Chapter 9
Green's Function-Dual Series Approach in Wave Scattering by Combined Resonant Scatterers

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1. INTRODUCTION

1.1 Historical Background

As is well known in general, rigorous scattering and diffraction theory is based on the theory of boundary-value problems, therefore the progress here is closely tied to that of this branch of mathematics. So, as two-dimensional wave scattering problems can be reduced to singular integral equations by using a generalized potential theory approach, the Fredholm's theory of integral equations is of great importance [1]. Besides, the theory of Fourier transforms and functions of complex variable enables one to solve integral equations of a certain class [2]. This approach is called the Wiener-Hopf (WH) technique; one delivers exact solutions of such canonical diffraction problems as that for a semiinfinite zero-thickness plate, and produces effective approximate solutions for a great number of modified geometries. All these problems are known to be rearrangable in the form of equivalent dual integral equations.

The development of the Riemann-Hilbert Problem (RHP) approach can be considered as another great event in diffraction theory, although it is still less known than WH one. This technique delivers exact solution of certain dual series equations. In the core of the approach there lies a problem about reconstruction of an analytical function $X(z)$ of complex variable $z = x + iy$ whose limiting values $X^{\pm}(z)$ from inside and outside of a closed finite or infinite curve L (in) satisfy the following condition

$$X^{+}(z_0) - A(z_0)X^{-}(z_0) = B(z_0), \quad z_0 \in L \tag{1}$$

with known functions $A(z_0)$ and $B(z_0)$ called the coefficient and the free term of RHP, respectively.

Actually, the solution of this problem was first given by Carleman [3] but thorough investigation of the RHP theory and general solution of (1) for L being an arbitrary curve can be found in the books by Muskhelishvili [4] and Gakhov [5]. Agranovich, Marchenko and Shestopalov [6] were the first who applied this approach to particular scattering problem

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