2011 IEEE AP-S Doctoral Research Award
Project Proposal

Resonant scattering of electromagnetic waves by finite periodic configurations of sub-wavelength circular wires

Denys M. Natarov

Laboratory of Micro and Nano-Optics
Institute of Radio-Physics and Electronics NASU
vul. Proskury 12, Kharkiv 61085, Ukraine

Abstract. It is proposed to study the diffraction of the electromagnetic waves by finite periodic configurations of sub-wavelength circular wires using accurate and efficient full-wave approach. Such configurations display very interesting phenomena: periodicity-caused resonances. If, additionally, the wires are made of noble metals, plasmon resonances are observed in the optical range and can be enhanced by the grating resonances. These effects have potential applications in optical antennas and biosensors; their study can be a subject of a paper for the *IEEE Transactions on Antennas and Propagation*.

Background and motivation. As known, material objects can exhibit localized surface-plasmon or quasistatic resonances at certain wavelengths for which the object permittivity is negative [1-3]. The physical mechanism behind these resonances is the presence of free electrons in the noble-metal nano-particles that may display collective oscillations giving a major contribution to the dielectric permittivity at optical frequencies. For sub-wavelength metallic objects, plasmon resonances result in powerful enhancement of scattered and absorbed light that is used in stained glasses and, more up-to-date, in the design of optical antennas and biosensors for advanced applications. Plasmon resonances have unique physical property: in the leading terms, the resonance frequencies depend on the object shape but do not depend on its dimensions [2]. As known, they may shift considerably if the noble-metal particles or wires are collected in closely spaced ensembles, e.g. dimers, trimers, and more complicated configurations [1,3,4].

Finite periodic structures made of many particles or wires are even more interesting objects of research as they are strongly wavelength-selective scatterers. As known, periodicity in *infinite* gratings of wires or particles leads to specific phenomena called grating resonances (a.k.a. geometric resonances) [4-6]. If the wire diameter $2a$ is a small fraction of the period p, then at the normal incidence their frequencies lie just below the frequencies of Rayleigh-Wood "anomalies" (branch points at $\lambda = p/m, m = 1, 2, \ldots$). The grating resonances are caused by the poles and lead to almost total reflection of the incident plane wave in narrow bands in the H and E-polarization cases [5,6]. However it seems that the question of how these resonances display themselves if a grating of wires is *finite* has not been studied so far.

Method of numerical solution. I solve the two-dimensional (2-D) problem of the plane-wave diffraction by the finite number of arbitrarily located parallel nanowires. The field function must satisfy the Helmholtz equation with corresponding wavenumbers inside and outside the cylinders, the tangential field continuity conditions, the radiation condition at infinity, and the condition of the local power finiteness. The full-wave solution can be traditionally obtained by expanding the field function in terms of the azimuth exponents in the local coordinates, using addition theorems for cylindrical functions, and applying the boundary conditions on the surface of all M wires. This leads to an infinite $M \times M$ block-type matrix equation where each block is infinite. However I emphasize that unlike earlier works dated from [7], I cast it to the Fredholm second kind matrix equation. It is only in this case that the solution of equation with each block truncated to finite order N converges to exact solution if $N \to \infty$. The results presented below were computed with $N = 4$; this provided 3 correct digits in the far-field characteristics of the gratings of silver wires with radii $a \leq 75$ nm and periods $p \geq 200$ nm. Note that denser gratings need larger values of N to achieve the same accuracy.

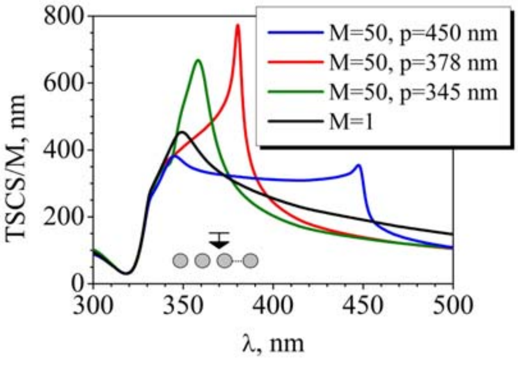

Fig. 1. TSCS per wire as a function of wavelength for the H-wave incident broadside ($\varphi_0 = \pi/2$) on the gratings of M = 50 silver nanowires with radii a = 60 nm.

Preliminary results. In the on-going research, I consider the scattering of the H-polarized plane waves by sparse ($p - 2a > a$) finite linear gratings of sub-wavelength silver nanowires with radii about tens of nanometers, in the visible band. In Figs. 1-3, the wavelength varies from 300 nm to 500 nm and the complex-valued dielectric function of silver has been borrowed from [8].

The most interesting question is what the peak values of the grating characteristics become if the plasmon and the grating resonance wavelengths coincide. Here, the main instrument of tuning them together is the period of the wire grating, because the plasmon resonance has almost fixed wavelength near to the single-wire value around 350 nm. For example, in Fig. 1

presented is the plot of the total scattering cross-section (TSCS) as a function of the wavelength for a stand-alone silver nanowire with radius 60 nm. It has broad maxima at 349 nm.

In Fig. 1, presented are also the plots of TSCS per wire (i.e. normalized by M) as a function of the wavelength for several sparse gratings of M = 50 silver wires with radii 60 nm. One can see that if the period is far from the plasmon resonance wavelength, the grating resonance has almost no effect on TSCS. This behavior changes drastically, however, if two resonances are tuned together. In such case there appears one much more intensive resonance. As visible, enhancement in per-wire TSCS can reach 1.7. The combined peak is red-shifted relatively to plasmon resonance of a single wire. As we have found, the optimal wire radii to observe the strongest per-wire TSCS enhancement lie in the range of 25 nm to 70 nm.

In Fig. 2, presented is relief of per-wire TSCS as a function of two parameters: wavelength and period, for gratings of 50 silver wires with radii 60 nm. In each case one can see a sharp "ridge" stretching along the line $\lambda = p$ that marks the grating resonance. Another, broader ridge stands at the fixed wavelength near to 350 nm – this is plasmon resonance. The areas of strongly enhanced TSCS are located at the junction of these two ridges.

In Fig. 3, presented is the near-field amplitude pattern for the 10-wire sparse grating with a = 60 nm at the wavelength of the combined resonance. The field hot-spot maxima are visible near the illuminated sides of the wires similarly to the single-wire case [1]. However besides of that one can clearly see two local standing waves near to the grating: one is above the illuminated side – it is formed by the incident plane wave and the reflected field. Another is an "orthogonal" wave standing along the wires; it is formed by two oppositely propagating ±1-st quasi-Floquet harmonics of the grating as the wavelength is near the value $\lambda = p$.

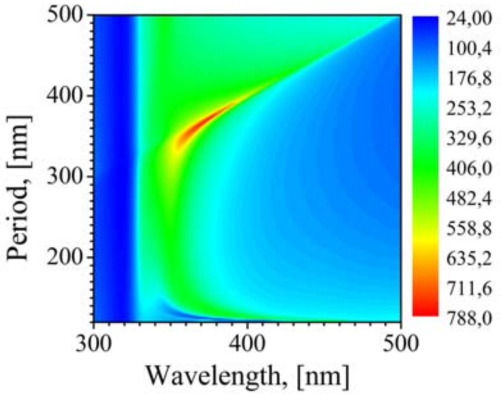

Fig. 2. TSCS per wire as a function of wavelength and period for the H-wave incident broadside ($\varphi_0 = \pi/2$) on the gratings of M = 50 silver nanowires with radii a = 60 nm.

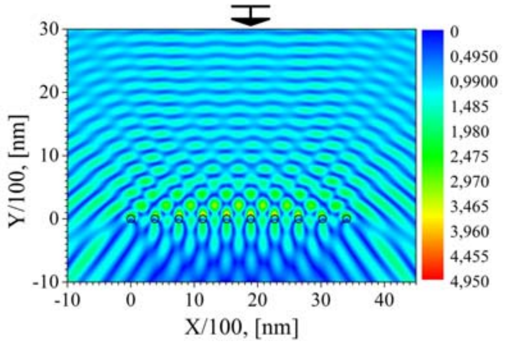

Fig. 3. Near-field amplitude pattern for the H-wave incident normally ($\varphi_0 = \pi/2$) at the grating of M = 10 silver nanowires with radii a = 60 nm, period p = 378 nm, in combined resonance at λ = 376.6 nm.

The further research will be concentrated on the investigation of the scattering of light by structures formed of finite number of nanowires grouped in various configurations such as discrete corner, cross and parabolic reflector.

My technical interests and skills. I have a background in the "ex-USSR-style" radio physics, being graduated from a well-reputed university in the East Ukraine. Traditionally, this implies education in applied physics with greater emphasis on the mathematical and computational aspects of the electromagnetic wave propagation and interaction with various objects. My interests, since my work on the M.S. thesis, have been with the scattering by single and multiple configurations of wires, plasmon and grating resonances, optical antennas, and lasing. I believe that this will enable me to work on the proposed project.

Award and project impact. The work on the proposed research project is important for my current PhD study. I strongly wish to develop my abilities and build a career in electromagnetics. I hope that IEEE AP-S research award will help me perform my research work, visit several scientific conferences, and get access to new educational materials.

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