Radiation of Electromagnetic Waves by a Spherical-Circular Misrostrip Antenna Excited with Elementary Dipoles Located on the Axis of Symmetry

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Abstract—Canonical-shape conformal printed antenna is considered, excited by elementary dipoles. Numerical solution is obtained by the method of analytical regularization. Radiation characteristics reveal interplay of several types of resonances.

Keywords-circular microstrip antenna; spherical substrate; analytical regularization; radiation resistance

I. INTRODUCTION

We investigate conformal microstrip antennas (MSAs) shaped as a perfectly electrically conducting (PEC) spherical disk placed on the spherical PEC "ground" with a dielectric substrate. The interest in these antennas is caused by the wide use of conformal MSAs in communication and radar systems [1,2]. Here, implementation of MSAs enables one to satisfy strict and contradictory demands to electrical and aerodynamic characteristics and also allows reducing their weight and cost.

We consider spherical-circular MSAs excited by two types of elementary dipoles: radial electric dipole (RED) (also considered in [8] however with incomplete numerical study) and tangential magnetic dipole (TMD) located on the antenna axis of symmetry.

II. SPHERICAL-CIRCULAR MSA EXCITED BY RED

Consider the axially symmetrical problem of the wave radiation by a spherical-disk MSA excited by a RED sitting on the surface of the PEC sphere (Fig. 1). This is a model of antenna fed with a coaxial probe. The spherical PEC ground has the radius a and the substrate has the radius c, thickness h, and relative permittivity \mathcal{E} . On the surface of the substrate there is a PEC zero-thickness spherical disk of the angular size $2\theta_0$. The formulation of the electromagnetic field problem includes Maxwell's equations off the conductors and boundaries, the boundary conditions on the PEC conductors, the tangential field components' continuity conditions on the dielectric boundaries, Silver-Muller radiation conditions, and the condition of local finiteness of the field energy. In the rotationally symmetric case of the axial RED excitation the problem simplifies because for complete description of electromagnetic field it is enough to introduce only one auxiliary function (e.g., electric Debye potential).

Our solution is based on the well developed method of analytical regularization (MAR) [3] and the Abel integral-



Figure 1: Geometry of a spherical-disk MSA excited by a RED

equation (IE) technique [4-8], used earlier in other relevant scattering problems. By substituting the electromagnetic field series expansions to the mixed-type boundary conditions, the boundary-value problem is reduced to the dual series equations (DSEs) in terms of associated Legendre functions. The regularization is realized by extracting, from the full DSE operator, a part that corresponds to the static problem for a PEC spherical disk in free space. Further the Meller-Dirichlet transformation is applied to the polynomials and associated Legendre functions for the conversion of each series equation to a homogeneous Abel IE. Further one can reduce them to a DSE in terms of trigonometric functions that is partially inverted by using the inverse discrete Fourier transform. The resulting expressions take the form of the infinite-order matrix equation of the second kind. One can verify that this is a Fredholm operator equation in the space of sequences l_2 , and hence solutions obtained after truncation to finite orders do converge to exact solution.

For the algorithm verification, the behavior of the relative error caused by the matrix truncation has been investigated. This value is shown in Fig. 2 as a function of the matrix



Figure 2: Relative computational error versus the matrix size for h/c=0.1, $\theta_0=18^\circ$, $\varepsilon=1.3$.

truncation order N for several MSAs. The error value depends on the electrical size of antenna and relative thickness of substrate. For achieving 10^{-3} accuracy, it is necessary to truncate the matrix at $N \ge k_0 c \sqrt{\varepsilon} + c / h + 10$.

The radiation resistance behavior has rather complicated resonance character (Fig. 3). Low-Q resonance at $k_0 c \approx 1$ (missing in [8]) is the dipolar resonance of the PEC spherical ground. If the frequency gets larger, the higher-Q resonances of the TM_{0m0} type are excited in the open cavity between the disk and the ground, where *m* corresponds to the number of variations in the current function along the disk radius. The same number of conical beams is observed in the far-field radiation pattern (RP). If the frequency increases still further and reaches the value approximately corresponding to $k_0 h \sqrt{\varepsilon} \approx \pi/2$, then periodical very high-Q resonances of the "whispering gallery" type are excited in the spherical dielectric substrate. Gradually these resonances become dominating feature of the frequency dependence (note that they were not studied in [8]).



Figure 3: Normalized radiation resistance versus normalized frequency, RED excitation. h/c=0.02, $\theta_0=18^\circ$, $\varepsilon=1.3$.



Figure 4: Normalized RPs in the ground-sphere resonance, TM₀₁₀ cavity resonance, and in the whispering-gallery-mode range (see Fig. 3), for h/c=0.02, $\theta_0 = 18^\circ$, $\varepsilon = 1.3$.

The field analysis in the far zone shows that RP has a conical shape with nulls along $\theta = 0$ and 180°. This is because RED does not generate the electric field directed along its axis and axially-symmetric MSA does not change this field type. At the frequencies below $k_0c < 2$, the radiation has a dipole character with predominant sidelobes looking at 90° and 270°.

It is necessary to note that besides the radiation into the forward (regarding to the disk) half-space, the analyzed antenna may have strong "parasitic" radiation into the backward half-space, in certain regimes. In fact, the main beam (RP maximum) "keel over" to the backward half-space always takes place if increasing the frequency. This phenomenon is caused by the oscillations of the "whispering gallery" type and by the currents generated due to these waves on the back side of the ground sphere. When increasing the disk angular size θ_0 , the resonances associated with the cavity under the disk move to the lower frequency range and get higher Q-factors, and their amplitudes rise. However the variation of this parameter has a little influence on the resonances associated with the spherical ground and the "whispering gallery" resonances. Increasing of the $\mathcal E$ forces the cavity resonances to move to the lower frequency range, their amplitudes rise, and the region of the "whispering gallery" resonances also moves to the lower frequencies. However, the \mathcal{E} variation has very small influence on the resonance associated with the spherical ground. Decreasing of the substrate thickness h/c leads to the cavity resonances growth, their Q-factors rise, and the range of "whispering gallery" resonances starts from the higher values of the normalized frequency.

III. SPHERICAL-CIRCULAR MSA EXCITED BY TMD

Consider now the radiation of waves by a spherical disk MSA excited by an elementary TMD located at the symmetry axis on the surface of PEC "ground" (Fig. 5). This dipole corresponds to the feeding of antenna by a slot in the ground conductor. The difference of this problem from the previous one is that here the electromagnetic field depends on the azimuth coordinate φ and has all six components.

As before, the corresponding conditions at the material boundaries, conductors, infinity and spherical disk edge are formulated for providing the uniqueness of the solution. Here, two Debye potentials, electrical and magnetic, are used to characterize the electromagnetic field. We reduce the boundary-value problem to two DSEs in terms of associated Legendre functions, coupled to each other by two constants that correspond to the fact that the spherical harmonics of the E and H-type do not scatter independently. The next step of solution is the regularization procedure of the coupled DSE sets. Here, Meller-Dirichlet transformation for the associated Legendre functions and the tangent and cotangent functions is applied. This allows obtaining two coupled sets of homogeneous Abel IEs and further two coupled DSEs in the



Figure 5: Geometry of a spherical-disk MSA excited by a TMD



Figure 6: Normalized radiation conductance versus normalized frequency, TMD excitation, h/c=0.02, $\theta_0=18^\circ$, $\varepsilon=1.3$.

trigonometric functions. Then the standard steps are followed similarly to the ones described earlier. The main difference of given equations from the ones found in the previous case consists in the necessity of finding two unknown constants, by means of which the sets of equations for the expansion coefficients of electric and magnetic fields are coupled. The final pair of coupled matrix equations of the second kind is a Fredholm operator equation in the space $l_2^2 = l_2 \times l_2$ and can be solved by the truncation. To achieve the relative accuracy of 10^{-3} in the solution, one has to take the truncation order of the each block of the resulting block-type 2x2 matrix as $N \ge k_0 c \sqrt{\varepsilon} + c/h + 10$.

The principal antenna characteristics are the RPs in two main planes, partial directivity factors in these two planes, and also the power and the conductance of radiation. In general, when analyzing the radiation of our MSA excited by TMD one observes the same phenomena as in the case of antenna excited by RED. These phenomena have the same regularities as in the previous problem that is demonstrated by the plots of the normalized radiation conductance versus the normalized frequency (see Fig. 6). From the RP analysis, one can see that the analyzed conformal MSA is able to radiate the main beam in the direction of the symmetry axis into the forward halfspace, i.e. at $\theta=0^{\circ}$ (Fig. 7). On the whole, the radiation of the antenna excited by TMD has more directional character than the same antenna excited by RED.

In particular, when the substrate is thin enough the dominant factor is the cavity resonances of the quasi- TM_{1m0} type (m=1,2,...), where the radiation power exceeds similar values in the low-frequency resonance of the PEC sphere and the "whispering-gallery" resonances by 2-3 orders of magnitude. Each sub-figure of Fig 7 contains the information about the RP shape in the both of the main planes: left half is for the E-plane and right half is for the H-plane RP. When the frequency comes to the next cavity resonance, a new sidelobe appears in RP in the E-plane.

As in the previous axially-symmetric problem, at the high enough frequencies the radiation goes predominantly into the



Figure 7: Normalized RPs in the ground-sphere resonance, quasi-TM₀₁₀, and in the whispering-gallery-mode range (see Fig. 6) for h/c=0.02, $\theta_0=18^\circ$, $\varepsilon=1.3$.

backward half-space, i.e. the RP "keel over" happens, caused by the interference of currents associated with the "whispering-gallery modes" in the substrate.

IV. CONCLUSIONS

Thus, we have developed a numerically exact analysis method and studied the associated wave phenomena for the spherical-circular MSAs excited by on-axis dipoles. The main features of these MSAs are the following: the low-frequency dipolar resonance of the PEC ground sphere, the resonances in the cavity between the PEC spherical disk and the ground, and the periodic high-frequency resonances of the quasi-surface modes of the "whispering gallery" type resulting in the "keel over" of the radiation patterns.

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