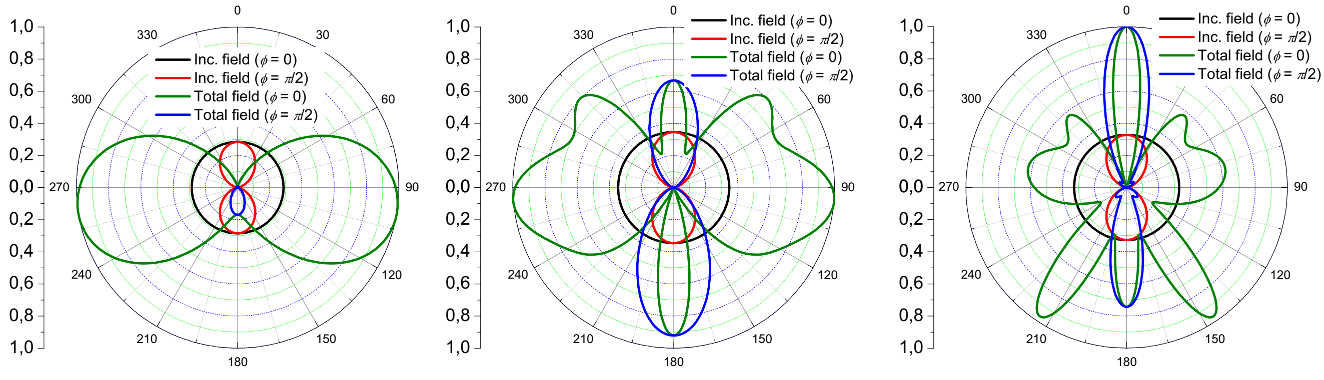


(a)

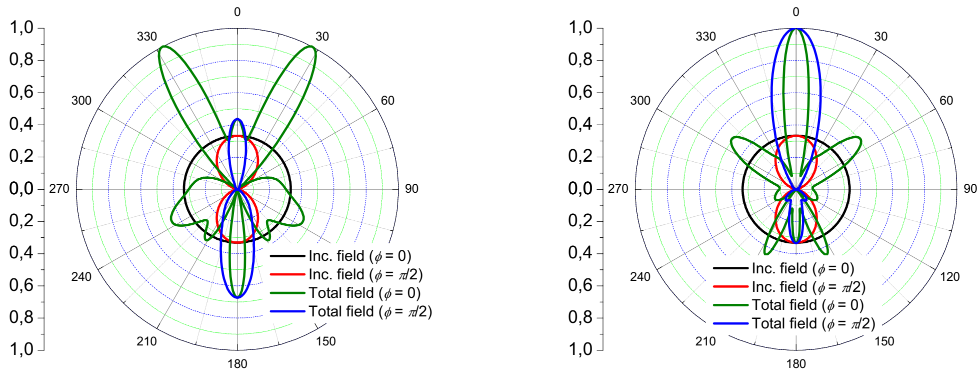


(b)

Figure 2. Normalized radiated power (a) and resistivities dependences as a functions of the normalized radius of the disk *ka*



(a) (b) (c)



(d) (e)

Figure 3. Normalized radiation patterns of incident and total field at points *ka* = 3.043 (a); *ka* = 4.6386 (b); *ka* = 5.836 (c); *ka* = 6.883 (d); *ka* = 7.681 (e) for dielectric disk material $\varepsilon_r = 100 + i$; $\mu_r = 1$ and geometry parameters $a./\tau = 200$; $d/a = 0.1$

A resonance of the other type appears only if the permittivity is large enough. It is marked by 6 in Fig. 2(a). This resonance corresponds to the disk thickness equal to half-wavelength in the dielectric material ($k\tau\sqrt{\varepsilon_r\mu_r} = \pi$) and is connected to a zero of the electric and magnetic resistivities (see Fig 2(b)). Therefore this is a dielectric layer transversal resonance. Radiation patterns of the incident and the total fields at this point are presented in Fig. 4.

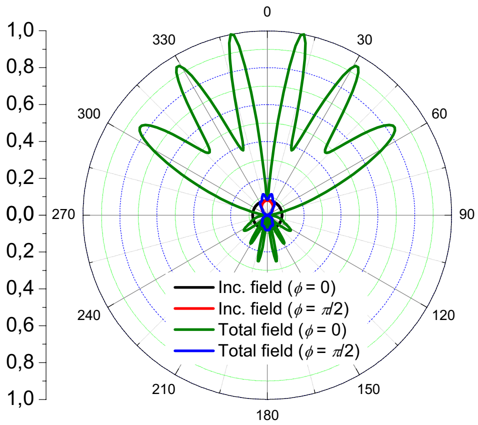


Figure 4. Normalized radiation patterns of incident and total field at point *ka* = 19.95 for dielectric disk material $\varepsilon_r = 1000 + i$; $\mu_r = 1$ and geometry parameters $a/\tau = 200$; $d/a = 0.1$

ACKNOWLEDGEMENT

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