The Method of Analytical Regularization in Wave-Scattering and Eigenvalue Problems: Foundations and Review of Solutions

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1. Abstract

On the rugged terrain of today's computational electromagnetics, the universal rope-way of MoM and industrial rock-climbing with FDTD electric hammers are necessary technologies. However, a free-style solo climb at the Everest of analytical regularization is still a fascinating achievement. Here, we discuss the foundations and state-of-the-art of the Method of Analytical Regularization (also called the semi-inversion method). This is a collective name for a family of methods based on conversion of a first-kind or strongly-singular second-kind integral equation to a second-kind integral equation with a smoother kernel, to ensure point-wise convergence of the usual discretization schemes. This is done using analytical inversion of a singular part of the original equation; discretization and semi-inversion can be combined in one operation. Numerous problems being solved today with this approach are reviewed, although in some of them, MAR comes in disguise.

2. Introduction

Three different IEEE periodicals, dealing with closely related problems, have come out of the printers almost simultaneously: *IEEE Transactions on Antennas and Propagation*, **45**, 3, 1997 ("Special Issue on Advanced Numerical Techniques in Electromagnetics"); *Computational Science and Engineering*, **4**, 1, 1997; and *IEEE Antennas and Propagation Magazine*, **39**, 1, 1997. This made possible a comparison of the trends and efficiency criteria of numerical methods in computational electromagnetics (CEM), on the one side, and micro-electro-mechanical systems (MEMS) and fluid dynamics, on the other side. Although the opening paper in the *Transactions* [1] is an entirely mathematical essay on integral equations in Sobolev spaces, in my opinion, this special issue leaves no doubt that in CEM, finite-difference and finite-element methods, combined with genetic algorithms when it comes to optimization, are rapidly taking over integral-equation analysis. Meanwhile, a systematic study of accuracy, not to talk about the comparative costs of different convergent schemes, is still not a primary concern, in visible contrast with fluid dynamics [2] and MEMS [3]. Even such a trivial remark as that of [2]–that when talking about numerical accuracy, it is necessary to use a *relative norm of error* as a function of, say, the number of mesh points, and to display it on a *logarithmic scale* to show clearly the range and the rate of convergence–seems to still not be a common practice of CEM publications. On the other hand, in MEMS, intrinsic limitations of finite methods are so clearly understood [3] that serious academic and commercial efforts are made to use improved boundary-integral-equation formulations wherever possible. In view of this, it seems that the following review of an alternative CEM experience may be useful.

Together with fluid dynamics, MEMS, and some other neighboring fields of engineering science, applied electromagnetics today is closely tied to progress in computer-aided modeling. Technologies for fabricating antennas, microwave circuits, and sources have developed rapidly, but this is not so with computational tools enabling quick desktop design and optimization. Inadequate simulation tools still force engineers to resort to costly physical prototyping, which may take weeks, or to rely on intuition. Meanwhile, an aggressive design strategy towards devising really "smart" antennas and circuits–multi-element, high-performance, and low-cost–calls for both faster *and* more-accurate algorithms. In the CEM community, this double challenge seems still not to be recognized, and what is paid attention to are usually only the tradeoffs between the efficiency (i.e., the computation time and memory) and versatility of a solver. The only, but remarkable, exception seems to be the design of complicated waveguide circuits (filters, diplexers, multiplexers, transitions, etc.), where the accuracy of the numerical modeling of each elementary resonant discontinuity is crucial for the design of the whole circuit [4-6].

The performance of antennas and many other microwave devices has to be analyzed in large or infinite domains. This leads to finding solutions to exterior problems for the electromagnetic fields and waves; additionally, in many applications, simulating the time-harmonic-field performance is crucial. Thus, one comes to wave scattering and radiation analysis based on the time-independent Maxwell and Helmholtz equations, in open domains. Although it is possible to solve them by using finite-difference discretizations of the partial differential equations, associated problems of domain truncation, "good" exterior meshing, and solving enormous matrices are hardly compatible with high accuracy. To avoid these, boundary-element and Green's function methods can be used, applied to integral-equation formulations. Of the advantages, two main points are to be emphasized: the radiation condition is automatically taken into account, and only the boundaries need to be discretized. However, this frequently generates ill-conditioned dense matrices, and so something should be done to adapt these formulations to meeting the combined challenge of the speed and accuracy of computations. Many textbooks and journal papers on computational electromagnetics deal with first-kind integral equations for determining the surface or polarization currents of two-dimensional and three-dimensional metallic or dielectric scatterers, respectively, given the incident field. Such equations are obtained from the boundary conditions, and normally have logarithmic or higher-order singular kernels. They are further discretized for a numerical solution by using subdomain (collocations) or entire-domain basis functions. Although this commonly brings meaningful and useful results, unfortunately, there are not any general theorems proving convergence, or even the existence of an exact solution, for such equations [7]. A rule-of-thumb of taking at least 10 mesh points per wavelength is only a rule-of-thumb, and by no means does it guarantee any number of correct digits.

A good demonstration of what may sometimes happen to such algorithms is given in [8]. By the simple example of two-dimensional plane-wave scattering from a tubular circular cylinder, it is shown that the Moment-Method and FDTD solutions can be 1000% or more in error in a typical resonant situation. The "pain points" of the conventional Moment-Method approach have been excellently reviewed in [7]; since then, essentially nothing has changed. The final statement of [7] is worth reciting: "It is misleading to refer to the result as *solution* when in fact it is *numerical approximation* with no firm mathematical estimate of nearness to solution." Mathematically, if the corresponding integral operator is viewed as a mapping between certain Sobolev spaces, the existence of a solution and the convergence of standard discretization schemes for some of the first-kind equations can be shown. (A Sobolev space has a scalar product and norm defined through both the function and its gradient; in electromagnetics, such a norm is easily identified with power.) Uniqueness is normally guaranteed by a sufficient set of boundary, edge, and radiation conditions. A simple example of this sort is the logarithmic-singular integral equation in the two-dimensional E-wave scattering from a PEC (perfectly electrically conducting) flat strip. However, in a practical sense this is not a great deal, as the condition number grows with the number of equations [9], thus making the matrix impossible to solve for an accuracy better than several digits and a scatterer greater than 10-20 wavelengths. Nearly the same can be said of the discretization of second-kind equations having strongly singular kernels. Although it is possible to eliminate ill-conditioning by using specialized discretization schemes, based on Sobolev-space inner products [1], this appears to have a limited range of application.

Meanwhile, there exists a general approach to obtaining second-kind integral equations of the Fredholm type, with a smoother kernel, from first-kind equations. Discretization of these new equations, either by collocation or by a Galerkin-type projection on a set of basis functions, generates matrix equations the condition number of which remains small when the number of mesh points or "impedance-matrix" size is taken to be progressively greater. The approach mentioned is collectively called the Method of Analytical Regularization (MAR). The term has apparently been introduced by Muskhelishvili [10]; sometimes *semi-inversion* is used as a synonym. It is based on the extraction and analytical inversion of a singular part of the original full-wave operator; however, in principle, it is possible to make a partial inversion numerically. It must be admitted that the whole idea of MAR can be traced back to the pioneering work of the founders of singular-integral-equation theory, Hilbert, Poincaré and Noether, well before the first appearance of a computer.

3. Foundations of MAR

The formal scheme of MAR is deceptively simple, and works as follows. Assume that the boundary conditions generate a first-kind integral equation. In operator notation, this can be written as

$$\hat{G}X = Y, \tag{1}$$

where X and Y stand for the unknown and given function, respectively. In wave-scattering problems, a direct analytical inversion of $\hat{G}$ is normally not possible, while a numerical inversion, as has been mentioned, has no guaranteed convergence. Split operator $\hat{G}$ into two parts: $\hat{G} = \hat{G}_1 + \hat{G}_2$. Assume now that the first of these has a known inverse, $\hat{G}_1^{-1}$. Then, by acting with this operator on the original equation, one obtains a second-kind equation:

$$X + \hat{A}X = B, \tag{2}$$

where $\hat{A} = \hat{G}_1^{-1}\hat{G}_2$ and $B = \hat{G}_1^{-1}Y$. However, this scheme is mathematically justified only if the resulting operator equation is of the Fredholm type. This means that the operator $\hat{A}$ must be compact on a certain Hilbert space H (i.e., must have a bounded norm $\|A\|_H < \infty$), and the right-hand side vector B must belong to the same space H. This inherently implies that the inverted operator $\hat{G}_1$ is singular, while $\hat{G}_2$ is regular. Then, all the power of the Fredholm theorems generalized for operators [10-12] can be exploited, proving both the existence of an exact solution, $X = (I + A)^{-1} B$ (I is the identity operator), and the point-wise convergence of discretization schemes in H, without resorting to residual-error estimations like for first-kind equations [1, 7]. Indeed, suppose that we have discretized the second-kind equation. Consider its "truncated" counterpart,

$$X^N + A^N X^N = B, \tag{3}$$

the matrix A^N for which is filled with zero elements off the $N \times N$ square. It is easy to show that the relative error, by the norm in H,

$$e(N) = \frac{\|X - X^N\|}{\|X\|} \le \left\|(I + A)^{-1}\right\| \left\|A - A^N\right\|, \tag{4}$$

is destined to go to zero with $N \to \infty$, as the first factor in the right-hand part above is a bounded constant, while the second is decreasing. Of course, in finite-digit arithmetic, this decrement is limited by the machine precision. The rate of decay of the function $e(N)$ determines the *cost of the algorithm*, and this can be different for different ways of selecting the invertible singular part, $\hat{G}_1$.

Here comes a key question: how to select the operator $\hat{G}_1$? It is apparently possible to point out at least three basic principles for extracting an invertible singular part of the original operator. These are extracting the *static part* (Laplace equation theory is simpler than the Helmholtz theory, and sometimes associated boundary problems can be solved analytically); extracting the *high-frequency part* (in fact, this is about half-plane scattering, which can be solved by the Wiener-Hopf method); and extracting the frequency-dependent part corresponding to a *canonical shape* (which is either a circle, in two dimensions, or a sphere, in three dimensions, solvable by separation of variables). A note can be made that a half plane is also a sort of canonical-shape scatterer, but it has an infinite surface in terms of any length parameter, including the wavelength.

Although inversion of the static or high-frequency part seems to be based on quite specialized functional techniques, it is useful to point out one general feature in all the above cases. If it is possible to find a *set of orthogonal eigenfunctions* of the separated singular operator $\hat{G}_1$, then the Galerkin-projection technique, with these functions as a basis, immediately results in a regularized discretization scheme (i.e., yields a Fredholm second-kind infinite matrix-operator equation). This is especially evident with the third way of extracting out an invertible operator, as in this case, the orthogonal eigenfunctions are just azimuthal exponents or spherical harmonics (products of the former with the Legendre functions). Such an eigenfunction-Galerkin projection, in fact, combines both semi-inversion and discretization in one single procedure. This was apparently first clearly formulated in [13] for eigenvalue problems in open domains, although the emphasis there was on the opportunity to obtain iterative solutions. It should be noted that, as frequently happens, in the neighboring area of elasticity theory, this latter technique has been in use before it was in electromagnetics [14]. One may easily see that it bridges the gap between MAR and conventional MoM solutions. Indeed, the intuitive idea that a judicious choice of expansion functions in MoM can facilitate convergence obtains the form of a clear mathematical rule: to have the convergence guaranteed, take the expansion functions as orthogonal eigenfunctions of $\hat{G}_1$. The procedure of finding such functions is called *diagonalization* of a singular integral operator. From the viewpoint of numerical analysis, semi-inversion plays the role of a perfect pre-conditioning of the original operator equation, the direct discretizations of which are ill-conditioned.

4. Two examples

To make this review more tutorial, two examples of MAR-based numerical solutions are very briefly presented here; details can be found in [15] and [16], respectively. Both of these are about scatterers in layered media. In the first case, it is the free-space canonical-shape inversion that is used; in the second case, it is the free-space static-part inversion.

4.1 Circular dielectric cylinder in layered medium

The geometry of the problem is shown in the insert of Figure 1. The incident field is specified by the excitation. If it is a guided surface mode of the dielectric substrate, then the problem serves as two-dimensional model of the whispering-gallery-mode dielectric resonator (DR), used as a band-stop filter. Considering, for definiteness, the case of H polarization, we present the scattered field in terms of the single-layer potentials. Transmission-type boundary conditions at the surface of the DR lead to the following set of integral equations:

$$\int_L \varphi(\vec{r}')G_\varepsilon(\vec{r},\vec{r}')dl' - \int_L \psi(\vec{r}')G_w(\vec{r},\vec{r}')dl' = H_z^{in}(\vec{r}), \quad (5)$$

$\vec{r} \in L$,

$$\lim_{r\to a-0}\frac{\partial}{\varepsilon\partial n}\int_L \varphi(\vec{r}')G_\varepsilon(\vec{r},\vec{r}')dl' - \lim_{r\to a+0}\frac{\partial}{\partial n}\int_L \psi(\vec{r}')G_w(\vec{r},\vec{r}')dl' = \frac{\partial H_z^{in}(\vec{r})}{\partial n}. \quad (6)$$

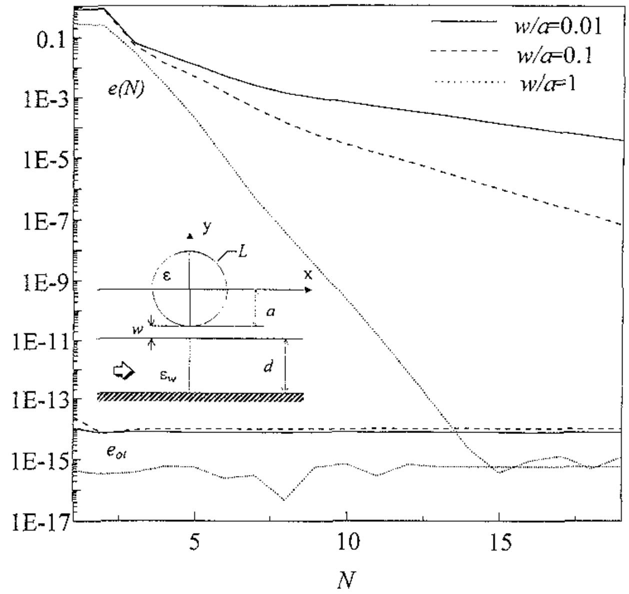

Figure 1. The MAR computation error versus the number of expansion functions (i.e., the matrix size), for the dielectric-slab surface wave scattering from a circular dielectric cylinder [15].

Here, φ and ψ are functions to be found, L is the contour of the DR cross-section, $\vec{n}$ is the outer normal unit vector, and H_z^{in} is the incident field. The kernels in Equations (5) and (6) depend on the uniform space Green's functions:

$$G_0 = i/4H_0^{(1)}\left(k|\vec{r}-\vec{r}'|\right),$$
$$G_\varepsilon = i\varepsilon/4H_0^{(1)}\left(k\varepsilon^{1/2}|\vec{r}-\vec{r}'|\right), \quad (7)$$

and the Green's function of the layered medium:

$$G_w = G_0 + G_s = G_0 + \int_{-\infty}^{\infty} f(kd,\varepsilon_w,h)e^{i\left(1-h^2\right)^{1/2}k(y+y')+ihk(x-x')}dh. \quad (8)$$

The integrand function in G_s is found analytically as a meromorphic function on the two-sheet Riemann surface of complex variable h; integration is done along the real axis of the "proper" sheet, bypassing the poles from the lower side. Now, introduce the two-component vector functions X and Y of unknown densities and the right-hand parts, respectively, and a 2×2 "matrix" kernel $\hat{G}$:

$$X = \begin{Bmatrix}\phi \\ \psi\end{Bmatrix}, \quad Y = \begin{Bmatrix} H_z^{in} \\ \dfrac{\partial H_z^{in}}{\partial n}\end{Bmatrix}, \quad \hat{G} = \begin{pmatrix}\hat{G}_\varepsilon & -\hat{G}_w \\ \varepsilon^{-1}\hat{G}'_{\varepsilon-} & -\hat{G}'_{w+}\end{pmatrix}, \quad (9)$$

where $\hat{G}_{\varepsilon,w} = \int_L \ldots G_{\varepsilon,w}dl'$, $\hat{G}'_{\varepsilon,w\pm} = \lim_{r\to a\pm 0}\frac{\partial}{\partial n}\int_L \ldots G_{\varepsilon,w}dl'$. Then we can rewrite Equations (5) and (6), in the operator notation, as one first-kind equation of the form of Equation (1): $\hat{G}X = Y$. Now, note that the kernel function G_w is a sum of the singular free-space term G_0 and the regular term G_s, given by the Fourier integral in Equation (8). This leads to the following decomposition of the whole "matrix" operator $\hat{G}$ into singular and regular parts:

$$\tilde{G} = \hat{G}_1 + \hat{G}_2 = \begin{pmatrix} \hat{G}_\varepsilon & -\hat{G}_0 \\ \varepsilon^{-1}\hat{G}'_\varepsilon & -\hat{G}'_{0+} \end{pmatrix} + \begin{pmatrix} 0 & -\hat{G}_s \\ 0 & -\hat{G}'_{s+} \end{pmatrix}. \quad (10)$$

In the case where L is a circle of radius a, the orthogonal eigenfunctions diagonalizing all four operators $\hat{G}_0$, $\hat{G}'_{0+}$, $\hat{G}_\varepsilon$, and $\hat{G}'_\varepsilon$ are easily found as the angular exponents $e^{in\phi}$, $n = 0, \pm 1, \pm 2, \ldots$. Indeed,

$$\int_0^{2\pi} e^{in\phi'} H_0^{(1)}\left(2ka \sin\frac{1}{2}|\phi - \phi'|\right) d\phi' = e^{in\phi} 2\pi J_n(ka) H_n^{(1)}(ka), \quad (11)$$

$$\lim_{r \to a \pm 0} \frac{\partial}{\partial n} \int_0^{2\pi} e^{in\phi'} H_0^{(1)}\left[k\sqrt{(r\cos\phi - a\cos\phi')^2 + (r\sin\phi - a\sin\phi')^2}\right] d\phi'$$
$$= e^{in\phi} 2\pi k \begin{cases} H_n'^{(1)}(ka) J_n(ka) \\ J'_n(ka) H_n^{(1)}(ka) \end{cases}, \quad (12)$$

were J_n and $H_n^{(1)}$ are the first-kind Bessel and Hankel functions of order n, respectively; the prime is for the derivative with respect to the argument.

So, in the absence of the slab, one has $\hat{G}_2 = 0$, and full inversion of the operator $\hat{G}$ is possible, leading–no surprise–to the well-known Mie-type series solution of the free-space circular-cylinder scattering. However, in the presence of the slab, inversion of $\hat{G}_1$ by means of the diagonalization procedure yields only a partial inversion of the full operator $\hat{G}$. Therefore such a specialized MoM-type projection results in the infinite matrix Equation (2), instead of the series solution [15]. It can be verified that this equation is of the Fredholm second kind in the space of the square-sumable number sequences l_2. The behavior of the computational error $e(N)$, in the sense of the l_2-norm, is presented in Figure 1 for this equation. The error is progressively minimized all the way to the machine precision by increasing N; the more the distance from the DR to the slab, the fewer expansion functions one should take to achieve the needed accuracy. Note also that the error in the energy-conservation law (also known as the Optical Theorem), e_{ot}, is always at the machine-precision level, thus being satisfied in a term-by-term manner.

4.2 Lossy circular disk patch on dielectric substrate

The geometry of the problem is shown in the inset of Figure 2. We suppose that a uniformly-resistive circular disk is coaxially excited by a vertical electric dipole, located at the ground plane. This problem can serve as the simplest printed-antenna model, although it has some real-life applications. The field in such a geometry is ϕ-independent, and can be expressed via a single potential function in the form of Fourier-Bessel transformation. On using the resistive boundary conditions at the disk and the free surface of the substrate, one arrives at dual integral equations, which collectively form a familiar operator equation of the first kind, $\hat{G}X = Y$:

$$\int_0^\infty x(\kappa) W(\kappa, ka, h/a, \varepsilon) J_1(\kappa\rho) d\kappa = y(\rho), \quad \rho = r/a < 1, \quad (13)$$

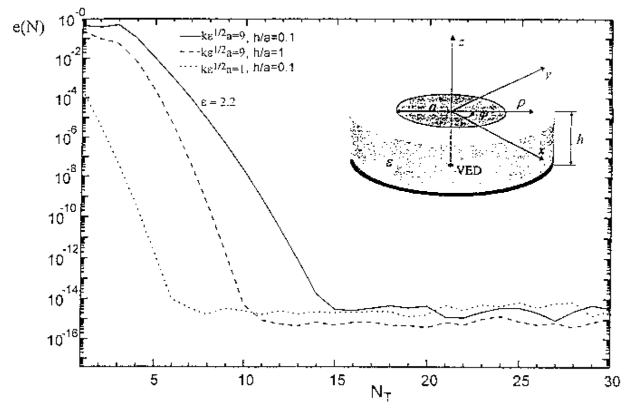

Figure 2. The same as in Figure. 2, but for the axisymmetric dipole excitation of a circular-patch antenna [16].

$$\int_0^\infty x(\kappa) J_1(\kappa\rho) d\kappa = 0, \quad \rho > 1. \quad (14)$$

Here $x(\kappa)$ is an unknown function, and

$$W(\kappa, ka, h/a, \varepsilon) = \kappa - \frac{\gamma\gamma_\varepsilon}{\gamma_\varepsilon + \gamma\varepsilon \coth(\gamma_\varepsilon h/a)} + ikaR, \quad (15)$$

with $\gamma = \left(\kappa^2 - k^2a^2\right)^{1/2}$, $\gamma_\varepsilon = \left(\kappa^2 - k^2a^2\varepsilon\right)^{1/2}$, and R is the resistivity normalized by the free-space impedance.

As one can see, Equation (15) is a meromorphic function of variable κ, with a finite number of real poles, and also with complex poles responsible for the surface and leaky waves, respectively. The right-hand part $y(\rho)$ is determined by the excitation, i.e., by the dipole location [16]. Now, note that in the case of $h/a \to \infty$ and $ka \to 0$, the weight function is $W(\kappa) \to W_0 = \kappa \frac{\varepsilon}{\varepsilon + 1}$. Fortunately, if W is replaced with κ, Equation (13) has a set of orthogonal eigenfunctions that diagonalize this equation. They are:

$$f_n = \left[(4n+5)/\kappa\right]^{1/2} J_{2n+5/2}(\kappa), \quad n = 0, 1, 2, \ldots. \quad (16)$$

Note also that Equation (14) is identically satisfied by the functions of Equation (16). Hence, if we take this set of functions as the expansion basis in the Galerkin discretization scheme, we obtain a Fredholm second-kind infinite matrix equation, $X + \hat{A}X = B$, that is equivalent to Equations (13) and (14) together, and has a solution in l_2. In Figure 2, the behavior of the computational error as a function of the number of expansion functions is shown. One can see that machine precision is achieved with only several expansion functions. This is not possible if one takes the so-called "cavity modes" as the expansion basis, or resorts to subdomain discretizations.

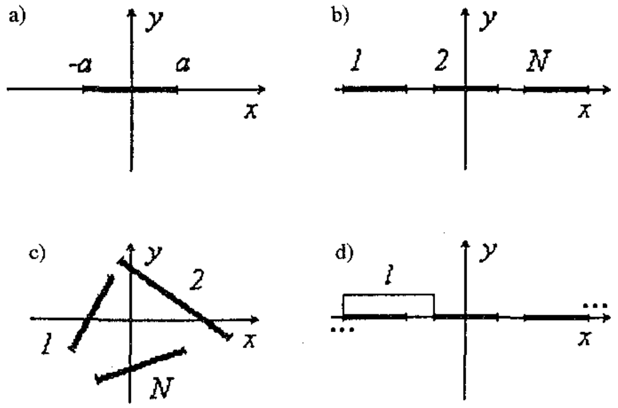

Figure 3. Two-dimensional flat-strip geometries: (a) a single strip, (b) flat, and (c) arbitrary finite arrays of strips; (d) an infinite periodic strip grating.

5. Review of solved problems

5.1 PEC screens–static part inversion

A variety of problems, solved by MAR with a static-part inversion, cover a wide class of two-dimensional-metal zero-thickness scatterers (screens). Among these, there is first of all a **PEC flat strip** and related geometries (Figures 3a-3d): finite collections of strips, infinite periodic strip gratings, strip irises in parallel-plate waveguide, and straight slots (single or multiple) in a PEC plane. This is due to the fact that an integral equation with a Cauchy kernel has a known inverse, which is used to convert an H-polarization-case electric-field integral equation into a Fredholm second-kind equation, since the static limit of the full-wave operator is reduced exactly to a Cauchy operator. The theory of this procedure has been developed by Muskhelishvili [10], Gakhov [17], Shtayerman [18], and Krein [19]; however, earlier results of Carleman, Keldysh, and Vekua were important. Interestingly, the same technique seems to have been developed independently by Hayashi [20]. The E-polarization integral equation is first differentiated to obtain the Cauchy kernel as a static limit, and then the same semi-inversion is used. Regularization can also be based on the analytical solution to a logarithmic-kernel integral equation (see [18, 21]). In fact, this mathematical approach seems to have been invented and re-invented several times within the last 50 years, in several equivalent formulations.

After analytical semi-inversion, one may apply different discretization schemes: this can be just a collocation method, but it can also be a variant of the Galerkin projection technique. In the former case, it is advantageous to first perform a transformation from the space domain to the Fourier-transform domain, as then the kernel is smoothed and, hence, the resulting equation can be more easily solved [22]. In the latter case, two choices of the basis functions are especially remarkable (equivalence between them exists based on re-expansions in terms of each other).

One is the set of strip or slot-supported Chebyshev polynomials of the first or the second kind, with a square-root weight or its inverse, depending on the polarization. This choice takes into account the electric-current edge behavior, and is convenient in the study of the scattering from both a single strip [23-26], finite collections of strips [27, 28], and infinite periodic strip gratings [29, 30]. Moreover, this projection can be applied directly to a first-kind equation, as these weighted polynomials form a set of orthogonal eigenfunctions of the static kernel (see the very last sentence of [24]); this was apparently done first in [23]. In fact, the so-called Richmond's edge-wave approximation [31] is simply the zeroth weighted-polynomial term of such a full projection scheme. This can explain why a convergence improvement is observed if one uses several edge-weighted terms in the Moment-Method expansions [32]. However, calculating the matrix elements in the Chebyshev discretization involves numerical integrations of the weighted products of trigonometric functions. This can be done more economically by adding and subtracting the asymptotic form of the integrand, or by reducing the integration to summing up certain series. A similar projection can be done after applying the Fourier transformation to the first- or second-kind integral equations [33]. The Chebyshev polynomials are then transformed to the Bessel functions, and one has to numerically integrate the oscillating products of these functions to fill the matrix.

The other choice is useful provided that the scatterer is a flat infinite **periodic strip grating** (Figure 3d), or a strip iris in a waveguide. This is to discretize the second-kind equation in terms of the full-period exponents [34, 35]; hence, the unknowns coincide conveniently with the Floquet-mode amplitudes. Such a scheme leads to matrix elements that are reduced to finite combinations of the Legendre polynomials, and thus no numerical integration is needed. Additionally, this scheme is equally efficient for arbitrary-strip-width-to-period ratios of the grating. In grating problems, such a discretization can be introduced from the beginning; then, matching the fields results in the so-called dual-series equations. That is why this technique is called the dual-series or the Riemann-Hilbert Problem (RHP) method in many publications. Indeed, the static part of an arbitrary two-dimensional H-wave field, scattered by a two-dimensional PEC screen, is represented by a Cauchy integral. Hence, determining it can be reduced to a RHP about recovering an analytic function from its limiting values on a curve given by the contour of the screen. A general solution to the RHP was given in [10, 17]. However, equivalently, the unknown function can be obtained directly by using the properties of inte-

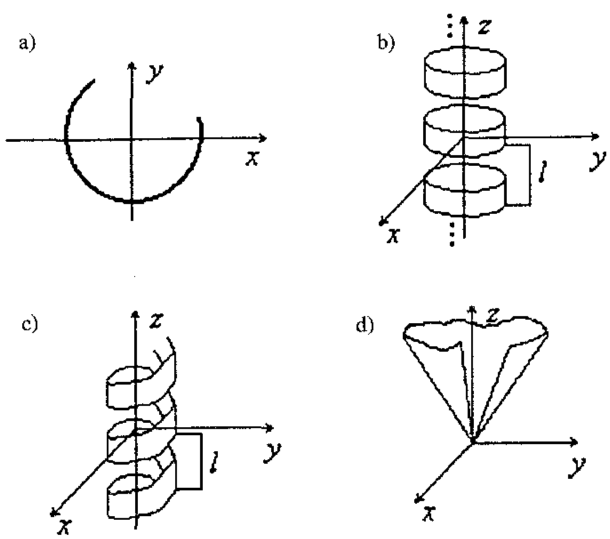

Figure 4. Generalized periodic-screen types of geometries: (a) A two-dimensional circularly curved strip, (b) a circular ring waveguide, (c) a circular helically-slit waveguide, and (d) an infinite slit cone.

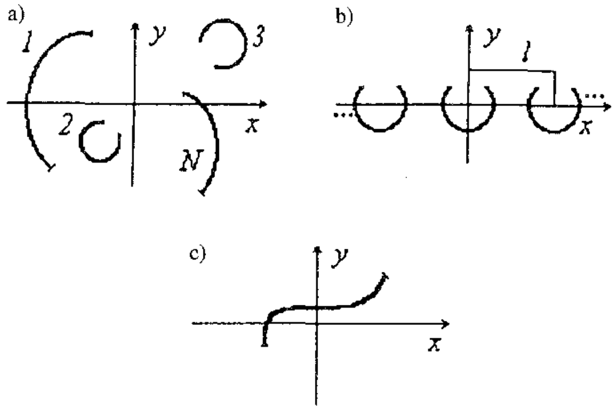

Figure 5. Two-dimensional modified curved-strip geometries: (a) a finite collection and (b) an infinite grating of circularly curved strips; (c) an arbitrarily curved strip.

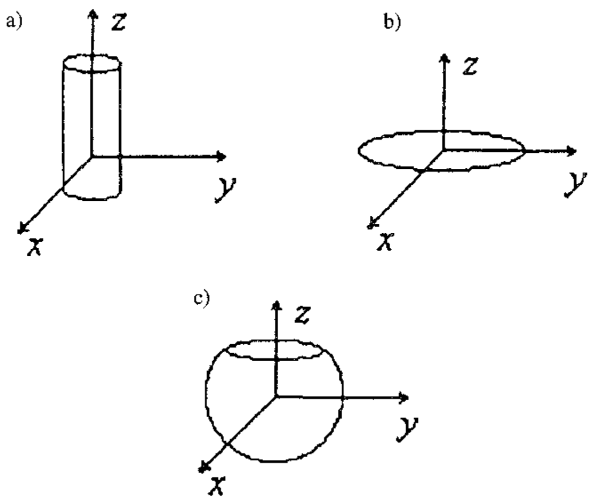

Figure 6. Axisymmetric three-dimensional screen types of geometries: (a) a finite hollow circular pipe, (b) a flat circular disk, and (c) a spherical-circular cap.

grals with the Cauchy or logarithmic kernels [18-20], i.e., without resorting to dual-series equations and the RHP at all. Such a technique was first developed in [36], and was recently re-invented again in [37]: both authors used the same book [18] as a basis.

Due to a topological analogy (periodicity along a coordinate surface), the same choice of exponents in the discretization works well for studying wave scattering from a **PEC axially slit circular cylinder** (i.e., a circularly curved strip in two dimensions) [38-42] and a **periodic transversely slit circular cylinder** [43] (Figures 4a, 4b). Moreover, a **helically slit circular cylinder** (Figure 4c), with a constant-width infinite slot [44], and an infinite **PEC axially slit cone** (Figure 4d), with a slot of constant angular width [45] have also been analyzed by this method, combined with a transformation to the helical coordinates and with the Kontorovich-Lebedev integral transform, respectively. A cylinder with N identical periodic slots was solved in [38]. Finite collections of axially-slit cylinders [46-48], and an infinite periodic grating of such cylinders [49] (Figures 5a, 5b), have been studied by combining the RHP technique with the addition theorems for cylindrical functions. An arbitrarily curved two-dimensional strip (Figure 5c) has been proposed to be studied by extracting out and inverting the static part of the problem, corresponding to a flat [50] or a circularly curved strip [51]. Numerical results are available only for an elliptically curved strip. In this case, matrix elements involve numerical integrations for the expansion coefficients of the difference of kernels, which becomes computationally expensive if the strip contour differs much from a straight interval or a circle, respectively.

Turning to three-dimensional axially-symmetric scatterers, single flat-strip MAR solutions are directly applicable to scattering by a **PEC finite circular pipe** (Figure 6a). The resulting second-kind equations have kernels that are similar to the flat-strip case, both in the space and Fourier-transform domains. Diagonalization of the static-part integral operators is again done in terms of the weighted Chebyshev polynomials and the corresponding Bessel functions, respectively. Published results relate mainly to axisymmetric excitations [52-55], with applications to accelerator drift tubes and dipole antennas. Similar solutions take place for a complementary geometry of a finite slot cut across an infinite circular waveguide.

In the case of a **PEC circular disk** (Figure 6b), a Fredholm second-kind integral equation was obtained in [56], in the Fourier-Bessel (also called the Hankel) transform domain, due to a Titchmarsh [57] inversion formula. Similarly to the above, it was solved by collocations. An eigenfunction-Galerkin solution, in the transform domain, was proposed in [58] by projecting onto a set of the Bessel functions of semi-integer index (Equation (16)). In this case, the matrix elements involve numerical integrations of the Bessel-function products. In the space domain, a Galerkin-type projection onto the Jacobi polynomials works out as well, because the latter form a set of orthogonal eigenfunctions of the static limit of the integral-equation operator associated with a free-space disk (the Bessel functions mentioned appear quite naturally as the transforms of these polynomials).

A **PEC spherical cap** of arbitrary angular width (a spherical shell with a circular aperture, Fig. 6c) is solved in a conceptually analogous way, by exploiting an exact solution of the Abel integral equation [57]. Here, projection is naturally done on the set of the Legendre functions of the axial-plane angular coordinate as an entire-domain expansion basis [59-62]. Then, the matrix elements are reduced to trigonometric functions, and do not involve numerical integrations. This solution has been extended to spheroidal rotationally-symmetric caps by using expansions of angular-spheroidal functions in terms of spherical functions [63]. A note should be made that both for a finite pipe, a disk, and a spherical cap, electromagnetic (i.e., vector) wave scattering leads to the coupled integral equations and further matrix equations for two potential functions, unless the excitation is azimuthally symmetric.

5.2 PEC screens–high-frequency-part inversion

As has been mentioned, in the core of this analysis there lies an analytical solution to the Wiener-Hopf integral equation [64] for the scattering by a PEC half-plane. This solution was used to obtain a Fredholm second-kind integral equation, in the Fourier-transform domain, to the problem of a wave scattering from a finite PEC flat strip [65, 66]. The kernel of this equation decays with increasing strip width, thus insuring that a numerical solution will be progressively more efficient for larger scatterers. A technique similar to [65] has been developed for scattering from a periodic strip grating [67], a disk [68], and a finite cone [69], here in combination with the Kontorovich-Lebedev integral transformation.

There is no doubt that these analyses can be potentially extended to curved screens, as semi-infinite curved scatterers have been solved by the Wiener-Hopf method in [70].

In fact, a MAR technique, developed in [71, 72] for the analysis of waveguide-bend discontinuities, is closely tied to the approach mentioned, although it is more conveniently formulated via a modified residue-calculus technique.

5.3 Non-PEC screens

A class of interesting problems is associated with zero-thickness screens supporting imperfect boundary conditions. There are basically three types of such conditions [73, 74]: resistive, thin-dielectric, and impedance-surface. The first two characterize transparent, and the last one, non-transparent, imperfect screens; resistive-screen scattering is a key problem. In the two-dimensional case, a resistive boundary condition, unlike a PEC, leads directly to a second-kind integral equation. However, the further treatment is completely different for the two polarizations. In the E case, this equation has a logarithmically-singular kernel, and so it is already a Fredholm one. Thus, non-zero resistivity plays the role of a regularizing parameter in Tikhonov's sense [75], and hence no other analytical regularization is needed and any reasonable discretization scheme converges. But in the H case, the original equation kernel still has a strong singularity, and thus must be regularized: the same schemes as in the PEC case work out. So far, MAR solutions based on the static-part inversion have been reported for the single flat resistive [76] and impedance [77] strips; for periodic resistive-strip gratings [78, 79], and in the latter paper for a dielectric-strip grating as well; and for a circular resistive strip [80]. Worth noting is that not only the scattering but also the absorption by imperfect screens was studied. High-frequency part inversion was used in [81, 82] to derive iterative solutions of the Fredholm second-kind integral equations for the scattering from imperfect flat strips.

All the above-mentioned solutions can also be generalized to variable-resistivity screens, at the expense of a certain loss in the convergence rate (i.e., an algorithm becomes more costly for a fixed accuracy). In [83], this was demonstrated in the analysis of a cylindrical-reflector antenna with a non-uniformly resistively loaded edge.

5.4 Canonical-shape inversion

It is well known that the scattering from a PEC, as well as from an imperfect or material circular cylinder in free space, is exactly solved by the Fourier, or separation-of-variables, method. It is reduced analytically to summing up infinite series of azimuthal harmonics (exponents), with cylindrical functions in the coefficients [84, 85]. Similarly, for three-dimensional free-space scattering from a spherical object, the Mie solutions, in terms of a series of spherical harmonics, are known [85]. This can be attributed to the fact that the corresponding integral equations have a set of azimuthal exponents or spherical harmonics as entire-domain orthogonal eigenfunctions of the full kernel. This can be used to develop a MAR solution in the **arbitrary smooth-surface scatterer** analysis (Figures 7a, 7b). Extracting out a canonical-shape part of the kernel function, and using the above functions as a Galerkin projection basis, one comes to a Fredholm second-kind matrix equation. MAR solutions of this sort have been obtained for the scattering from smooth PEC and dielectric cylinders in free space and in plate-parallel waveguides [86], and for inhomogeneous spherical particles [87].

A special two-dimensional shape is a **PEC polygonal cylinder** (Figure 8a). It can be viewed as a finite collection of flat strips, and thus a single-strip MAR analysis can be used as a reference. The fact that the strips are joined by the edges, and thus have a modified edge behavior of the field, is not very important, as it is less singular than in the single-strip case. In the eigenfunction-Galerkin scheme, this can be accounted for by choosing the Gegenbauer or Jacobi polynomials of a needed index as expansion functions, instead of the Chebyshev polynomials. MAR solutions to this problem have been published based on the static-part inversion [88], and on the analytical solution to a flat strip as a degenerate form of an elliptic cylinder [89]. A dual counterpart of this geometry is a two-dimensional model of a waveguide multi-arm junction, where the elementary scatterer is an infinitely-flanged slot (Figure 8b). A single-flanged-slot analysis with a static-part inversion was done in [90, 91], and step discontinuities in waveguides (Figure 8c) were solved in [92, 93]. A junction analysis, based on the elliptic-function solution to a single slot, was considered in [94].

Worth mentioning is that the method of [88] has been extended in [95] to treat polygonal cylinders with circularly curved facets (Figure 8d).

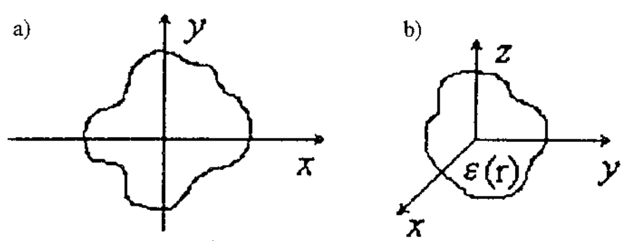

Figure 7. Arbitrary-shape smooth geometries: (a) A two-dimensional metal or material scatterer, (b) a three-dimensional inhomogeneous dielectric particle.

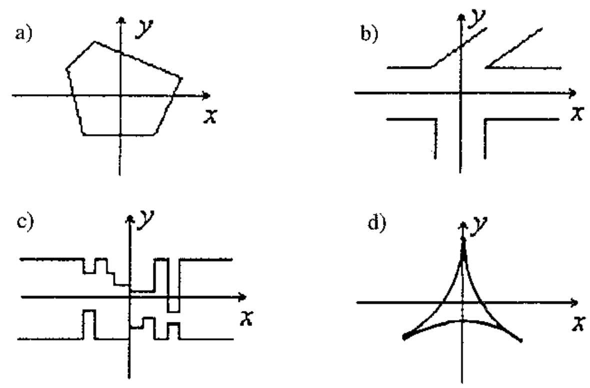

Figure 8. Piecewise-smooth two-dimensional geometries: (a) a flat-facet polygonal cylinder, (b) a waveguide joint, (c) a stepped waveguide circuit, and (d) a polygonal cylinder with circularly curved facets.

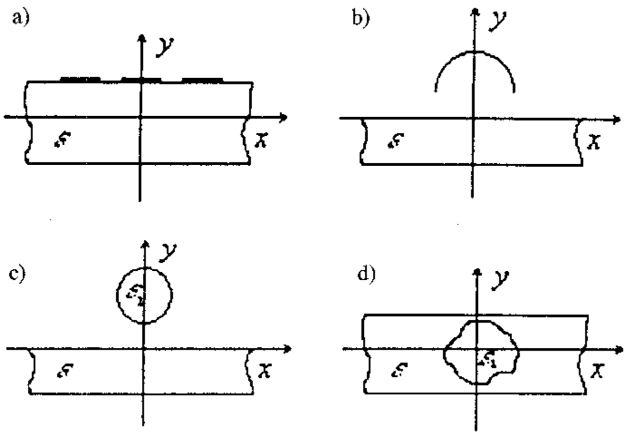

Figure 9. Two-dimensional geometries of scatterers in flat-layered media: (a) a finite strip array on a dielectric substrate, (b) a circularly curved strip and (c) a circular dielectric cylinder near a dielectric slab, and (d) a smooth cylindrical inhomogeneity in a slab.

5.5 Scatterers in layered media

Extensions of all the above-mentioned MAR solutions to similar scatterers embedded into a **flat-layered medium** with piecewise-constant material parameters are possible. This is due to the fact that the corresponding Green's function (available analytically in the transform domain) has the same type of singularity as a free-space counterpart. Extracting out this main part from the kernel of an integral equation, and handling it in the same way as for free-space scattering, leads to a regularized second-kind equation with a modified smooth kernel. The latter accounts for the continuity conditions across the layered-medium boundaries. The matrix elements of the discretized equation obtain additional terms, associated with the effect of the layers. MAR solutions of this type have been reported for the flat [96-98] and circular PEC strips [99-101], on and near to a material interface (Figures 9a, 9b), and in a dielectric-slab waveguide. Besides, in two dimensions, circular [102-104] and arbitrarily smooth cylinders [105] in a layered medium (Figures 9c, 9d) have been solved; applications to the CAD of surface-wave band-stop filters were studied in [15]. In combined geometries, the expansion of the cylindrical waves in terms of plane waves (given by the Fourier integrals) is involved, and hence the matrix elements contain numerical integrations. In three-dimensions, plane-wave scattering by a PEC disk, on or near an interface, has been analyzed in [56].

In the case of circular-cylindrical and spherical open-screen scatterers, MAR solutions are easily modified for inhomogeneous coaxial and concentric **cylindrically and spherically-layered media** [106-111] (Figures 10a, 10b). Non-coaxial geometries, such as a confocal resonator with an inhomogeneity [112] and a reflector in a radome [113] (Figures 10c, 10d), have been solved as well, although new infinite series appear, due to using the addition theorems for cylindrical functions. The MAR approach of [96] has been extended in [114] to the case of a PEC strip buried in a dielectric circular cylinder (Figure 11a). As an example of a three-dimensional mixed geometry, an elegant MAR-type solution has been given in [115] for guided-mode scattering from a spherical particle in a circular dielectric waveguide (Figure 11b), based on the expansion of spherical wave functions in terms of cylindrical vector wave functions.

A more complicated case of a mixed layered geometry is the scattering of waves from localized scatterers in **periodic media**, for example, near a periodic surface. This analysis is based on the generalized Fourier-integral transformation, taking into account that in the transform domain, the field is represented as a series in the Floquet-Rayleigh space harmonics. A two-dimensional model of an open resonator, formed by a circularly-curved PEC strip and a periodic flat-strip grating (Figure 11c), was studied in [99, 116] (static-part inversion), and a circular cylinder above a sinusoidal interface (Figure 11d) was considered in [117] (free-space cylinder inversion).

5.6 Analytical solutions

As has been emphasized, MAR solutions, based on the Fredholm second-kind matrix equations, have a guaranteed point-

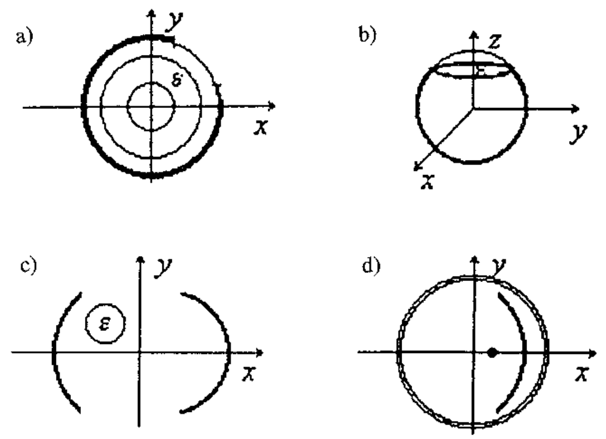

Figure 10. Geometries of scatterers in circular-cylindrical and spherical layered media: (a) a circular slit cylinder and (b) a spherical-circular cap with coaxial and concentric material fillings or coatings, (c) a two-dimensional open resonator of two circularly curved strips with a circular dielectric rod, and (d) a two-dimensional circular-reflector antenna inside a circular dielectric radome.

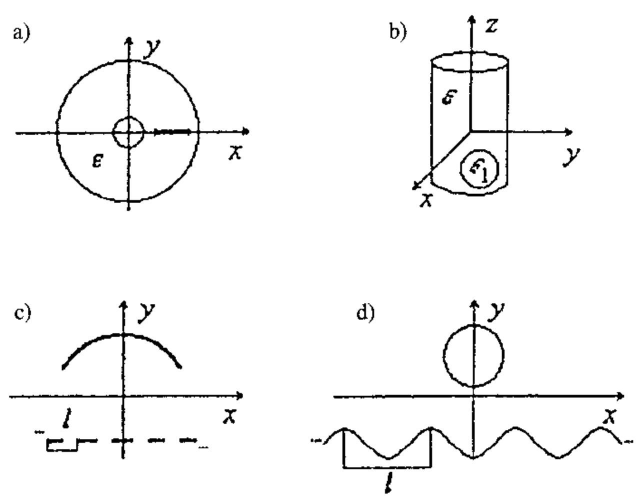

Figure 11. Combined geometries: (a) a flat strip and (b) a spherical particle inside a circular dielectric cylinder, (c) a circularly curved strip near an infinite flat-strip grating, and (d) a circular cylinder over a periodic interface.

wise convergence, and thus a controlled accuracy of numerical results. Depending on the nature of the inverted part, the number of equations needed for a practical two-to-three-digit accuracy is usually slightly greater than, respectively, the electrical dimension of the scatterer (see Figures 1 and 2), or its inverse value, or the normalized deviation of the surface from the canonical shape, in terms of both distance and curvature. In fact, the norm of the compact operator $\|A\|_H$ is always proportional to one of the above-mentioned values, denoted as, say, κ. This enables one to exploit an important feature of the Fredholm second-kind equations. Provided that $\|A(\kappa)\|_H < 1$, which can always be satisfied for a small enough κ, an iterative solution to the above equations is given formally by the Neumann-operator series

$$X = \sum_{s=0}^{\infty} \left(\hat{A}\right)^s B, \qquad (17)$$

which converges to the exact solution, by norm in H. Hence, one can avoid inverting Equation (2), at least in a certain domain of parameters. For example, in [65] it was demonstrated that the low-frequency and high-frequency MAR integral equations in PEC flat-strip scattering have overlapping domains of the Neumann series convergence. This is the greatest hit of MAR, as it completely eliminates the need of solving a matrix. Besides numerical efficiency, this has another attractive consequence. On expanding Equation (17) in terms of the power series of κ, one obtains, analytically, rigorous asymptotic formulas for the low-frequency or high-frequency scattering, or the scattering from a nearly-canonical object. Such asymptotics have been published for PEC flat [65, 118-121] and circular [40, 80] strips, finite pipes [53, 122], disks [56, 68], a spherical cap [59], and PEC and imperfect strip gratings [67, 79]. What is worth noting is that this can be done for various excitations specified by B: plane or cylindrical waves, a complex source-point beam, a surface wave in the layered-media scattering, etc. For dielectric, material, or chiral scatterers, another small parameter can be used in the asymptotic solution: this is the contrast between a scatterer and a host medium, in terms of material constants. In [102] and [115], approximate solutions of this kind were obtained for cylindrical and spherical inhomogeneities in the slab and fiber waveguides, respectively.

5.7 Eigenvalue problems

These problems are closely tied to the wave-scattering problems. They can be classified as either natural-frequency or natural-wave problems, although other eigen-parameters can be considered. The natural-wave problems appear only in the analysis of infinite cylindrical geometries, assuming a traveling-wave-field solution (i.e., $\sim e^{-ikct+i\beta z}$). Correspondingly, the complex parameter, the eigenvalues of which are to be determined, is either the normalized frequency, k, or the modal wavenumber, β (the propagation constant). What is important, in either case, is that a MAR solution leads to a homogeneous equation analogous to the scattering problem:

$$X + \hat{A}(k,\beta) X = 0. \qquad (18)$$

This is a Fredholm operator equation in H, with compact operator A normally being a continuous function of the geometrical parameters and a meromorphic function of the material parameters, frequency, and modal wavenumber. Hence, due to the Steinberg

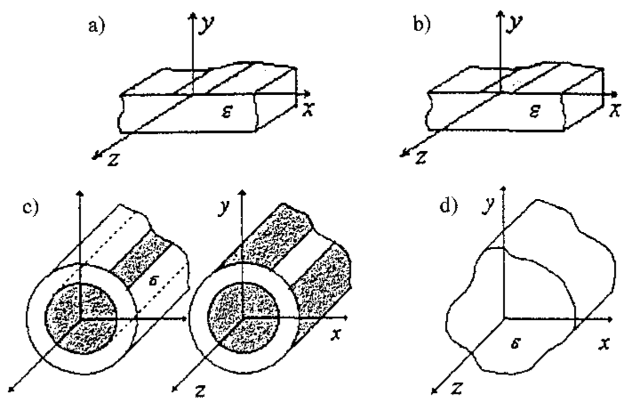

Figure 12. Regular waveguide geometries: (a) planar slot and (b) microstrip lines, (c) circular cylindrical microstrip and slot lines, and (d) an arbitrary-cross-section dielectric waveguide.

theorems [12], it is guaranteed that the eigenvalues form a discrete set on a complex k plane (in the three-dimensional case), or on a logarithmic Riemann k or β surface (in the two-dimensional case and for natural waves). There are no finite accumulation points; eigenvalues can appear or disappear only at those values of the other parameters where continuity or analyticity of A is lost. Moreover, after discretization, the determinant of the infinite-dimensional matrix $Det(I + A)$ exists as a function of a parameter, and its zeroes are the needed k or β eigenvalues. The latter are piece-wise continuous or piece-wise analytic functions of geometrical and material parameters: these properties can be lost only at the points where two or more eigenvalues coalesce. From a practical viewpoint, it is important that eigenvalues can be determined numerically: the convergence of discretization schemes is guaranteed, the number of equations needed being dependent on the desired accuracy and the nature of the inverted part. No spurious eigenvalues appear, unlike many approximate numerical methods. Note that nothing of the above can be established for an infinite-matrix equation of the first kind, which is common in conventional Moment-Method analyses.

Additionally, if a corresponding parameter κ is small, then the determinant is quasi-diagonal, and the eigenvalues of k or β can be obtained in the form of an asymptotic series. Such an analytical study has been done for a PEC axially-slit cylinder [123, 107] and a spherical cap [60], assuming a narrow slot or a small circular aperture, respectively. These asymptotics serve as a perfect starting guess when searching for eigenvalues numerically, with a Newton or another iterative algorithm. MAR-based numerical analyses of natural-frequency problems have been published for a two-dimensional model of an open PEC two-mirror resonator [124], and for flat-strip gratings [125]. Natural-wave problems have been studied with the MAR for the dominant modes of a planar strip and slot lines (Figures 12a, 12b) in [126-129], for principal and higher-order modes of circular-cylindrical strip and slot lines (Figure 12c) in [130-132], for arbitrary-cross-section dielectric waveguides (Figure 12d) in [13, 133], and recently for generalized slot lines [134] and Goubau-type striplines [135]. Formal MAR solutions (homogeneous matrix equations) have also been published for periodic waveguides: a PEC circular waveguide with periodically-cut transverse slots [136], a helically-slit guide [137], and a PEC plane-strip grating on a non-reciprocal substrate [137].

6. Conclusions

Summarizing, by using the MAR it is possible to overcome many difficulties encountered in conventional Moment-Method solutions. The merits of the MAR are numerous: exact solution existence is established, a numerical solution can be as accurate as the machine's precision, rigorous asymptotic formulas can be derived. Computationally, the MAR results in a small matrix size for a practical accuracy, and sometimes no numerical integrations are needed for filling the matrix. Thus, the *cost of MAR algorithms is low* in terms of both CPU time and memory. A common feature is that both power conservation and reciprocity are satisfied at the machine-precision level, independently of the number of equations, whatever it is. The condition number is small and stable, not growing with mesh refinement or increasing with the number of basis functions. The latter fact means that conjugate-gradient numerical algorithms are very promising, even in spite of a possible squaring of the condition number; this has already been emphasized in [139]. Using fast iterative methods, applied to the MAR matrix equations with static semi-inversion, it is probably possible to perform an accurate full-wave desktop analysis of the Arecibo reflector. However, the same thing can be done more economically by using a high-frequency semi-inversion, although this has not yet been extended to curved screens. Worth noting is that although the advantages of analytical regularization are obvious, a fully numerical eigenfunction-Galerkin scheme can work out as well. This technique was recently developed in patch-antenna analysis [140]: static eigenfunctions were pre-computed by a conventional rooftop MoM scheme, and then used as a global expansion basis in a dynamic solution. A full description of this technique is given in [141], where favorable convergence properties and high numerical efficiency are noted.

From a practical viewpoint, it is also important that the accuracy of the MAR is uniform, including resonances, both in near-field and far-field predictions. Here, one must be reminded that near sharp resonances, conventional Moment-Method and FDTD solutions suffer heavy inaccuracies [8], which cannot be removed, in principle. All this makes MAR-based algorithms perfect candidates for CAD software in the numerical optimization of multi-element two-dimensional and three-dimensional scatterers in the so-called resonant range, where interaction between separate elements plays an important role. In fact, this is already used in waveguide circuit optimization [5]; not only simple geometries, but quite complicated two-dimensional models of reflector antennas, open resonators, and open waveguides have been accurately studied in [112, 113], showing a variety of features not predicted by approximate techniques. The "demerits" of the MAR can be seen in more-painful mathematical work and greater human-time expenditures. Generally, this leads one to an old dilemma between specialized and universal algorithms. However, a tradeoff between efficiency and versatility is not enough: first of all, both algorithms must be convergent. In the neighboring fields of engineering science that rely on numerical simulations, today it is accuracy that plays a decisive role, followed by the cost comparison between equally accurate algorithms [2, 3]. In view of the increased pressure of CAD and CAE demands, probably the same should be done, sooner or later, in computational electromagnetics.

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8. Post-Conclusion: An editorial comment

Taking advantage of the *Magazine* article style, it seems possible to make one remark of a non-electromagnetic character. A close view of the partial list of MAR-related publications, below, shows that most of the authors are of Soviet (now Ukrainian and Russian) origin. This is not by chance: this is only the visible (i.e., available in English) top-of-iceberg list of publications. The roots of this phenomenon appear to be amazingly deep. In the USSR, the number of scientists was much greater than the number of computers, which used to be second derivatives of IBM and CDC main frames. However, antenna-design engineers and simulation scientists faced the same problems as their colleagues in the USA, Europe, and Japan. How could they cope with these problems, without sophisticated hardware? This review gives a partial reply: by means of a deeper, on average, usage of analytical and specialized-function-theoretic methods. The same is valid even more in gas and fluid dynamics, plasma science, and nuclear fusion. So, adding the ambitious word *new* to the title of a new MAR-related paper (as in [37, 54, 110]) can be risky, without a glance to the "East." One may ask here: and why is it the USSR had failed to develop adequate computers? To my belief, the explanation lies not in the economy. Landslide implementation of desktop PCs and workstations in research and development in the West was a logical product of progress in a relatively free society. But in the USSR, it was something intolerable. Computers did not obey Party discipline. Instead, they promised the terrifying prospective of many thousands of educated dissidents able to write and print millions of critical pages. That is why "cybernetics," as computer science was called in the USSR, together with genetics, were labeled early-on the two "call-girls in the service of capitalism," and severely suppressed until the 60s. (In the USSR's number-one university, Moscow State University, a department of cybernetics was opened only in 1969.) Oppositely, mathematics and physics were considered regular and useful ladies; after successful development of nuclear weapons and ballistic missiles, they were promoted by all means. So, finally, there is no surprise that genetic algorithms come to electromagnetics from the "West." But equally, there is no surprise that MAR comes from the "East." Both, apparently, have come to stay.

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Introducing Feature Article Author

Alexander I. Nosich was born in 1953 in Kharkov, Ukraine. He graduated from the Radio Physics Department of Kharkov State University in 1975. He received the PhD and DSc degrees in radio physics from the same university in 1979 and 1990, respectively. Since 1978, he has been on the research staff of the Institute of Radiophysics and Electronics (IRE) of the Ukrainian Academy of Sciences, in Kharkov. In 1992-1998, he held research fellowships and visiting professorships at Bilkent University, Ankara, Turkey; Kumamoto University, Gifu University, and Chuo University, Tokyo, Japan; University of Rennes 1, France; and Torino Polytechnical University, Italy. Currently, he is with the IRE, Kharkov, as a leading scientist in the Department of Computational Electromagnetics.

His research interests include analytical regularization, free-space and open-waveguide scattering, complex mode behavior, radar-cross-section analysis, and antenna simulation. In 1995, he was the organizer and Chairman of the IEEE AP-S East Ukraine Chapter, the first one in the former Soviet Union. Since 1995, he has been on the editorial board of *Microwave and Optical Technology Letters*. In 1990-1998, he was the organizer and TPC Co-Chairman of the series of international conferences Mathematical Methods in Electromagnetic Theory (MMET), held in Ukraine.

Report on the ANTEM '98 Symposium

The ANTEM (Antenna Technology and Applied Electromagnetics) Symposium has been held biannually since 1986. On August 10-12, 1998, the seventh ANTEM Symposium was held at the Chateau Laurier Hotel in Ottawa, Ontario, Canada. This was the second time the Symposium has been held in Ottawa, following a very successful year in 1994. The symposium had 156 scheduled papers, with one-third of them from outside Canada. The conference was quite successful, with over 175 participants from 15 countries besides Canada attending and enjoying the beautiful city of Ottawa. The conference continued to be one of the most economical, due to sponsorship from 10 Canadian companies.

Symposium participants presented their work on the latest developments in antenna technology and applied electromagnetics, in areas including personal communications, satellite antennas, and microwave electronics. This year, the customary sessions by international researchers in the antenna and electromagnetic fields and the special session profiling Canadian industry, held for the second time this year, were complemented by three keynote addresses and one special plenary session.

Emerging areas of research

Emerging areas of research were highlighted with presentations by three international leaders in their fields:

"Micromachining for Microwave and Millimeter-Wave Systems," Prof. Gabriel M. Rebeiz, Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA

"Recent Progress in Reflector Antenna Analysis and Design," Dr. Trevor S. Bird, CSIRO Telecommunications and Industrial Physics, Epping, Australia

"Emerging Array Antenna Technologies at JPL," Dr. John Huang, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, USA

Emerging wideband wireless delivery systems

"Emerging Wideband Wireless Delivery Systems" was the title and theme of a special plenary session, with representatives from major industry leaders in satellite and terrestrial wideband delivery systems. The session focused particularly on requirements placed on the RF technology for the terminals for emerging broadband wireless systems.

Satellite Wideband Service Delivery Systems

- Teledesic: Hugh Malone, "A Motorola Ka-Band Wideband Low Earth Orbit System"
- Astrolink: Kungta K. Chow, "A Lockheed-Martin Ka-Band Wideband GEO System"
- Skybridge: Pierre Fraise, "An Alcatel-Space Ku-Band Wideband Low Earth Orbit System"

Terrestrial Point-to-Multipoint Wideband Wireless Delivery Systems

- Lucent Broadband Networks Division (ex HP): Gary Labelle, "A Local Multipoint Distribution System"
- Nortel Broadband Wireless Access Division: Paul Astell, Steve Rayment, Michael Van Der Tol, "Wideband Ka-Band Delivery Systems"

Proceedings and future plans

The proceedings of the symposium are available from the Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, Manitoba, Canada, R3T 5V6. ANTEM 2000 is planned for August 2000, in its birthplace of Winnipeg, Manitoba, Canada. The organizing committee promises to ensure that ANTEM 2000 will be a major event, worthy of international participation.

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